Damage Detection on Structures using Transfer Matrix with Lumped Crack Properties

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***Abstract*—A novel lumped crack transfer matrix (LCTM) and state vector based method is proposed in this paper for identification of damage in the beam like structures. Transfer matrix (TM) is a square matrix which contains all structural and crack parameters such as crack depth and its location. State vector at a node is the sum of internal and external contributions of displacements, forces and moment at that node on the structural element, when it is multiplied with TM the state vector at the adjacent point can be obtained. A cracked beam element is assumed as two intact beam elements are connected by a hinge or torsional spring. The crack is modelled as an element of zero length and mass but has elastic properties. Hence the LCTM for the cracked beam element is obtained by series multiplication of LCTMs for the intact beam elements and that for the crack element. State vector is formed at one node known as initial state vector from which state vectors at other nodes are predicted by multiplying with TM for the predicted values of crack parameters. Displacement responses are measured at a few nodes in the structure. The mean square error between measured and predicted responses is minimized using a heuristic optimization algorithm with crack depth and location as the optimization variables. Two numerical examples a cantilever and sub-structure of a frame with nine members are solved with multiple cracks in an element. The TM algorithm is also validated experimentally. The main advantage in this method is one or more cracks in the single element can be identified.**

Keywords – transfer matrix for cracked element; state vector; crack detection; successive identification; particle swarm optimization;

I. Introduction

The engineering structures are very often undergo periodic or repeated cyclic loading which cause unexpected fatigue failure even if there is a small crack. Hence, it is mandatory that such structures must undergo structural health monitoring process periodically in which the magnitude and location of the crack can be identified and the remaining life of the structure can also be predicted. Generally, the crack detection process such as ultrasonic methods, optical methods, radiography, magnetic field methods, eddy-current methods and thermal field methods are used for damage detection. They are highly expensive and require that the vicinity of the damage is known a priori. Further, the structural element to be inspected is accessible. Hence, the vibration based crack detection methods deserve further investigation. Crack in a structural element increases the local flexibility which is the function of crack depth and its location. Gounaris and Dimarogonas[1] developed a compliance matrix for cracked beam element by assuming the crack increases the flexibility due to strain energy concentrations in the vicinity of the crack tip under load. Ibrahim[2] formulated new stiffness matrix using TM theory for beam element with elasto-plastic crack where the crack was modelled as massless torsional spring with flexibility equal to additional flexibility due to the crack presence in the element. Krawczuk *et al.*[3] developed new finite element matrices for cracked beam elements using elasto-plastic fracture mechanics. Finite element procedure was applied with suitable boundary conditions at the crack. Viola *et al.*[4] developed a new prismatic Timoshenko beam element with crack and the effect of crack in stiffness and mass matrices were investigated. Khiem and Lien [5] used TM for determination of natural frequency of beam with multiple open edge cracks.

Krawczuk [6] detected the crack in beam like structures using wave propagation method using Genetic Algorithm (GA) and gradient search technique. Tee *et al.*[7] identified crack on 50 degree of freedom(DOF) shear model using Observer Kalman filter Identification and Eigen Realisation Algorithm (OKID/ERA) based Condensed Model Identification and Recovery technique with global and sub-structural approach using GA. Prashanth and Shankar [8] detected damage on structures using a two stage artificial neural network technique. Damage on a six story shear building, a nine member frame structure and thirty member frame were identified. Varghese and Shankar [9] identified cracks in sub-structure of cantilever using multi-objective optimization approach with combined acceleration matching and transient power flow balance matching. Most of the above algorithms need mathematical model of the complete structure and need to solve the second order differential equations by using suitable

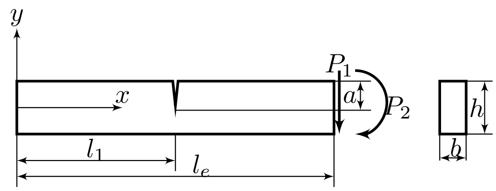


Fig. 1. Cracked beam element

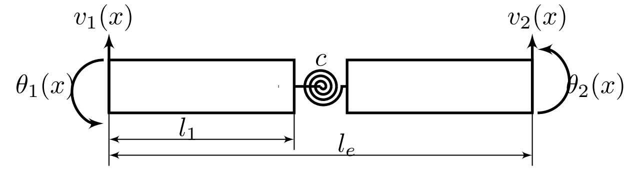


Fig. 2. Equivalent model of cracked beam element

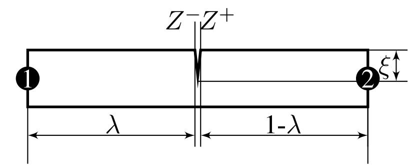


Fig. 3. Cracked beam element

numerical technique. The size of the system matrices is in the order of $n \times n$, where n is the total number of degrees of freedom (DOF) of the structure. The size of the matrices depends on the number of DOF. Even for a modern computer, the computational speed for solving large matrices is challenging. In the above algorithms, for each iteration, the large system matrices need to be solved which consumes more computational time. As an alternative, TM based method is introduced in this paper. The size of the TM does not increase with respect to the total number of DOF of the model, thus reducing the computational effort.

To the best of Author's knowledge, Nandakumar and Shankar[10] identified structural parameters of a ten DOF lumped mass system and a cantilever using lumped mass transfer matrices first time in the literature. They developed an improved consistent mass TM which was used for parameter identification of beam like structures, and proved that it is superior to lumped mass TM. Later Nandakumar and Shankar[11] developed TM including damping parameters and extended the identification into damping parameters of the structures. In general, damage detection methods have a limitation in that there is only one crack in an element. In this paper, a TM for a bending cracked beam element has been developed for more than one cracks present in an element.

II. Cracked beam element

The cracked beam element shown in the Fig.1 is modelled as a torsional spring of flexibility c which connects the two intact portions of the beam element as shown in Fig.2 and provides additional flexibility to the beam element. Let l_e is the length of the element, l_1 is the location of the crack from its left end node and a is the crack depth from the top of the crack section. The cross sectional dimensions of the beam is $b \times h$. The normalised crack depth(ξ) and location (λ) are defined as $\xi = a/h$ and $\lambda = l_1/l_e$ respectively. P_1 and P_2 are the shear force and moment applied at the right node on the cracked element. The flexibility due to the crack in the beam element can be obtained by Castigliano's theorem:

$$c_{ij} = \frac{1-\nu^2}{E} \frac{\partial^2}{\partial P_j \partial P_i} \int_A \left(\sum_{k=1}^{2} K_{I,k} \right)^2 dA \tag{1}$$

The stress intensity factor at any crack depth α is given by $K_I = \sigma\sqrt{\pi\alpha} f(\alpha/h)$, where σ is bending stress at the cracked section, E is Young's modulus of the beam material, I is area moment of inertia of the section and $f(\alpha/h)$ is geometric correction function. The geometric correction function is given by [3]

$$f(\alpha/h) = \sqrt{\frac{tan(\pi\alpha/2h)}{\pi\alpha/2h}} \frac{0.923 + 0.199[1 - sin(\pi\alpha/2h)]^4}{cos(\pi\alpha/2h)} \tag{2}$$

$$c = \frac{6\pi h(1-\nu^2)((1-\lambda)l_e)^2}{EI} \int_0^{\xi} \bar{\alpha} f(\bar{\alpha})^2 d\bar{\alpha} \tag{3}$$

A. Lumped Crack Transfer Matrix Formulation

The LCTM for the cracked beam element is the multiplication of three transfer matrices corresponds to the above said portions. Let $[T_L]$ and $[T_R]$ be the TM for the left and right intact portions of the beam element respectively. $[T_c]$ be the TM for the lumped crack portion in the beam element. The transfer matrices $[T_L]$ and $[T_R]$ for the intact portion of beam element, obtained from consistent mass TM [10]. The crack is assumed as an element which does not have any length and mass but provides an additional flexibility c to the cracked beam element. The TM for the torsional spring is formulated from the static equilibrium equations of the spring as given in [12].

$$[T_c] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -c & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The overall LCTM $[T_{2,1}]$ for the cracked beam element which converts the state vector at the node 2 to state vector at the node 1 can be obtained.

$$[T_{2,1}] = [T_L][T_c][T_R] \tag{5}$$

In general, for a beam element with n number of cracks as shown in Fig.4, the overall LCTM for the element is

$$[T_{2,1}] = [T_1][T_{c1}][T_2][T_{c2}][T_3] \dots [T_n][T_{cn}][T_{n+1}] \tag{6}$$

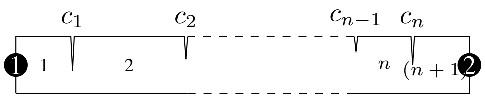


Fig. 4. Beam element with multiple cracks

where $[T_1], [T_2], \ldots [T_{n+1}]$ are TMs for intact portions and $[T_{c1}], [T_{c2}], \ldots [T_{cn}]$ are TMs for cracks. The TM for multiple elements is calculated from elemental TMs. The state vector is estimated across n elements and state vector at node 1 $\{X_1\}$ is known, then the internal response vector at the $(n+1)^{th}$ node $\{X_{n+1,i}\}$ is given by

$$\{X_{n+1,i}\} = \left(\prod_{k=1}^{n} [T_{(n+1-k),(n+2-k)}]\right)\{X_{1,i}\} + \sum_{j=1}^{n}\left(\prod_{k=1}^{n+1-j}[T_{(n+1-k),(n+2-k)}]\right)\{X_{j,e}\} \qquad (7)$$

where $\{X_{j,e}\}$ is external force vector at j^{th} node.

III. Crack Detection by Lumped Crack Transfer Matrix Method

Cracks in a structure are identified using the newly developed LCTM for cracked beam element by assuming the mass and flexural rigidity of the beam are known and the crack present in the structure is open crack. The unknown parameters are normalized cracks depth (ξ) and their locations (λ) in the beams. The structure is excited by a known harmonic force at a node. The acceleration responses are measured at few nodes on the structure and shear force and bending moment responses are measured at the node where initial state vector is formed. The measured accelerations are converted into displacement by numerical integration. Since the crack parameters in the LCTM are unknown, they are searched by Particle Swarm Optimization (PSO) algorithm within the feasible range of zero and one. It is assumed that all the elements have two open edge cracks. The zero value of identified normalised crack depth shows the undamaged state of an element. The crack parameters are identified by successive identification strategy [10] of LCTM algorithm.

A. *Successive crack detection*

In this strategy, the parameters of element(s) are identified between the nodes where the initial state vector is formed and the nearest node where displacement measurements are taken. The mean square error between the predicted displacement responses for searched values of crack parameters and measured displacement responses at that location is minimized. The crack parameters for which the mean squared error between measured and predicted responses is minimum are the identified parameters of crack.

The error function is given by

$$\varepsilon = \frac{\sum_{j=1}^{L} |v_m(j) - v_e(j)|^2}{L} \qquad (8)$$

where $v_m(j)$ and $v_e(j)$ are measured and predicted displacement responses respectively at j^{th} time step. L is the total number of time steps. This procedure is repeated until parameters of all the elements are identified. Parameters are identified from both complete and incomplete measurements. Complete measurement means that the translational acceleration response is measured at all nodes and angular acceleration is measured only at the initial node. The complete measurement is not always practical for large structures due to the requirement of large number of sensors. Hence, parameters are identified by measuring responses at selected nodes only, which is known as incomplete measurement.

IV. Numerical Examples

The crack detection algorithm based on LCTM is applied on a cantilever with cracks at different locations and a substructure of a nine member frame structure. It is assumed that the mass and flexural rigidity of each element are known priori, each element assumed to have two cracks, absence of a crack gives zero for crack depth and its location. All measured responses are numerically simulated by MATLAB using Newmark's constant acceleration scheme for a length of 3 s with time step of 0.001 s. In order to simulate experimental error, the I/O responses are polluted with a zero mean Gaussian white noise of 5% of the RMS value of unpolluted responses.

A. *Example-1: Cantilever with multiple cracks*

A uniform slender cantilever of cross section 50×5 mm and length of 520 mm which was used by Viola *et al.*[13] is considered here with four cracks. The Young's modulus of the beam material (E) is 206 GPa and its density is 7850 kg/m^3. The cracks of depth 0.5 mm, 1.5 mm, 0.4 mm and 2.5 mm are located at 120 mm, 150 mm, 380 mm and 395.2 mm respectively from the fixed end of the cantilever. The normalized crack depths are $\xi_{c1} = 0.1$, $\xi_{c2} = 0.3$, $\xi_{c3} = 0.08$ and $\xi_{c4} = 0.5$ and the normalised crack locations in global structure are $\lambda_{c1} = 0.23$, $\lambda_{c2} = 0.288$, $\lambda_{c3} = 0.73$ and $\lambda_{c4} = 0.76$ respectively. The first natural frequency of the cracked beam is 15.07 Hz. The cantilever is divided into six elements. Two cracks are located in element 2 and two cracks are located in element 5 as shown in Fig.5. The normalized crack locations in the element 2 with respect to left end of the element are λ_{e21}=0.384 and λ_{e22}=0.73 and the same in the element 5 are λ_{e51}=0.384 and λ_{e52}=0.56. ($e21$ represents crack 1 in element 2 - $C1$). The free end of the cantilever is applied with an harmonic excitation of $F(t) = 10sin(2\pi 10t)$ N. Acceleration responses are measured at all nodes and angular acceleration response is measured at free end node only. The initial state $\{X_7\} = \{v_7(t), \theta_7(t), M_7(t), V_7(t)\}^T$ is formed at the free end since the bending moment and shear force is zero. The parameters

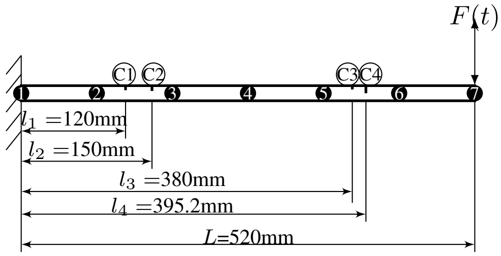
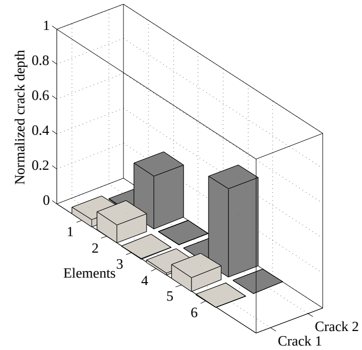


Fig. 5. FE model of cantilever with multiple cracks

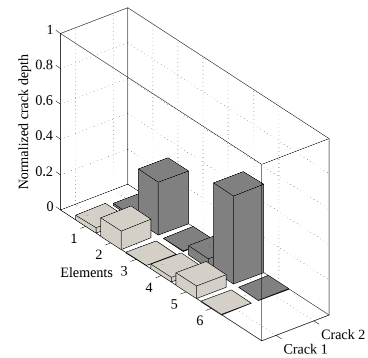
TABLE I. NORMALIZED GLOBAL CRACK LOCATION IN THE CANTILEVER

Crack Number	Exact Location	Identified crack location (λ)			
		Complete measurement		Incomplete measurement	
		Noise free	5% Noise	Noise free	5% Noise
C1	0.231	0.2303	0.2289	0.229	0.2252
C2	0.288	0.2885	0.29	0.2893	0.2807
C3	0.731	0.7322	0.7248	0.7288	0.7369
C4	0.76	0.7587	0.7624	0.7544	0.7516

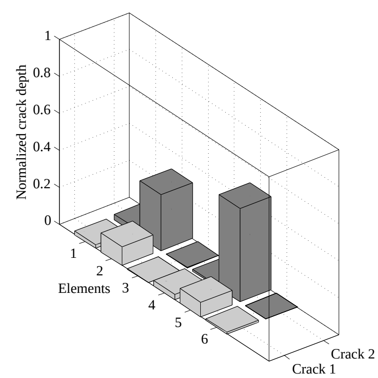
are searched by PSO with 50 swarm size and 30 iterations in each identification cycle. The identified crack depths in each element with complete set of measurement are shown in Fig.6(a) and 6(b). The absolute percentage of error in identified crack depth with complete set of measurement in the element 2 are 0.3% and 0.06% and in the element 5 are 0.32% and 0.086%. The corresponding percentage of error in crack location from the left end of the cantilever in the element 2 are 0.19% and 0.08% and in the element 5 are 0.2% and 0.18%. The absolute percentage of error in identified crack depth with complete set of measurements with 5% noise are 5.46% and 0.53% in element 2 and 11.31% and 0.12% in element 5. The corresponding error in the identified locations measured from the left end of the cantilever are 0.78% and 0.53% in element 2 and 0.82% and 0.32% in element 5. The identified normalized crack locations from the left end of the cantilever is tabulated in the Table.I. The total time taken for convergence is 11 s. The problem is next solved using the displacement responses measured at nodes 2, 4, 6 and 7 only. The PSO parameters are 50 swarm size and 100 iterations in each identification cycle (total 400). The total time taken for the convergence is 15s. The identified crack depth in each element with incomplete set of measurement are shown in Fig. 6(c) and 6(d). The percentage of absolute error in identified crack depth are 2.46% and 1.57% in element 2 and 3.15% and 1.01% without noise. The same in identified crack locations from the left end of the cantilever are 0.78% and 0.28% in element 2 and 0.27% and 0.74% in element 5. In case of measurements with 5% noise level, the percentage of error in crack depth are 8.46% and 4.02% in element 2 and 11.31% and 2.04% in element 5. The same in crack locations are 2.43% and 2.71% in element 2 and 0.84% and 1.1% in element 5. The percentage of error is comparatively high



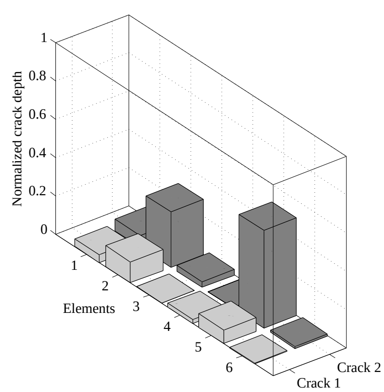
(a) Crack depth(No Noise, complete)



(b) Crack depth(5% Noise, complete)



(c) Crack depth(No Noise, Incomplete)



(d) Crack depth(5% Noise, Incomplete)

Fig. 6. Identified crack depth of cantilever

at cracks with small magnitudes since they show very small amount change in the dynamic responses.

1) Comparison with other damage detection methods: Viola *et al.*[13] identified crack parameters in the same cantilever with single crack of depth 2.5 mm and at a distance 395.2 mm from its fixed end, using a frequency domain method from experimentally obtained measurements of unspecified noise content. The percentage of error in identification was 2.5% in crack depth and 0.53% in crack location in global structure. The same problem was also solved by Cibu Varghese and Shankar [14] using combined acceleration and power flow matching method. They identified the crack parameters with the percentage of error of 0.92% in crack depth and 0.5% in location respectively without noise and 1.72% and 2.8% respectively with 5% Noise. The LCTM identified the crack ($C4$) of the same magnitude and same location along with other three cracks. The crack parameters of crack $C4$ is identified with an error of 0.08% in depth and 0.17% in global location with uncontaminated responses and 0.12% in depth and 0.32% in global crack location with 5% noise contaminated responses. The method proposed by Viola *et al.* has a constraint that there should be only one crack in the cantilever. Similarly the combined acceleration and power flow matching method has a limitation that there should be only one crack in each element. This LCTM method has an advantage that it can able to identify multiple cracks in single element.

B. Example-2:Sub-Structure of Frame Structure

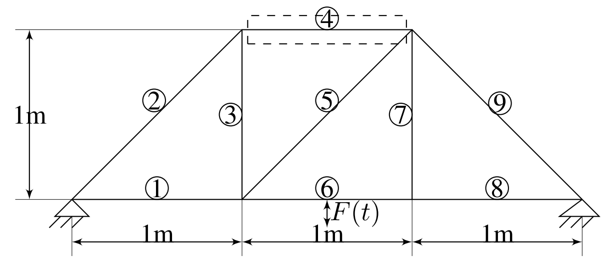
A frame structure with nine slender beam members is supported as shown in Fig.7(a) used in [11]. The cross section of each member is 12 $\times$ 6 mm. The density of the frame material is 7800 kg/m^3 and its Young's modulus (E) is 200 GPa. The flexural rigidity of the each member is 43.2 N.m^2. The first natural frequency of the structure is 11.87 Hz. Four open edge cracks of depth 0.3 mm, 1.5 mm, 3 mm and 2 mm are considered at a distance of 362.5 mm, 387.5 mm, 600 mm and 650 mm respectively from the left end of the member 4 as shown in Fig.7(b). The normalized crack depth for the above cracks are ξ_{c1}=0.05, ξ_{c2}=0.5, ξ_{c3}=0.25 and ξ_{c4}=0.33 and their normalized locations from the left end of the member 4 are λ_{c1}=0.3625, λ_{c2}=0.3875, λ_{c3}=0.6 and λ_{c4}=0.65. It is proposed to detect the cracks locally in the sub-structure (member 4) of frame which is shown in side the dotted box in Fig.7(a). The sub-structure considered is the middle portion of member 4 which has a length of 875 mm and is divided into seven finite elements. First two cracks lie on the element 3 and the remaining two cracks lie on the element 5. The normalized locations of them from the left end of the respective elements are $\lambda_{e3.1}$=0.3, $\lambda_{e3.2}$=0.4, $\lambda_{e5.1}$=0.6 and $\lambda_{e5.2}$=0.7 ($\lambda_{e3.1}$ means that crack 1 is in the element 3). The damping effect in the structure is assumed as Rayleigh's damping with damping ratio of 1%.

The structure is excited by a harmonic force of $10sin(2\pi \times 10t)$ N at the midpoint of the member 6 which is outside of the sub-structure considered. Hence the measurement of input force is not required for identification. The initial state vector $\{X_8\} = \{v_8(t), \theta_8(t), M_8(t), V_8(t)\}^T$ is formed at node 8 by measuring bending moment and shear force using strain measurements. Translational acceleration is measured at all nodes while angular acceleration, bending moment and shear force responses are measured at starting node 8 only. The bending moment and shear force responses are measured by measuring corresponding strains at the node 8 using strain gauges. It is assumed that EI of starting node is known. They are calculated from measured strains using the relations [11].

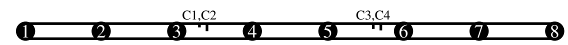
$$M(t) = \frac{2EI\epsilon_B(t)}{h} \tag{9}$$

$$V(t) = \frac{4EI\epsilon_S(t)}{h^2(1+\nu)} \tag{10}$$

where $M(t)$ and $V(t)$ are bending moment and shear force respectively, $\epsilon_B(t)$ and $\epsilon_S(t)$ are strain due to bending and shear respectively, ν is Poisson's ratio. Dynamic strain $\epsilon_B(t)$ at the starting node is measured at the top and bottom of the node 8, and the mean value is considered for calculation of bending moment. The algorithm works as explained in the previous example. The PSO searches the parameters within the feasible range of zero and one with swarm size of 50 and 50 iterations in each cycle. The element wise identified crack parameters with complete set of measurement are shown in Fig.9. The percentage of absolute error in identified crack depth are 1.45%, 0.07%, 0.72% and 1.1% in the order of cracks shown in Fig.7(b) without noise. The same are 8.8%, 0.76%, 1.978% and 1.83% with 5% noise level.



(a) Global Structure



(b) Sub-structure-Member 4

Fig. 7. Frame Structure

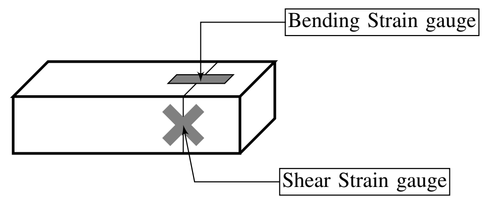
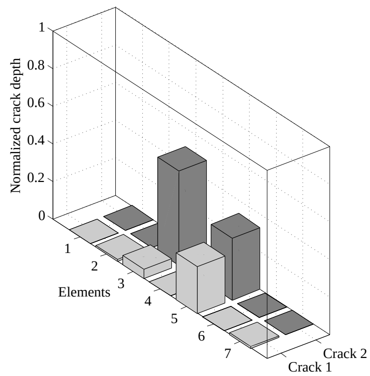
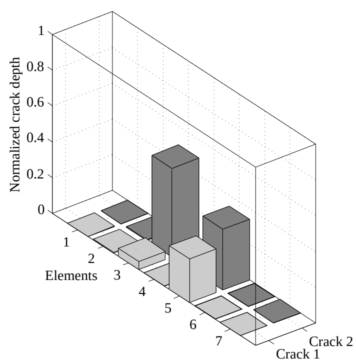


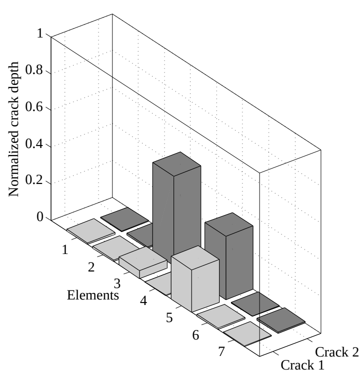
Fig. 8. Strain gauge arrangements at starting node



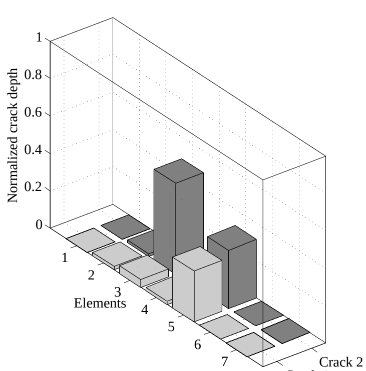
(a) Crack depth(No Noise, Complete)



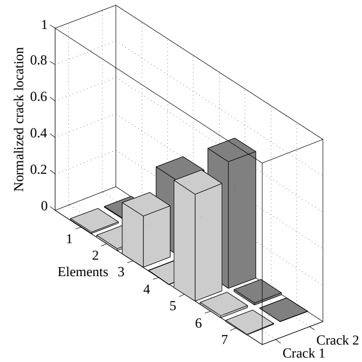
(b) Crack depth(5% Noise, Complete)



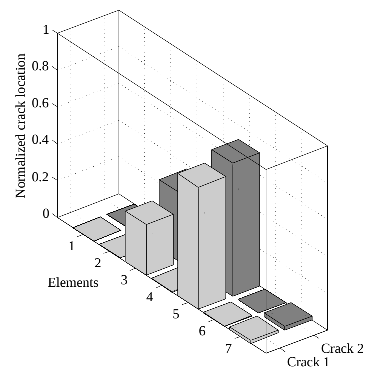
(c) Crack depth(No Noise, Incomplete)



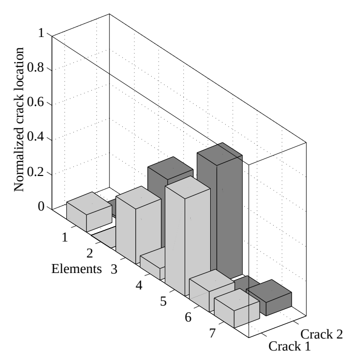
(d) Crack depth(5% Noise, Incomplete)



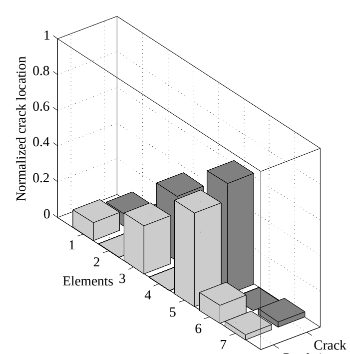
(e) Crack location(No Noise, Complete)



(f) Crack location(5% Noise, Complete)



(g) Crack location(No Noise, Incomplete)



(h) Crack location(5% Noise, Incomplete)

Fig. 9. Identified crack parameters of SS of frame structure

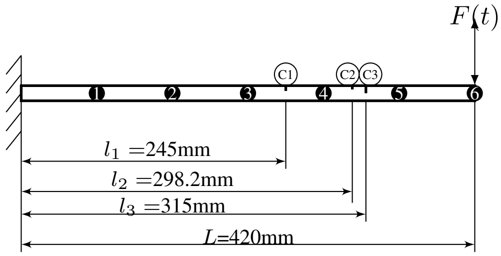
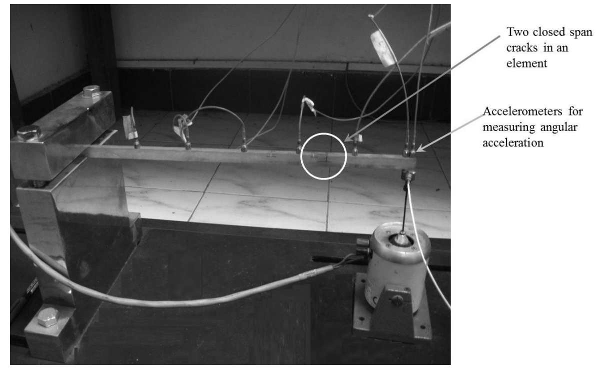


Fig. 10. FE model of cantilever experiment

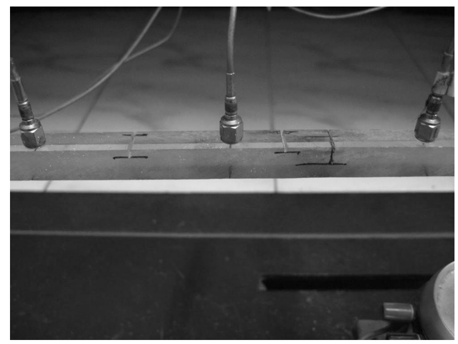
The error in identified global location of the cracks are 0.62%, 0.23%, 0.23% and 0.13% in the same order without noise and 0.91%, 0.84%, 1.26% and 0.45% with 5% noisy measurement. The mean computational time is 8s. The problem is repeated with only four set of measurements at nodes 1, 3, 5 and 8. Since the number of unknowns are more in one cycle, number of iterations are increased to 200 for the same 50 number of swarm size in PSO. The identified element wise results are shown in Fig.9. The percentage of absolute error in identified crack depth are 11.9%, 2.46%, 6.75% and 3.42% in the order of cracks shown in Fig.7(b) without noise. The same are 17%, 2.8%, 8.9% and 6.39% with 5% noise level. The error in identified global location of the cracks are 0.71%, 0.63%, 0.69% and 0.3% in the same order without noise and 1.19%, 1.3%, 1.48% and 1.58% with 5% noisy measurement. The mean computational time is 18s. In this example also, it can be seen that the crack with small magnitude is identified with more error. However, the LCTM method is suitable to locate the multiple cracks in an element.

V. Experimental Validation-Cantilever

A cantilever made up of acrylic material of length 420 mm and cross section of 12×12 mm is taken and it was divided into six finite elements of each of length 70 mm as shown in Fig.10. The Young's modulus of the cantilever material was estimated to be 3.9 GPa from a simple bending test and the density was measured to be 1190 kg/m^3. The area moment of inertia for the cross section is I = 1.728×10^{-9} m^4. The flexural rigidity (EI) of each element is calculated as 6.739 N.m^2. The experimental setup is shown in Fig.11(a). Cracks were introduced in the fourth and fifth elements as shown in Fig.11(b). The crack depths of the cracks $C1$, $C2$ and $C3$ are 3 mm, 1.5 mm and 6 mm respectively. The normalized crack locations of the above cracks from the fixed end of the cantilever are λ_{c1}=0.583, λ_{c2}=0.71 and λ_{c3}=0.75 respectively and the normalized crack depths are ξ_{c1}=0.25, ξ_{c2}=0.125 and ξ_{c3}= 0.5 respectively. The crack $C1$ lies in the element 4 and other two cracks lie in the element 5. The element wise crack locations are $\lambda_{e4.1}$ =0.5, $\lambda_{e5.1}$=0.26 and $\lambda_{e5.2}$=0.5. The natural frequencies for the first mode of the structure was calculated using above values of mass and stiffness parameters as 19.89 Hz. There were seven DYTRAN miniature accelerometers of 2 gm mass with sensitivity of 107 mV/g and acceleration range of 50g. Each



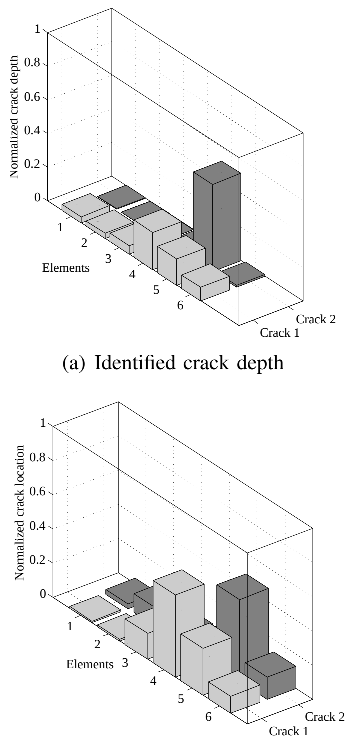
(a) Cantilever



(b) Cracks in the cantilever

Fig. 11. Experimental set-up of cantilever

node was fixed with one accelerometer to measure transverse acceleration and the free end node was fixed with two accelerometers very close to each other as shown in Fig.11(a) to measure angular acceleration. The center distance between two accelerometers fixed at the free end was dx=7 mm. The cantilever was excited at the free end with an harmonic input force of $F = 1.5sin(2\pi 80t)$ N by a LDS permanent magnet 20 N modal shaker with a maximum displacement of 5 mm, with an operating frequency range of 5 Hz - 13 kHz. The input force was measured by a KISTLER force transducer with sensitivity of 90.4 mV/lbf and range of 50 lbf. The acceleration responses measured at all six nodes and input force response were acquired with a sampling frequency of 1000 Hz using DEWE 43 8 channel data acquisition card. From the acquired data, a portion of data length 3 s was considered for damage identification. The translational acceleration at the free end (Node 6) is the mean of $\ddot{v}_7(t)$ and $\ddot{v}_6(t)$. All the measured accelerations are converted into displacement responses. The initial state vector is formed at the free end as $\{X_6\} = \{v_6(t), \theta_6(t), 0, 0\}^T + \{0, 0, 0, F(t)\}^T$. The normalized crack depth (ξ) and location (λ_e) are the unknowns to be identified in each element. The parameters were searched between the feasible search range of zero and one by PSO. The swarm size is 50, number of iterations for each cycle is 100. The identified element wise crack parameters are shown in Fig.12. The percentage of absolute error in identified normalized crack depth in the order of



(a) Identified crack depth

(b) Identified crack location

Fig. 12. Experimentally identified crack parameters of cantilever

TABLE II. NORMALIZED GLOBAL CRACK LOCATION(λ) IN THE CANTILEVER

Crack Number	Exact Location	Identified location	% of error
C1	0.583	0.58	-0.62
C2	0.71	0.712	0.31
C3	0.75	0.746	-0.46

cracks shown in Fig. 10 are 12.24%, 25.9% and 3.52% respectively. The absolute percentage of error in identified element wise crack location for the above cracks are 4.32%, 5.07% and 4.14% respectively. The mean CPU time required for convergence was 10s. The global crack location and percentage of error in identified global crack position measured from the left end of the cantilever are shown in Table. II. The crack with small magnitude ($C2$) is identified with more error. Hence the LCTM method is validated experimentally.

VI. CONCLUSION

A new LCTM based damage detection algorithm for beam like structure is presented in this paper. The LCTM for the cracked beam element is formed from TMs for intact beam elements and lumped crack elements. The initial state vector is obtained at one node on the structure by measuring displacement, bending moment and shear force responses. A different strategy is used for nine member structure for obtaining the initial state vector from strain measurements. Normalized crack depth and its location are the parameters to be identified. Two cracks with close span are assumed in each element. The successive identification strategy adopted here is fast and accurate. Two numerical examples were studied with complete and incomplete set of measurements. They are cantilever with multiple cracks and a nine member structure with multiple cracks. the typical error in identified crack parameters ranging from 0.07%(without noise) to 17%(with 5% noise). Further the LCTM method is validated experimentally by identifying cracks of varying depth and location in a cantilever. This algorithm is capable of identifying multiple cracks in an element with accuracy comparable to existing literature. Further, this algorithm is suitable for local damage identification in a large structure.

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