Evaluation of Dynamics of Soft Contact Rolling using Multibond Graph Approach

[1]Anand Vaz
[1]Department of Mechanical Engineering
Dr. Ambedkar National Institute of Technology
Jalandhar-144011, Punjab, India.
Email: anandvaz@ieee.org

[2]Anil Kumar Narwal, [3]K. D. Gupta
[2,3] Department of Mechanical Engineering
Deenbandhu Chhotu Ram University of Sc. and Technology
Murthal-131039, Sonepat (Haryana), India
Email: anilnarwal10@yahoo.com

***Abstract*—The dynamics of soft contact between a rigid body rolling over a soft material is quite challenging to be solved. Contact area and force distribution at the contact interface changes as the rigid body rolls over the soft material. The soft contact dynamics is modeled using multi bond graph approach integrated with finite element method. Stiffness and Inertia matrices for the soft material are calculated using finite element method. Stiffness and inertia matrices are used as C-field and I-field respectively in the bond graph model. In this work, as an example a circular disc rolling over a layer of silicon rubber is considered. The disc is moved with controlled force using proportional and derivative controller. Bond graph structure for the system is developed. MATLAB code is generated directly from the bond graph model. Model is validated with simulation which determines soft material deformation, contact area and forces distribution.**

Keywords*—Bond graph, Causality, FEM, Simulation, Soft contact Rolling,

I. Introduction

First contact model was developed by Hertz in 1892 which relates radius of contact with normal force. It determines radius of contact is proportional to the normal force raised to power 1/3. The surfaces considered in the model were linear elastic with small contact. Later Johnson-Kendall-Robert [1] included the effect of adhesive forces in the contact model. Xydas and Kao [2] developed power law theory for the soft contact between a rigid body and a nonlinear elastic soft finger.

The contact between two objects may be considered either a point contact or an area contact. The point contact is an ideal approximation. Area contact is more realistic which produces force distribution over the contact area and also results in development of moments. The existing approaches for the contact modeling can be broadly classified in three categories: constraint based, impulse based and penalty based approach [3-4]. Constraints based approach uses optimization of linear complementarity problem and limited to simple geometry. Impulse based approach is more suitable for impact contact [5]. Penalty based model constraints inter penetration of contact surfaces by inserting a virtual spring and dashpot in between. Spring produces force directly proportional to depth of penetration [6-7]. Most of the existing work solves the contact problem when two contact bodies are rigid and in static equilibrium.

In this work, a circular disc and a piece of silicon rubber are taken as a rigid body and a soft material respectively. The soft contact dynamics are obtained for two cases. First is when the disc is merely placed on the soft material. Second is when the disc is rolled on the soft material in controlled manner. Bond graph structure representing rigid body dynamics is established. The soft material stiffness and mass matrices are calculated using finite element method (FEM) [8]. The bond graph model for the soft material is developed by considering stiffness and inertia matrices as **C**-field and **I**-field respectively. Subsystems bond graph models are integrated through contact interface. For computational simplicity, model is developed for planar case. The soft material is subjected to plane stress condition.

The bond graph model developed is applicable for rigid bodies of all geometry. Cause and effect based representation facilitates the better understanding of the soft contact dynamics. Once the bond graph is developed on flow map basis, the dynamics is obtained from the model or vice versa. In this paper, development of the bond graph model is explained in section 2. Simulation results are presented in section 3 followed by concluding remarks at the end.

II. Bond Graph Modeling

A bond graph structure for the rigid disc is developed on the basis of flow mapping [9-13]. The model represents the kinematics of the disc. The disc is placed on the soft material layer. Moving frame of reference *1* is attached at the center of the disc. Velocity of any point P_i on the disc is given as

$$ {}_0^0\dot{\bar{r}}_{P_i} = {}_0^0\dot{\bar{r}}_C - [\,{}_C^0\bar{r}_{P_i}\times]\,{}_0^0\bar{\omega}_B. \quad (1)$$

Where ${}_0^0\dot{\bar{r}}_C$ is the velocity of the center of mass (CM) of the disc, ${}_0^0\bar{\omega}_B$ is the angular velocity of the disc observed and expressed in inertial frame *0*. Translational velocity ${}_0^0\dot{\bar{r}}_C$ of the disc is represented at junction $1_{{}_0^0\dot{\bar{r}}_C}$ and angular velocity

${}^{0}_{0}\bar{\omega}_B$ is given at junction $1_{{}^{0}_{0}\bar{\omega}_B}$. The velocity ${}^{0}_{0}\dot{\bar{r}}_{P_i}$ is given at junction $1_{{}^{0}_{0}\dot{\bar{r}}_{P_i}}$ as shown in Fig. 1. Translational inertia of the disc is modeled as **I**: *m*. Rotational inertia of the disc is modeled as **I**: $\left[{}^{0}_{C}I_D\right]$, where $\left[{}^{0}_{C}I_D\right]$ is inertia tensor of the disc about its CM and expressed in inertial frame. Transformer element $MTF:\left[{}^{0}_{C}\bar{r}_{P_i}\times\right]$ relates angular velocity of the disc to translational velocity of point P_i with respect to CM.

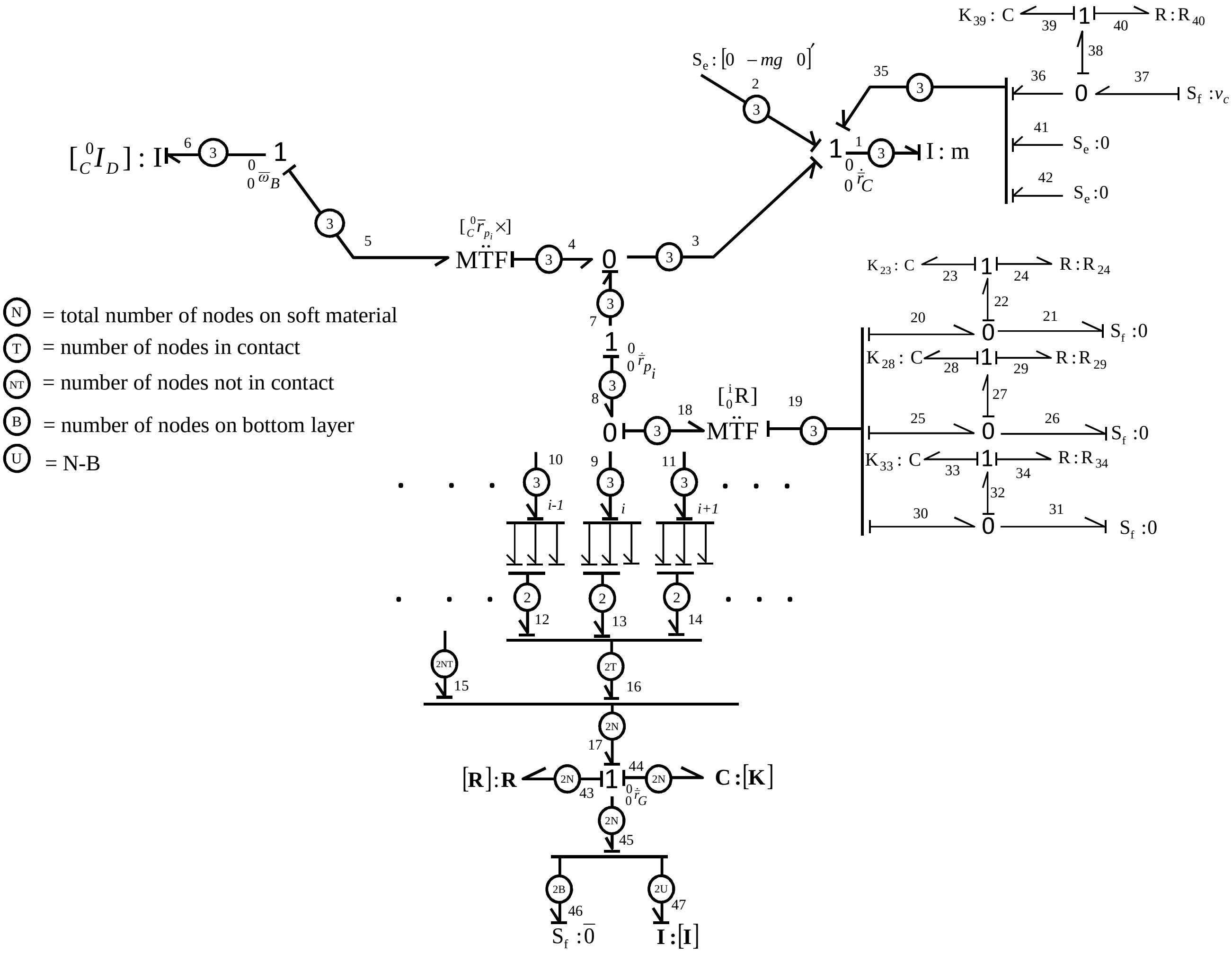

Fig. 1. Bond graph model for soft contact between a rigid body and the soft material.

For computational simplicity, the soft contact is modeled for planer case. The soft material is assumed to be in plane stress condition. The soft material properties determine the material deformation and contact area for a given disc mass. Stiffness and mass matrices for the soft material are calculated using finite element method. The soft material is discretized in to number of quadrilateral elements as shown in Fig. 2.

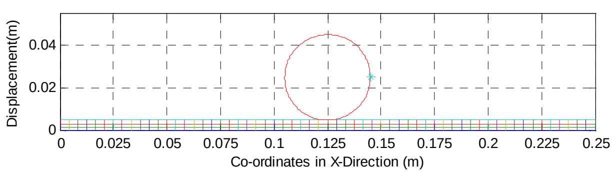

Fig. 2. Discretized soft material with the disc placed on it

Displacement field is calculated using linear shape functions. Elements' stiffness matrices are calculated using principle of virtual work and is given as

$$K^e = t_e \int_{-1}^{1}\int_{-1}^{1} B^T D B \left|J\right| \, d\xi \, d\eta \,. \qquad (2)$$

Where J is Jacobian matrix and D is material matrix. Local stiffness matrices are assembled into global stiffness matrix. Mass matrix is obtained using same shape functions. Damping of the soft material is assumed to be constant. Stiffness, mass and damping matrices obtained above are used as **C**-field, **I**-field and **R**-field respectively in the bond graph structure. The soft material is discretized in to N number of nodes. For planer case, each node has 2 degree of freedom. **C**-field and **R**-field are modeled as **C:**

$[K]$; $[K] \in \mathbf{R}^{2N \times 2N}$ and **R:** [R]; $[R] \in \mathbf{R}^{2N \times 2N}$ respectively. Bottom layer of the soft material is made to be fixed by appending $S_f : \bar{0} \in \mathbf{R}^{2B}$ to junction $1_{{}^0_0\dot{\bar{r}}_{G_i}}$ corresponding to these nodes. Inertia of the bottom nodes B do not contribute to the system dynamics. Inertia of remaining nodes is modeled as **I:** [I]; $[I] \in R^{2(N-B) \times 2(N-B)}$.

To model the soft contact, if the bond graph model for the disc is attached directly to the soft material bond graph, the inertia of contact nodes attains derivative causality. Momentum states of contact nodes inertia depend upon other states of the system. To remove derivative causality, a virtual spring and damper is inserted between point P_i on the disc and contact node S_i on the soft material. The resulting contact interface is the manifestation of penalty approach as shown in Fig. 3.

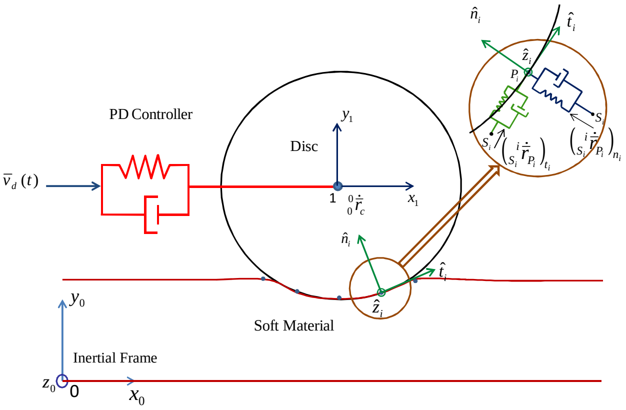

Fig. 3. Contact interface of a virtual spring and a damper inserted both in tangential and normal direction. PD controller produce controlled force according to desired displacement trajectory.

Junction $0_{{}^0_{S_i}\dot{\bar{r}}_{P_i}}$ represents the relative velocity of P_i with respect to contact node S_i on the soft material. To compute normal and tangential forces at contact nodes, normal and tangential unit vector at each contact node is calculated. $\left[{}^0_i R\right]$ defines orientation of i^{th} moving frame with respect to inertial frame. **MTF**:$\left[{}^0_i R\right]$ expresses relative velocity ${}^0_{S_i}\dot{\bar{r}}_{P_i}$ in i^{th} moving frame as given by

$$ {}^i_{S_i}\dot{\bar{r}}_{P_i} = [{}^0_i R]^T \; {}^0_{S_i}\dot{\bar{r}}_{P_i} \tag{3} $$

Relative velocity and displacement between node P_i and S_i is obtained. An algorithm for detecting nodes come in contact with the disc is developed. Position of CM of the disc with respect to each node on the top layer of the soft material is calculated. Any i^{th} node comes in contact if

$$ {}^0_i\bar{r}_C \leq radius \tag{4} $$

Where ${}^0_i\bar{r}_C$ is the position vector of CM with respect to any i^{th} node on the top layer of the soft material. $C: K_{28}$ and $R: R_{29}$ model normal contact. S_i node remains in contact till $C: K_{28}$ element is in compression.

Normal reaction at the point of contact is given as,

$$ {}^iF_{N_i} = e_{27} = e_{28} + e_{29} \tag{5} $$

Contact condition is defined as

If ${}^iF_{N_i} \leq 0$, then

i^{th} node is in contact with the disc

else

node on the soft material is not in contact with the disc,

$\Rightarrow \; {}^iF_{N_i} > 0$;

normal reaction ${}^iF_{N_i} = 0$.

Friction force along common tangent to the contact point is modeled by elements $C: K_{23}$ and $R: R_{24}$. Tangential force acting on the i^{th} node is given as,

$$ {}^iF_{t_i} = {}^iF_{t_K} + {}^iF_{t_R} \tag{6} $$

Where, ${}^iF_{t_K} = e_{23}$ and ${}^iF_{t_R} = e_{24}$

If ${}^iF_{t_i} \leq \mu_S \, {}^iF_{N_i}$, then

friction force ${}^iF_{R_i} = {}^iF_{t_i}$

else,

friction force ${}^iF_{R_i} = sign\ ({}^i_{S_i}\dot{r}_{P_{it}})\ \mu_K \ {}^iF_{N_i}$

where, ${}^i_{S_i}\dot{r}_{P_{it}}$ is tangential component of the relative velocity ${}^i_{S_i}\dot{\bar{r}}_{P_i}$.

Total effort ${}^i\bar{F}_{S_i}$ at contact interface expressed in the i^{th} moving frame at node S_i is

$$ {}^i\bar{F}_{S_i} = [\ {}^iF_{R_i} \quad {}^iF_{N_i} \quad {}^iF_{Z_i}\ ]^T \tag{7} $$

Total force ${}^0\bar{F}_{S_i}$ at node S_i, expressed in inertial frame, is

$$ {}^0\bar{F}_{S_i} = \left[{}^0_i R\right] {}^i\bar{F}_{S_i} \tag{8} $$

Force acting on point P_i on the disc is given as

$$ {}^0\bar{F}_{P_i} = -\,{}^0\bar{F}_{S_i} \tag{9} $$

If T number of nodes are in contact with the disc at any instant, then the total force ${}^0\bar{F}_B$ on the disc, is given as

$$\frac{d\,{}^0\overline{p}_B}{dt} = {}^0\overline{F}_B = \left(\sum_i^T {}^0\overline{F}_{P_i}\right) + \overline{e}_2 + \overline{e}_{35} \qquad (10)$$

where $\overline{e}_2 = \{0 \;\; -mg \;\; 0\}^T$ is weight of the disc and $\overline{e}_{35}$ is controlled force produced by proportional and derivative controller which depends upon desired velocity trajectory .

Total moment produced by the contact forces is

$$\frac{d\,{}^0\overline{P}_B}{dt} = {}^0_C\overline{\tau}_B = \sum_i^T \left[{}^0_C\overline{r}_{P_i} \times\right]{}^0\overline{F}_{P_i} \qquad (11)$$

The bond graph model completely describes kinematics as well as dynamics of the soft contact interaction. MATLAB code is generated directly from the bond graph model. Simulation results are explained in the next section

III. SIMULATION RESULTS

The bond graph model is simulated for two cases. (1) The disc is place on the soft material and it attains static equilibrium (2) The disc is made to roll over the soft material with controlled force generated by proportional and derivative controller. A thin layer of silicon rubber of 0.25m length and 0.005m thickness is taken. For silicon rubber Young Modulus of elasticity E = 1.7 MPa and Poisson's ratio (ν) = 0.48 is taken. The soft material is meshed in to 3 layers as shown in Fig. 4.

Case 1: The disc of different weight is placed at the center of the soft material and allowed to attain static equilibrium. The model is simulated for 2 seconds. The soft material is deformed and the disc form area contact with the soft material as shown in Fig. 4(a). Normal and tangential forces are also determined as shown in Fig. 4(b). Normal force ${}^iF_{N_i}$ is maximum at the center of the contact area and it reduces outward. Maximum deformation for the discs of different weights is determined as given in table 1.

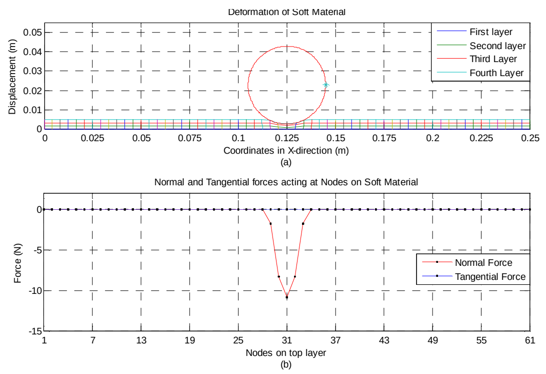

Fig. 4. (a) Deformation of the soft material when the disc attains static equilibrium (b) Distribution of normal and tangential forces.

TABLE I. FORCES VERSES MAXIMUM DEFORMATION AT THE CENTER OF CONTACT AREA

***Force* (N)**	***Deformation* (mm)**
0.0000	0.00000
9.8100	0.13000
19.6200	0.20000
29.4300	0.28000
39.2400	0.33000
49.0500	0.39000
58.8600	0.42000
68.6700	0.49000
78.4800	0.53000
88.2900	0.57000
98.1000	0.61000

Figure 5 shows a plot between force verses maximum deformation of the soft material at the center of contact area.

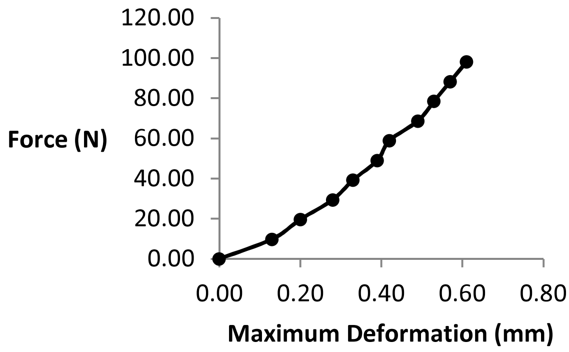

Fig. 5. Force verses maximum deformation at the centre of contact area

Case(2): The disc is made to move by controlled force acting at the center of the disc. The controlled force is produced by PD controller based on the desired displacement trajectory. The trajectory of the disc is such that the disc move X_T distance in time T but have no velocity, acceleration and jerk at the start and end and is defined as,

$$x_d(t) = 0\,, \qquad t \le 0 \qquad (12)$$

$$= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7\,, \quad for \;\; 0 < t < T \qquad (13)$$

$$= X_T\,, \qquad t > T \qquad (14)$$

Where, $a_0, a_1, a_2, ..., a_7$ are constant coefficients. The coefficients are calculated using conditions such that $\dot{x}(t)$, $\ddot{x}(t)$, $\dddot{x}(t)$ are zero at time t = 0 and at time t = T.

$C:K_{39}$ and $R:R_{40}$ element model PD controller. $x(t)$ is the actual displacement and $x_d(t)$ is the desired displacement. Displacement and velocity error are given as,

$$\varepsilon(t) = x_d(t) - x(t) \tag{15}$$

$$\dot{\varepsilon}(t) = \dot{x}_d(t) - \dot{x}(t) \tag{16}$$

The PD controller generates force F_C which is given as,

$$F_c = k_{39}\varepsilon + R_{40}\dot{\varepsilon} \tag{17}$$

Horizontal force F_C acts at the center of the disc. The disc rolls over the soft material. Figure 6 (a) shows translational momentum of the disc. Figure 6(b) shows angular momentum of the disc. Negative angular momentum is due to clockwise rotation of the disc about z axis.

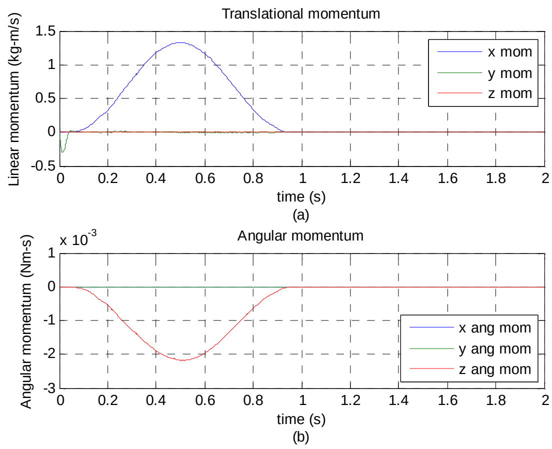

Fig. 6. (a) Translational momentum of the disc w.r.t. time; (b) Angular momentum of the disc w.r.t. time

The simulation results of soft material deformation and forces distribution both in tangential and normal direction during the rolling of the disc over the soft material are shown by series of snapshots captured at specified time in Fig. 7(a)-(i).

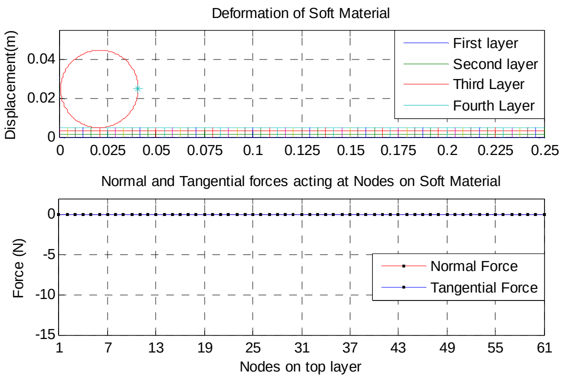

Fig. 7(a). The disk position and force distribution at time t = 0 s.

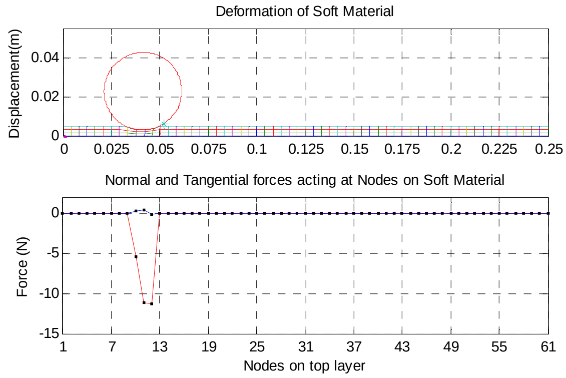

Fig. 7(b). The disk position and force distribution at time t = 0.2846s.

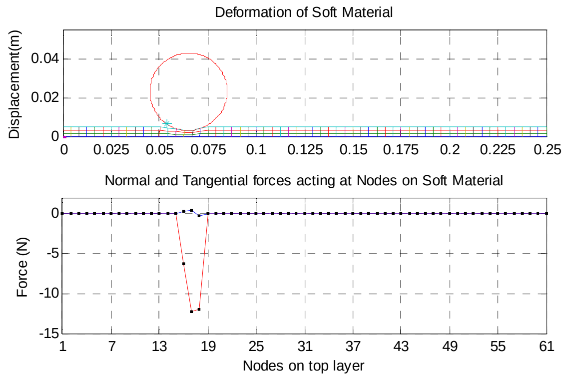

Fig. 7(c). The disk position and force distribution at time t = 0.3652s.

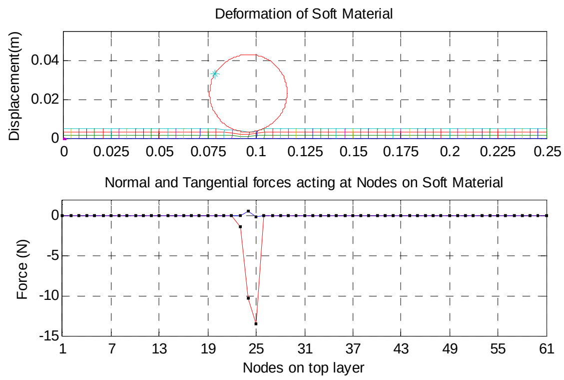

Fig. 7(d). The disk position and force distribution at time t = 0.4441s.

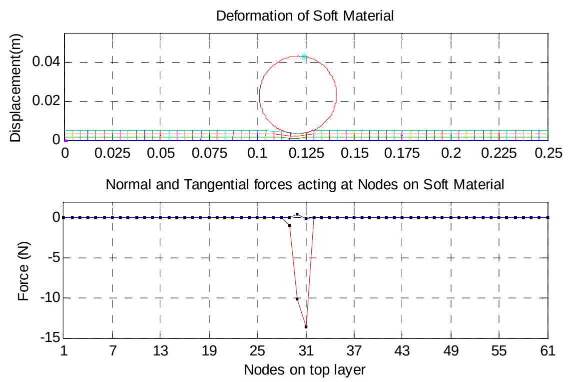

Fig. 7(e). The disk position and force distribution at time t = 0.5017s.

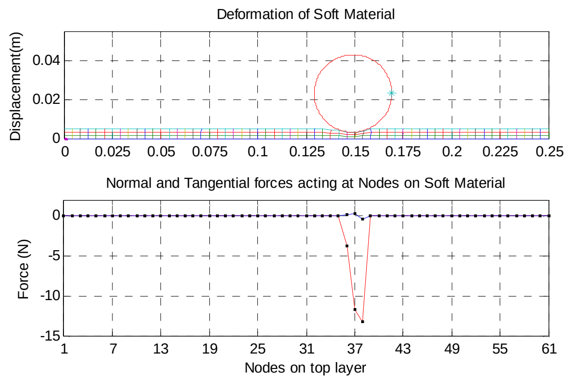

Fig. 7(f). The disk position and force distribution at time t = 0.5665s.

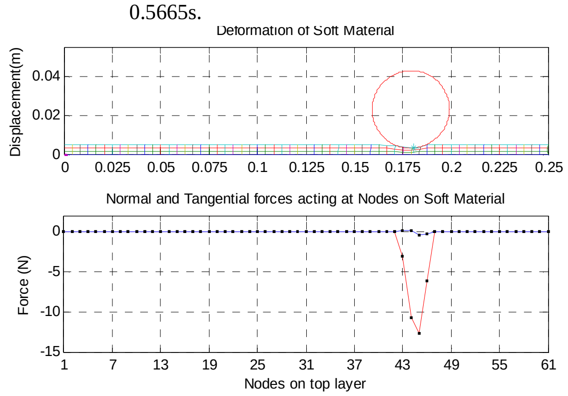

Fig. 7(g). The disk position and force distribution at time t = 0.6451s.

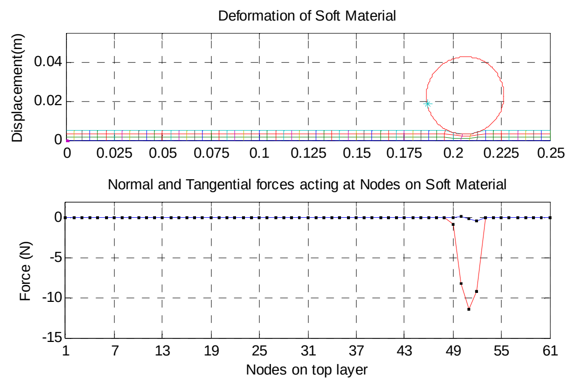

Fig. 7(h). The disk position and force distribution at time t = 0.7473s.

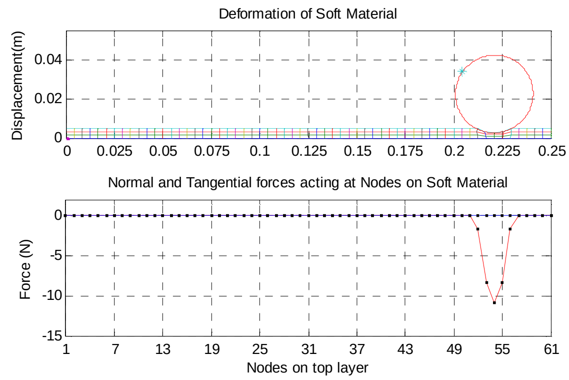

Fig. 7(i). The disk position and force distribution at time t = 1.9538s.

IV. CONCLUSION

The bond graph model developed above solves the various issues of the soft rolling contact. Causal based representation of the bond graph structure facilitates the better understanding of the contact dynamics. The model calculates both contact area and force distribution for the soft contact. The deformation of layers of the soft material is also obtained from the model. The soft contact is modeled by discretizing only the soft material. Discretization of the rigid disc is not needed. The model is general, not restricted to any specific geometry. With high computation capability the model can be extended to 3-dimensional case also.

V. REFERENCES

[1] K. L. Johnson, K. Kendall and A. D. Roberts, surface energy and the contact of elastic solids, Proc. R. Soc. London A 324 1971, pp. 301-313.

[2] N. Xydas, I. Kao, Modeling of contact mechanics with experimental results for soft finger, IEEE/RSJ International Conference on Intelligent Robots and System, Victoria, B. C. Canada, October 1998, pp. 488-493.

[3] Katsu Yamane and Yoshihiko Nakamura, Stable penalty based model of frictional contact, , IEEE International Conference on Robotics and Automation, Orlando, Florida, May 2006, pp. 1904-1909

[4] Yin-Tien Wang, Vijay Kumar and Jacob Abel, Dynamics of rigid bodies undergoing frictional contact, , IEEE International

Conference on Robotics and Automation, Nice, France, May 1992, pp. 2764-2769.

[5] David Baraff, Analytical method for dynamic simulation of non-penetrating rigid bodies, Computer Graphics, Proceeding SIGGRAPH 1989, Volume 23, Number 3, July 1989, pp. 223-232

[6] D. Marhefka and D. Orin, simulation of contact using a nonlinear damping model, in Proceedings of the 1996 IEEE Internationa lConference on Robotics and Automation, Minneapolis, MN, 1996, pp. 1662–1668

[7] Y. Hwang, E. Inohira, A. Konno, and M. Uchiyama, An order n dynamic simulator for a humanoid robot with a virtual spring-damper contact model, in IEEE International Conference on Robotics and Automation, Taipei, Taiwan, September 2003, pp. 31–36.

[8] Belegundu, Chandrupatla, Introduction to Finite Elements in Engineering, Prentice Hall of India Private Limited, 2002.

[9] A. Vaz, S. Hirai, Modeling of hand prosthesis with word bond graph object, Proceedings of The International Conference on Integrated Modeling & Analysis in Applied Control & Automation (IMAACA 2004), Oct. 28-31, Genoa, Italy 2 (2004), pp. 58-67.

[10] A. Vaz, S. Hirai, Modeling contact interaction of hand prosthesis with soft tissue at the interface, IEEE, International Conference on Systems, Man and Cybernetics (2003), pp. 4508-4513.

[11] A. Vaz, S. Hirai, Bond graph modelling of a hand prosthesis during contact interaction, proceedings of 2003 IASTED Int. Conf. on Applied Simulation and Modelling (ASM 2003), September 3-5, Marbella, Spain (2003) 313 – 318.

[12] D.C. Karnoop, D.L. Margolis, R. C. Rosenberg, System: Modeling and Simulation of Mechatronic System, third edition, Wiley-Interscience, 2000

[13] A. Mukherjee, R. Karmakar, Bond Graph in Modeling, Simulation and Fault Identification, I.K. International Pvt. Ltd., 2006.