Trajectory Control of a 3-link Planar Manipulator using Virtual Link Based Controller

M. K. Sutar¹, P. M. Pathak², N. K. Mehta³, A. K. Sharma⁴

Mechanical & Industrial Engineering Department

Indian Institute of Technology Roorkee

Roorkee, India

¹mihirkumarsutar@yahoo.in, ²pushpfme@iitr.ac.in, ³mehtafme@iitr.ac.in, ⁴akshafme@iitr.ac.in

Abstract— A hyper-redundant manipulator has the advantage that it can be used in constraint space; however the control of hyper-redundant manipulator is difficult. This paper presents a concept of virtual link based two degrees of freedom (DOF) controller. With the proposed controller, manipulator of any number of links can be reduced to either a 2-link virtual manipulator or one real and one virtual link manipulator. To illustrate the methodology, a 3-link planar hyper-redundant manipulator has been considered. Simulations have been carried out to validate the efficacy of the controller. The developed control scheme can be used in trajectory control of manipulators used for surgical applications i.e., *in-vivo* robot.

Keywords— *trajectory control; virtual link approach; bond graph.*

I. INTRODUCTION

Hyper-redundant manipulators have greater number of degree of freedom (DOF) than required to reach the point of interest; this improves its dexterity [1]. So hyperredundancy helps in exploring the abdominal cavity during biopsy in a better way as it renders better dexterity and maneuverability to the robotic arm. Therefore it is an advantage to use a hyper-redundant manipulator for invivo surgical applications like stomach tissue biopsy. The term in-vivo is used in literature for the condition when a tool is in contact with a living tissue. However controlling of the hyper-redundant manipulator is a challenging task and is one of the major concerns of many researchers worldwide. During biopsy it is required to move the manipulator in the abdominal cavity environment and to place its tip at the desired point of interest. For this a proper control scheme for trajectory control is required. Robots having several redundant DOF are termed as 'hyper-redundant' robot. They are quite alike to snake, tentacles or elephant trunks [2].

Chirikjian [3, 4] has significant contribution in the inverse kinematic theory of hyper-redundant manipulators. Uphoff and Chirikjian [5] have proposed an algorithm for solving the inverse kinematic problem of a discretely actuated hyper-redundant manipulator. Ning and Worgotter [6] have formulated a bang-bang control strategy for a wire driven hyper- redundant chain robot called '3D-trunk', to achieve a minimum time solution. A new geometrical method for solving the inverse kinematicsof hyper-redundant manipulator has been proposed by Yahya et al. [7]. Sreenivasan et al. [8] have presented a classical planar curve, known as tractrix for and visualization simulation of hyper-redundant manipulators. For trajectory planning a technique called closed-loop pseudoinverse method in combination with genetic algorithms has been presented by Marcos et al. [9]. Boudec et al. [10] have presented an adaptive control with optimization scheme for hyper-redundant manipulator called as Articulated Nimble Adaptable Trunk (ANAT). A geometrical approach to solve the inverse kinematics for a continuum robot has been presented by Nepalli et al. [11]. Analysis was performed for each section of the continuum robot, and then the summation gave the position of the end effector.

Zanganeh and Angles [12] have proposed a splinebased solution method for inverse kinematic analysis of hyper-redundant manipulator. Liljeback et al. [13] in their attempt have reviewed various literatures related to the modeling, design aspects and control strategies of snake robots. Conkur [14] has presented a new method of path planning of hyper-redundant manipulator. Dinulescu et al. [15] have described the image based and closed-loop control system of a hyper redundant manipulator. Sutar et al. [16] have presented a kinematic analysis of the 4-DOF hyper- redundant in-vivo robot. A 'virtual link' concept to obtain inverse kinematic solution of discrete planar hyperredundant manipulator has been proposed by [17], which was effectively applied to 7-9 DOF manipulators. Model planar linkages using much simpler methods in bond graph form has been explained in [18].

This paper presents a trajectory control scheme for a 3link planar hyper-redundant manipulator. The 3-link planar manipulator was modeled using bond graph, and a controller, using the virtual link concept has been formulated. A relational approach between the real and virtual links was established and the virtual link based controller was used to send effort signals to all the joints of 3-link planar manipulator upon receiving a corrective signal from PI controller. Different desired trajectory shapes were used to check the efficacy of the controller. Simulations have been performed and the efficacy of the controller was found as par with the expected.

The structure of the paper is as follows: *section II* contains the modeling of 3-link planar manipulator, *section III* includes the bond graph modeling while *section IV* elaborates the virtual link based control. Simulation and animation results are discussed in *section V* and *section VI* covers the concluding remark and the future scopes.

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II. MODELING OF 3-LINK PLANAR MANIPULATOR

Modeling of 3-link planar manipulator involves the linear and rotational dynamics of the links. For simplicity it is assumed that the 3-link manipulator has three revolute joints and is an open kinematic chain type. Fig. 1 shows the schematic diagram of the 3-link manipulator. In this figure {0} represents the inertial frame, {1} frame is located at the base of first link. Frame {2} and {3} are located at the base of link 2 and 3 respectively. Frame {4} has been located at the tip of the manipulator. Let L_1 , L_2 and L_3 be the length of the first, second and third link respectively. Angle θ_1 , θ_2 and θ_3 represent the joint rotations of the first, second and third joint respectively.



Fig. 1. Schematic representation of 3-link manipulator

III. BOND GRAPH MODELING

Kinematic analysis of the robot has been performed and various transformer moduli used in the bond graph were derived to draw the complete bond graph model. Complete bond graph model of the 3-link planar manipulator is shown in Fig. 2. The translational dynamics of each link segment shown on the left hand side of the model is resolved into X and Y components. Each link has the Center of Gravity (CG) velocity which depends on the link inertia. Therefore in the bond graph model I elements are attached at each velocity junctions representing the CG velocity of each link. The tip velocities are indicated on the top of the bond graph model. Integrators are used to integrate the actual velocity components to obtain the tip positions in X and Y directions i.e., Xtip_act and Ytip_act. The reference position of the end-effector tip is denoted by *Xtip_ref* and *Ytip_ref*.

I element in the joint represents the joint inertia, and the R element on each joint represent the joint resistance and integrators are used on each joint to evaluate the angular rotation of each joint. Pads are used to avoid any differential causality in the model. The Denavit Hartenberg (D-H) parameter [19] for the 3-link planar manipulator is shown in Table I. Generalized form of homogenous transformation matrix can be expressed as in (1).

From the D-H parameters shown in Table I the overall transformation matrix was derived as;

 ${}^{0}_{4}T = {}^{0}_{1}T {}^{1}_{2}T {}^{2}_{3}T {}^{3}_{4}T$

TABLE I.	D-H PARAMETERS OF THE 3-LINK PLANAR
	MANIPULATOR

S. No		Parameters				
i	α_{i-1}	<i>a</i> _{<i>i</i>-1}	d_i	θ_{i}		
1	0	0	0	θ_1		
2	0	L ₁	0	θ_2		
3	0	L_2	0	θ_3	1	
4	0	L ₃	0	0		
${}^{i-1}_{i}T =$	$\begin{bmatrix} c heta_i \\ s heta_i c lpha_{i-1} \\ s heta_i s lpha_{i} s lpha_{i-1} \\ 0 \end{bmatrix}$	$-s heta_i \\ c heta_i clpha_{i-1} \\ c heta_i slpha_{i-1} \\ 0$	$\begin{array}{c} 0\\ -s\alpha_{i-1}\\ c\alpha_{i-1}\\ 0 \end{array}$	$\begin{bmatrix} a_{i-1} \\ -s lpha_{i-1} d_i \\ c lpha_{i-1} d_i \end{bmatrix}$	(1	

So, the overall transformation can be deduced as;

$${}^{D}_{4}T = \begin{bmatrix} c_{123} & -s_{123} & 0 & L_{1}c_{1} + L_{2}c_{12} + L_{3}c_{123} \\ s_{123} & c_{123} & 0 & L_{1}s_{1} + L_{2}s_{12} + L_{3}s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 (2)

The position vectors for different values of i are as follows;

$${}^{0}P_{1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{T}; {}^{1}P_{2} = \begin{bmatrix} L_{1} & 0 & 0 \end{bmatrix}^{T};$$
$${}^{2}P_{3} = \begin{bmatrix} L_{2} & 0 & 0 \end{bmatrix}^{T}; {}^{3}P_{4} = \begin{bmatrix} L_{3} & 0 & 0 \end{bmatrix}^{T}$$
(3)

The angular velocities for different values of i are as follows:

The velocities of CG of the links obtained from the kinematic analysis can be expressed as;

$${}^{1}\left({}^{0}V_{g_{1}}\right) = \begin{bmatrix} -0.5L_{1}s_{1}\dot{\theta}_{1} \\ -0.5L_{1}c_{1}\dot{\theta}_{1} \\ 0 \end{bmatrix}$$
(5a)

$${}^{2} \left({}^{0}V_{g_{2}} \right) = \begin{bmatrix} -L_{1}s_{1}\dot{\theta}_{1} - 0.5L_{2}s_{12}\left(\dot{\theta}_{1} + \dot{\theta}_{2}\right) \\ L_{1}c_{1}\dot{\theta} + 0.5L_{2}c_{12}\left(\dot{\theta}_{1} + \dot{\theta}_{2}\right) \\ 0 \end{bmatrix}$$
(5b)
$${}^{3} \left({}^{0}V_{g_{3}} \right) = \begin{bmatrix} -L_{1}s_{1}\dot{\theta}_{1} - L_{2}s_{12}\left(\dot{\theta}_{1} + \dot{\theta}_{2}\right) \\ L_{1}c_{1}\dot{\theta} + L_{2}c_{12}\left(\dot{\theta}_{1} + \dot{\theta}_{2}\right) \\ 0 \\ 0 \end{bmatrix}$$
(5c)
$$-0.5L_{3}s_{123}\left(\dot{\theta}_{1} + \dot{\theta}_{2} + \dot{\theta}_{3}\right) \\ +0.5L_{3}c_{123}\left(\dot{\theta}_{1} + \dot{\theta}_{2} + \dot{\theta}_{3}\right) \end{bmatrix}$$

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Fig. 2. Bond graph of 3-link planar manipulator

And the tip velocity can be expressed as;

$${}^{4} \left({}^{0}V_{4} \right) = \begin{bmatrix} -L_{1}s_{1}\dot{\theta}_{1} - L_{2}s_{12} \left(\dot{\theta}_{1} + \dot{\theta}_{2} \right) - L_{3}s_{123} \left(\dot{\theta}_{1} + \dot{\theta}_{2} + \dot{\theta}_{3} \right) \\ L_{1}c_{1}\dot{\theta} + L_{2}c_{12} \left(\dot{\theta}_{1} + \dot{\theta}_{2} \right) + L_{3}c_{123} \left(\dot{\theta}_{1} + \dot{\theta}_{2} + \dot{\theta}_{3} \right) \\ 0 \end{bmatrix}$$

$$(6)$$

The transformer moduli obtained from above analysis were used to draw the bond graph and they are listed in Table II. Soft pads are artificial compliances used in bond graph to ovoid differential causality, they usually avoid algebraic loop while deriving equations.

The bond graph sub model of the soft pad is shown in

Fig.2. The virtual link based controller has been discussed in the next section.

IV. VIRTUAL LINK BASED CONTROLLER

For trajectory control, a virtual link based controller has been developed. In the present approach i.e., the virtual link based controller, the number of real links present in the controller are reduced to minimum number of fictitious links, called virtual links. This proposed control scheme is applicable to any number of planar hyper-redundant manipulator links.

A generalized solution technique for any number of planar links (more than one) has been discussed before its implementation to the 3-link planar hyper-redundant manipulator. For simplicity a 5-link planar manipulator example shown in Fig. 3-6, has been considered.

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A. Step I

In the first step shown in Fig. 3 two real links L_5 and L_4 are replaced with a fictitious link of varying length L_{v_1} . The joint angle of the virtual link L_{v_1} can be expressed as $(\zeta_2 + \theta_4)$, where θ_4 is the joint angle of real link L_4 with respect to its previous link L_3 , ζ_2 is the angle between L_4 and L_{v_1} , and ζ_1 is the angle between L_5 and L_{v_1} From the geometrical relations, the angles ζ_1 and ζ_2 can be deduced as;

$$\zeta_1 + \zeta_2 = \theta_5 \tag{7}$$

And virtual link L_{ν_1} can be expressed as;

$$L_{\nu_1}^2 = L_4^2 + L_5^2 - 2L_4L_5\cos(\pi - \theta_5)$$

Simplifying above equation we have;

$$L_{\nu_1} = \sqrt{L_4^2 + L_5^2 + 2L_4L_5\cos\theta_5} \tag{8}$$

B. Step II

In the second step shown in Fig. 4, the fictitious link L_{ν_1} and real link L_4 are replaced with an imaginary link L_{ν_2} where ζ_3 is the angle between L_{ν_1} and L_{ν_2} , ζ_4 is the angles between L_3 and L_{ν_2} . Geometrical relation of the angles can be expressed as:

$$\zeta_3 + \zeta_4 = \zeta_2 + \theta_4 \tag{9}$$

Length of $L_{\nu_{\gamma}}$ can be expressed as:

$$L_{\nu_{2}}^{2} = L_{\nu_{1}}^{2} + L_{3}^{2} - 2L_{\nu_{1}}L_{3}\cos\left\{\pi - (\zeta_{2} + \theta_{4})\right\}$$

thus $L_{\nu_{2}} = \sqrt{L_{\nu_{1}}^{2} + L_{3}^{2} + 2L_{\nu_{1}}L_{3}\cos(\zeta_{2} + \theta_{4})}$ (10)

C. Step III

In the third step shown in Fig. 5, the fictitious link L_{ν_2} and real link L_2 are replaced with an imaginary link L_{ν_2} .

where ζ_5 is the angle between L_{ν_3} and L_{ν_2} , ζ_6 is the angles between L_2 and L_{ν_2} .

Geometrical relation of the angles can be expressed as: $\zeta_5 + \zeta_6 = \zeta_4 + \theta_3$ (11)

The fictitious link L_{ν_3} length of can be expressed as: $L^2_{\nu_3} = L^2_{\nu_2} + L^2_2 - 2L_{\nu_2}L_2 \cos\{\pi - (\zeta_4 + \theta_3)\}$

or
$$L_{\nu_3} = \sqrt{L_{\nu_2}^2 + L_2^2 + 2L_{\nu_2}L_2\cos(\zeta_4 + \theta_3)}$$
 (12)

D. Step 4

The real links of the 5-link manipulator has been converted to one virtual and one real link, shown in Fig. 6. The length of the virtual link is expressed in (12) and the joint angle of the link is $(\zeta_6 + \theta_2)$. In a similar fashion the virtual link based controller to control the 3-link planar manipulator in a 2-dimensional workspace has been designed, which will provide the necessary torque required to each joints for a given end-effector trajectory.

Different geometrical relations required for obtaining the transformer moduli, to construct the bond graph model, are explained with the help of Fig. 7. Controller is composed of two links (one real and one imaginary link) and two revolute joints. The motion variables i.e., the joint angle $(\theta_1, \theta_2, \zeta_1)$, define the manipulator configuration.

In the virtual link based controller an imaginary link of length L_{ν} has been used instead of two actual links of length L_2 and L_3 . The length of the virtual link from the geometrical relation can be deduced as:

$$L_{\nu}^{2} = L_{2}^{2} + L_{3}^{2} - 2L_{2}L_{3}\cos(\pi - \theta_{3})$$
(13)

Further simplifying (13) the following relation can be deduced as:

$$L_{\nu} = \sqrt{L_2^2 + L_3^2 + 2L_2L_3\cos\theta_3}$$
(14)

Thus we can have the following geometrical relation:

$$\zeta_1 + \zeta_1 + (\pi - \theta_3) = \pi$$

 $\zeta_1 = \zeta_2 = \theta_3/2$ (15)

(: Assuming that the real links have equal in length, $\zeta_1=\zeta_2$)

Modulus	Expression	Modulus	Expression
μ_{1}	$-L_1 \sin \theta_1 - L_2 \sin(\theta_1 + \theta_2) - 0.5 L_3 \sin(\theta_1 + \theta_2 + \theta_3)$	μ_7	$L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2)$
μ_2	$-L_1\sin\theta_1 - L_2\sin(\theta_1 + \theta_2) - L_3\sin(\theta_1 + \theta_2 + \theta_3)$	μ_8	$L_1 \cos \theta_1 + 0.5L_2 \cos(\theta_1 + \theta_2)$
μ_3	$L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$	μ_9	$-0.5L_1\sin\theta_1$
μ_4	$L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + 0.5 L_3 \cos(\theta_1 + \theta_2 + \theta_3)$	μ_{10}	$-L_1\sin\theta_1$
μ_5	$-L_1\sin\theta_1 - 0.5L_2\sin(\theta_1 + \theta_2)$	μ_{11}	$L_1 \cos \theta_1$
μ_{6}	$-L_1\sin\theta_1 - L_2\sin(\theta_1 + \theta_2)$	μ_{12}	$0.5L_1\cos\theta_1$

TABLE II. VARIOUS TRANSFORMER MODULI FOR THE BOND GRAPH OF 3-LINK MANIPULATOR

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Fig. 7. Schematic diagram of virtual link based controller

Thus the virtual manipulator link length can be expressed as given in (14) and its joint angle can be expressed as $(\zeta_1 + \theta_2)$ with respect to link L_1 . The tip position of the virtual manipulator in *X* and *Y* directions can be expressed as:

$$X_{tip_vir} = L_1 \cos \theta_1 + L_v \cos(\theta_1 + \theta_2 + \zeta_1)$$
(16a)

$$Y_{tip_vir} = L_1 \sin \theta_1 + L_v \sin(\theta_1 + \theta_2 + \zeta_1)$$
(16b)





The corresponding tip velocities can be obtained by differentiating the above equation and can be expressed as:

$$\dot{X}_{iip_{-}vir} = -L_1 \dot{\theta}_1 \sin \theta_1 - L_v (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\zeta}_1) \sin(\theta_1 + \theta_2 + \zeta_1)$$
(17a)

$$\dot{Y}_{tip_vir} = L_1 \dot{\theta}_1 \cos \theta_1 + L_v (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\zeta}_1) \cos(\theta_1 + \theta_2 + \zeta_1)$$
(17b)

The transformer moduli obtained from (17) are as follows:

$$\mu_{13} = -L_1 \sin \theta_1 - L_\nu \sin(\theta_1 + \theta_2 + \zeta_1) \tag{18a}$$

$$\mu_{14} = L_1 \cos \theta_1 + L_y \cos(\theta_1 + \theta_2 + \zeta_1) \tag{18b}$$

$$u_{15} = -L_1 \sin \theta_1 \tag{18c}$$

$$\mu_{16} = L_1 \cos \theta_1 \tag{18d}$$

These transformer moduli have been used to construct the bond graph model of the virtual link based controller. The bond graph model of the controller is shown in Fig. 8. The controller joints are represented by 1 junction and the individual joint positions can be traced with the integrator used therein. The difference in the reference and actual velocity of the manipulator is termed as the error. The error signal is sent to the PI controller and it minimizes the error and sends it to the virtual link based controller. Controller receives the signal from the PI controller and supplies the required amount of torque to the 3-link planar hyper-redundant manipulator to obtain the desired tip

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trajectory. The modulus value at the 1_{ζ_1} junction has been taken as 0.5 as per the relation obtained in (15).



Fig. 8. Bond graph model of virtual link based controller

V. SIMULATION AND ANIMATION RESULTS

Three types of end-effector trajectories have been considered to validate the control strategy. First trajectory is a circle, the second one is an eight shape trajectory and third one is a knot- shaped trajectory based on Lissajous curve [20]. The initial configuration of the 3-link planar manipulator is shown in Fig. 9.



Fig. 9. Initial configuration of 3-link manipulator

TABLE III. PARAMETERS USED IN SIMULATION

Parameters	Value		
Joint resistance	$R_j = 0.001$ Nm/(rad/sec)		
Link length	<i>L</i> = 0.044 m		
Joint inertia	$I_{j=}0.001 \text{ Kg/m}^2$		
Amplitude	$A = 0.01 \mathrm{m}$		
Frequency	w = 10 rad/sec		
Mass of link	m = 0.01 kg		
Acceleration due to gravity	$g = 9.81 \text{ m/s}^2$		
Proportional gain	$K_p = 0.5$		
Integral gain	$K_i = 0.01$		
Lissajous constants	A=1, B=1, $\delta = \pi/2$, a/b = 0.5		

All the joints were kept at an initial position of 0.69813 rad. Parameters used in the simulation are enlisted in Table III. The three different cases considered, to check the efficacy of the proposed control scheme, are as discussed in this section.

A. Case I

Let us consider the reference trajectory for the 3-link planar manipulator tip in X and Y direction is assumed to be a circle of radius A. Then the tip coordinates are

$$X_{tip_ref} = A\cos(\omega t) + X_0 \tag{19a}$$

$$Y_{tip_ref} = A\sin(\omega t) + Y_0$$
(19b)

where (X_0, Y_0) is the center of the reference circular trajectory, *A* is the amplitude of circle, *w* is the angular velocity and *t* is the time in sec. Hence the corresponding reference velocity trajectory can be expressed as:

$$\dot{X}_{tip ref} = -A\omega\sin(\omega t) \tag{20a}$$

$$Y_{iip_ref} = A\omega\cos(\omega t) \tag{20b}$$







Fig. 11. Xtip and Ytip error vs. time for circular trajectory

Fig. 10 shows a comparative trajectory plot between the reference and actual tip trajectory, whereas Fig. 11 indicates the X_{tip} and Y_{tip} positional error with respect to time for the circular trajectory.

B. Case II

Let us assume the reference trajectory in X and Y directions is a simple eight shape trajectory and the tip coordinates are denoted as follows:

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$$X_{tip_ref} = 1 + 0.1 \sin(4\pi f_0 t) + 0.4\pi f_0 \cos(4\pi f_0 t) -1.6\pi^2 f_0^2 \sin(4\pi f_0 t)$$
(21a)

$$Y_{tip_ref} = 0.1\cos(2\pi f_0 t) - 0.2\pi f_0 \sin(2\pi f_0 t) -0.4\pi^2 f_0^2 \cos(2\pi f_0 t)$$
(21b)

The corresponding reference velocity trajectory can be expressed as:

$$\dot{X}_{tip_ref} = 0.4\pi f_0 \cos(4\pi f_0 t) - 1.6\pi^2 f_0^2 \sin(4\pi f_0 t) - 6.4\pi^3 f_0^3 \cos(4\pi f_0 t)$$
(22a)

$$\dot{Y}_{iip_ref} = -0.2\pi f_0 \sin(2\pi f_0 t) - 0.4\pi^2 f_0^2 \cos(2\pi f_0 t) + 0.8\pi^3 f_0^3 \sin(2\pi f_0 t)$$
(22b)





Fig. 13. Xtip and Ytip error vs. time for 8-shape trajectory

Fig. 12 shows the comparative tip trajectory between the reference and actual tip trajectory. Fig. 13 indicates the X_{tip} and Y_{tip} positional error with respect to time respectively.

C. Case III

Let us assume the reference trajectory in X and Y directions is a knot-shape trajectory based on the Lissajous curve [20] and the tip coordinates are denoted as follows:

$$X_{tip ref} = A\sin(at + \delta) \tag{23a}$$

$$Y_{tip ref} = B\sin(bt) \tag{23b}$$

where, A and B are constants and a/b is a ratio to be maintained i.e., a/b = 0.5 for this particular knot-shape.

The corresponding reference velocity trajectory can be expressed as:

$$\dot{X}_{tip_ref} = Aa\cos(at) \tag{24a}$$

$$\dot{Y}_{tip_ref} = Bb\cos(bt) \tag{24b}$$



Fig. 14. Reference vs. actual tip trajectory for Lissajous curve knot-shape



Fig. 15. Xtip and Ytip error vs. time for knot-shape trajectory

Fig. 14 shows the comparative tip trajectory between the reference and actual tip trajectory. Whereas Fig. 15 indicate the Xtip and Ytip positional error w.r.t time respectively. The competence of the controller can be evaluated from the comparison of given reference and actual tip trajectory obtained. The positional errors recorded in above three cases are: in case I Xtip error was 0.0033 mm and Ytip error was 0.0035 mm, in case II Xtip error was 0.00074 mm and Ytip error was 0.0023 mm and in case III Xtip error was 0.0033 mm and Ytip error was 0.0025 mm, after some initial disturbances. Fig. 16 shows the animation of case I, Fig. 17 indicates the animation of case II and Fig. 18 shows the animation of case III.

VI. CONCLUSIONS AND FUTURE SCOPE

A control scheme for 3-link planar redundant manipulator has been presented in this paper. The scheme is based on the model-based virtual link approach. Competence of the controller has been verified by considering different cases. Three cases have been considered, a circular trajectory, an 8-shaped trajectory and a knot-shaped trajectory based on Lissajous curve. Slight

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variations of actual tip trajectory from reference tip trajectory have been observed in all three cases.

The simulation results indicate that the proposed controller will be very effective for controlling the tip trajectory of the manipulator. In controller design authors have not considered the dynamic equivalence of actual links. However since the designed controller is based on kinematic equivalence, it will behave properly at low speed when Coriolis and centrifugal/centripetal forces will not be high.

In subsequent analysis, the present work will be extended for controlling the 4-DOF hyper-redundant *in-vivo* manipulator in 3-dimensional Cartesian space coordinate.



Fig. 16. Animation for a circular trajectory



Fig. 17. Animation for a 8-shape trajectory



Fig. 18. Animation for a knot-shape trajectory

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