

# Study On Mesh Power Losses In High Contact Ratio (HCR) Gear Drives

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**Abstract**—Continuous demands for higher efficiency gear drives need an understanding on the role of power loss that depends on the frictional forces. The load dependent tooth mesh power losses (i.e., sliding and rolling power losses) are the dominant power loss components at heavy loads with low or moderate pitch line velocities. Hence, these two power losses are given more importance in this study to evaluate the performance of the HCR gear drive. The calculation of sliding power loss is carried out based on the Elastohydrodynamic lubrication (EHL) model proposed by Xu et al.[1], but with the consideration of load sharing ratio(LSR). Higher contact ratio is achieved through addendum enlargement in this work. The comparative study of normal and high contact ratio gears is discussed for better understanding on gear mesh losses on HCR gear drive.

**Keywords** – Power loss; Efficiency; Finite element analysis; High contact ratio; Spur gear

## I. INTRODUCTION

The efficiency of power transmission system is considered as one of the important design factors due to the following reasons ([2]),

- 1) Efficient power transmission systems ensure fuel economy.
- 2) With less fuel consumption, less pollutant gases and particulate are emitted to the environment.
- 3) Since power losses amount to heat generation within the gearbox, which leads to several gear failure modes such as scoring and fatigue.
- 4) Improved efficiency of a gearing system can reduce the requirements on the capacity of the lubrication system and the gearbox lubricant and thereby reducing the operation costs of the system.
- 5) Efficiency prediction can assist in estimating the power requirements during the design stage of a machine and thus ensures reliable operation of the system. It can also assist in estimating the power output for a given power input.

The total power loss of the gearbox is attributable to sliding and rolling frictional losses between the gear teeth, windage losses due to complex interactions with the air surrounding the gears and oil splashing and churning losses inside the gearbox as well as the losses associated with the bearings and seals. While churning and windage losses are mostly geometry and speed related, friction losses are mainly associated with sliding velocities and load. Gears are usually operated under mixed EHL condition, where the lubricant film thickness is comparable to the surface asperity heights such that actual metal-to-metal contact are possible.

Friction can be stated as the resistance to motion between two surfaces in relative sliding and rolling under dry or lubricated contact conditions. Lubricant applied to a contact significantly alters the contact conditions and hence reduces the friction. Applied load, speed, parameters related to contact geometry, surface roughness and lubricant as a whole helps to define the lubrication conditions. As these factors are changing instantaneously during a mesh cycle, the friction will also change accordingly.

Friction at a gear mesh has two components: sliding friction and rolling friction. Sliding friction is a direct result of the relative sliding between the two contacting surfaces. The magnitude of the sliding frictional force depends on the coefficient of friction between the contacting surfaces.

Where as the hydrodynamic rolling (or pumping) loss is the power required to entrain and compress the lubricant to form a pressurized oil film, which separates the gear teeth. Based on disk machine data, Crook [3] found that the rolling loss was simply a constant value multiplied by the EHL central film thickness. Most of the literature uses the central film thickness model proposed by Hamrock and Dowson [4] adjusted with for thermal effects using Cheng's thermal reduction factor ([5]) to estimate rolling power loss.

Martin [6] has provided an extensive review of friction predictions in gear teeth published in boundary lubrication, mixed and EHL regimes.

In some of the earlier works, the sliding frictional loss was calculated by using average frictional coefficient value in a mesh cycle ([7], [8],[9],[10] and [11]).

The differences between the sliding force treatment of Buckingham's and Merritt's equations were explained by Yada [12].

In many of the works, the variation in the coefficient of friction during a mesh cycle was calculated based on the empirical equations derived by several authors like Benedict and Kelly [13], ODonoghue and Cameron [14], Kelly and Lemanski [15], Drozdov and Gavrikov [16], Xu et al. [1] etc.

Anderson and Loewenthal [17] proposed a method for predicting the power loss and efficiency of a steel spur gear set of arbitrary geometry supported by ball bearings. The method algebraically accounts for losses due to gear sliding, rolling, and windage and incorporates an expression for ball-bearing power loss. This method provides an accurate estimate of spur-gear-system efficiency at part load as well as full load. They have used Benedict and Kelly [13] frictional coefficient equation to calculate the sliding frictional loss. They have extended their work to explore the effect of parameters on HCR gear performance with the assumption that the teeth are rigid and equal load sharing for simultaneously contacting pairs ([18]).

Heingartner and Mba [19] modified the above approach to calculate the sliding and rolling friction losses in helical gear.

Arto and Asko [20] used the Benedict and Kelly model to estimate the sliding frictional loss. They have also included the load dependent bearing power loss to calculate the total power loss.

Benedict-Kelley's formula was evaluated by considering the oil sump temperature and the ambient pressure, but several parameters such as piezoviscosity or limiting shear stress are ignored. The influence of surface roughness was also discarded in their equation, which is realistic for fully flooded conditions, but certainly questionable when mixed lubrication prevails and direct asperity interactions cannot be avoided. Another shortcoming of their equation is due to the  $\log_{10}$  in the analytical expression, which (a) leads to infinite friction near pitch points and (b) for high slide-to-roll conditions, underestimates friction forces.

Due to the above shortcomings, Dib et al. [21] proposed a new traction law based on measurements from a two-disc machine which accounts for lubricant properties and surface finish and integrated in a 3-D dynamic model of gears with consideration of tooth friction. They have simulated the model with no friction, constant frictional coefficient, friction based on Benedict and Kelly [13] equation as well as Kelly and Lemanski [15] equation. They claimed that their model yields best results for both low and higher speeds. They have extended their study to predict the power losses in high-speed gears ([22]).

Xu et al. [1] proposed a model to evaluate the friction related mechanical efficiency losses for parallel-axis gear pairs. Their model combined a gear load distribution model, a friction coefficient model and a mechanical efficiency formulation to predict the instantaneous mechanical efficiency of a gear pair under typical operating, surface and lubrication conditions. They have also derived new friction coefficient formula by performing a multiple linear regression analysis to a large number of EHL model simulations representing various combinations of all key parameters influencing the friction coefficient. The new friction coefficient formula was shown to agree with the measured traction data. They have also highlighted the influence of key basic gear geometric parameters, tooth modifications, operating conditions, surface finish and lubricant properties on mechanical efficiency.

Kuria and Kihui [2] used the Xu et al. [1] model for determining the overall efficiency of a multistage tractor gearbox including all gear pairs, lubricant, surface finish related parameters and operating conditions. Rolling friction and windage losses were also included in their study and found that the overall efficiency varies over the path of contact of the gear meshes ranging between 94% and 99.5%.

Even though the availability of literatures on the performance of the normal contact ratio (NCR) gear drives is abundant, the performance study of the HCR gear drives has been rarely found. Also normally the higher contact ratios are achieved by way of enlarging the addendum beyond the standard values, this increases the sliding power losses. Hence a thorough efficiency analysis has to be made for HCR gear drives.

This work explores the performances of both the NCR and HCR spur gears in terms of load dependent mesh power losses by considering the LSR between the simultaneously meshing teeth pairs. The typical contact points along the path of contact and load sharing between simultaneously engaging pairs of NCR and HCR gear drives are evaluated based on the author's previous works ([23] and [24]).

## II. POWER LOSS CALCULATION

The different kinds of power losses that occur within the gear box can be grouped into two categories, one as load dependent power loss ( $P_{load}$ ) and the other as load independent power loss ( $P_{no\ load}$ ). Hence, the total power loss is considered as

$$P_{Total} = P_{load} + P_{no\ load} \quad (1)$$

Load dependent power loss has got the contribution from tooth mesh power loss ( $P_{Mesh}$ ) and bearing power loss ( $P_{b,load}$ ). Tooth mesh power loss consists of sliding frictional power loss components ( $P_s$ ) and rolling power loss components ( $P_R$ ).

The load independent power loss has got the contribution from gear windage loss ( $P_w$ ), oil churning loss ( $P_c$ ), load

independent bearing loss ( $P_{b,no\ load}$ ) and seal loss ( $P_{seal}$ ). Thus the total power loss is restated as

$$P_{Total} = (P_{Mesh} + P_{b,load}) + (P_w + P_c + P_{b,no\ load} + P_{seal}) \quad (2)$$

where,

$$P_{Mesh} = P_s + P_R \quad (3)$$

As the load acting on the gear tooth varies continuously during a mesh cycle, the load sharing based load dependent power loss calculation has to be done to estimate the gear drive efficiency to a reasonable accuracy. The load sharing ratio by a pair in the point  $i$  ( $(LSR)_i$ ) is evaluated using multi pair contact model(MPCM)[24].

#### A. Sliding Power Loss Calculation

1) *Sliding Force*: Sliding frictional force is recognized as one of the vital sources of power loss in gear drive at any contact point ( $i$ ), which is a function of normal load ( $F_n$ ) at that contact point and it is given as

$$(F_s)_i = \mu_i (LSR)_i (F_n)_i \quad (4)$$

where  $\mu_i$  is the instantaneous coefficient of friction.

A large number of empirical formulae found in the literature to determine  $\mu_i$  were obtained by curve fitting of measured data collected from twin-disk type tests. As the selection of model that calculates  $\mu_i$  can significantly affects the system losses, a suitable model has to be assumed. The formula developed by Benedict and Kelley [13] is conveniently modified in this work to incorporate the load sharing effect and the modified equation is given as

$$\mu_i = 0.0127 \log_{10} \frac{29.66 (LSR)_i (F_n)_i}{v_o (v_s)_i (v_T)_i^2} \quad (5)$$

where,

$v_o$  is the absolute viscosity in cPs and the rolling and sliding velocities are respectively given by  $v_T$  and  $v_S$  in  $m/sec$  as

$$(v_T)_i = [(v_p)_i + (v_g)_i] \quad (6)$$

$$(v_s)_i = [(v_p)_i - (v_g)_i] \quad (7)$$

where  $(v_p)_i$  and  $(v_g)_i$  are the sliding velocity of pinion and gear respectively at the contact point  $i$ . The Eq. 5 shows higher values near the pitch point for  $\mu_i$ , but it is experimentally proved by Xu (2005) [25] that this value is zero at pitch point (Figure 1), where slide to roll ratio (SR) is zero.

The model proposed by Xu et al. [1] includes the key parameters like sliding velocity, contact pressure, the surface roughness, lubricant dynamic viscosity, radius of curvature and entrainment velocity that are influencing friction between the contact surfaces of the gear and it is given as

$$\mu_i = e^{f(SR_i, (P_H)_i, v_o, s)} (P_H)_i^{b_2} |SR_i|^{b_3} (V_e)_i^{b_6} v_o^{b_7} R_i^{b_8} \quad (8)$$

in which

$$f(SR_i, (P_H)_i, v_o, s) = \left( \frac{b_1 + b_4 |SR_i| (P_H)_i \text{Log}(v_o) + b_5 e^{-|SR_i| (P_H)_i \text{Log}(v_o)} + b_9 e^s}{b_5 e^{-|SR_i| (P_H)_i \text{Log}(v_o)} + b_9 e^s} \right) \quad (9)$$

and  $P_H$  is maximum Hertzian contact pressure, which is evaluated by using MPCM in this work.

The entraining velocity  $(v_e)_i$  in  $m/sec$  is given by

$$(v_e)_i = \frac{1}{2} [(v_p)_i + (v_g)_i] \quad (10)$$

The slide- to -roll ratio (SR) is given by

$$(SR)_i = \frac{(v_s)_i}{(v_e)_i} \quad (11)$$

in which

$s$  is the RMS composite surface roughness in  $\mu m$  and  $b_i = - 8.92, 1.03, 1.04, - 0.35, 2.81, - 0.10, 0.75, - 0.39,$  and  $0.62$  for  $i = 1$  to  $9$ , respectively.

The coefficient of friction as per Eqs. 5 and 8 simulated along the path of contact for NCR gear using MATLAB for an input speed ( $N_{in}$ ) as 1500 rpm, power ( $P_{in}$ ) as 2.5 kW and for the lubricants of specified properties (Table I) is plotted in Figure 1 for comparison.

TABLE I. PROPERTIES OF THE LUBRICANT ([25])

| Properties                              | Value                   |
|---|-------------------------|
| Inlet temperature (K)                   | 373.15                  |
| Viscosity (Pa s)                        | 0.0065                  |
| Density ( $kg/m^3$ )                    | 813                     |
| Pressure viscosity coefficient (1/Pa)   | $1.2773 \times 10^{-8}$ |
| Temperature viscosity coefficient (1/K) | 0.0217                  |
| Thermal conductivity (W/mK)             | 0.1176                  |
| Coefficient of thermal expansion (1/K)  | $6.53 \times 10^{-4}$   |
| Specific heat (J/kgK)                   | 2000                    |

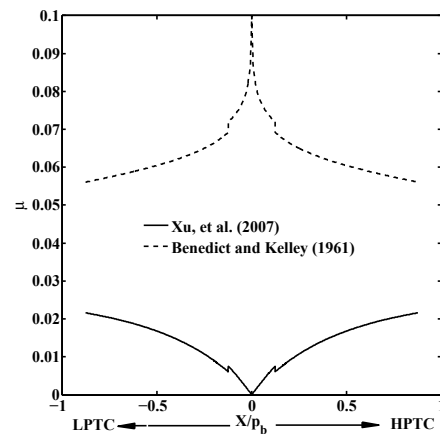
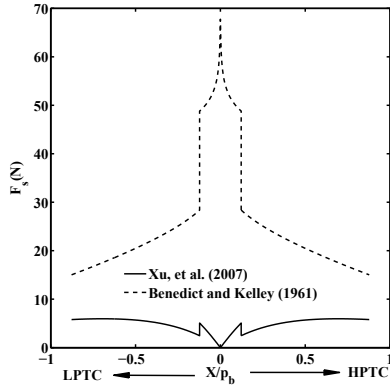


Fig. 1. Comparison of friction models (number of teeth=50, module=1, pressure angle =20°, cutter tip radius=0.3m, backup ratio=2.2, gear ratio=1 and contact ratio=1.755)

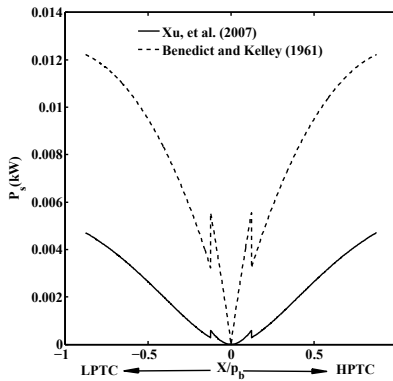
2) *Sliding Power Loss*: The instantaneous sliding power loss in kW is given by

$$(P_s)_i = 10^{-3}(F_S)_i(v_S)_i \quad (12)$$

The instantaneous sliding force and power loss calculated through the respective formulae are shown in Figures 2(a) and 2(b). It is observed that the model proposed by Benedict and Kelly [13] predicts more sliding frictional force at pitch point, where there is no sliding which is unreasonable. However, it correctly predicts zero power loss at this point.



(a) Sliding force



(b) Sliding power loss

Fig. 2. Sliding frictional force and power loss

### B. Rolling Power Loss in NCR Gears

1) *Rolling force ( $F_R$ )*: As the EHL film thickness ( $h_L$ ) developed between the gear teeth surfaces at any contact point in a mesh cycles controls the rolling force ( $F_R$ ), the  $h_L$  and  $F_R$  are calculated at all contact points along the path of contact using the formula referred by Anderson and Loewenthal [18]. The instantaneous rolling force at any point  $i$  due to build up of EHL film is given by

$$\text{Rolling force}(F_R)_i = 9X10^7(\phi_t)_i(h_L)_i b \quad (13)$$

where  $\phi_t$  is the thermal reduction factor to account for the effect of temperature rise at high speed conditions and  $b$  is

the face width of the gear tooth the film thickness ( $h_L$ ) used in the Eq.13 is expressed as

$$(h_L)_i = 2.69U_i^{0.67}G^{0.53}W_i^{-0.067}(1 - 0.61e^{-0.73K_i})R_i \quad (14)$$

where,  $R_i$  is the effective radius in the direction of rolling and  $K_i$  is the ellipticity parameter

$U$  is the dimensionless speed parameter and it is given as

$$U_i = \frac{(v_e)_i v_o}{E'R_i} \quad (15)$$

in which,

$$E' = \frac{2}{\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}\right)} \quad (16)$$

the material parameter( $G$ ) is expressed as a product of  $E'$  and the pressure viscosity coefficient ( $\alpha$ )

$$G = E'\alpha \quad (17)$$

the load parameter  $W_i$  is given as

$$W_i = \frac{F}{E'R_i^2} \quad (18)$$

The calculated instantaneous EHL film thickness is plotted in Figure 3. An abrupt drop in the lubricant film thickness observed near the pitch point of the NCR gear is due to a sudden increase in the load at this region.

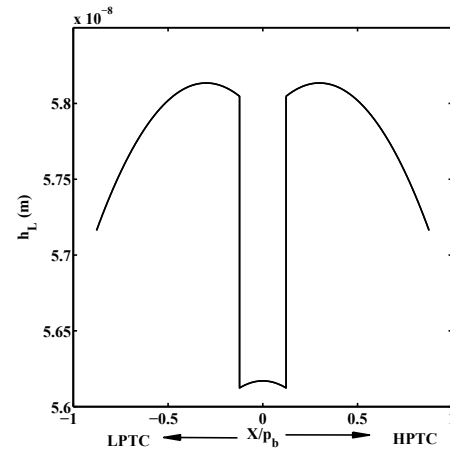


Fig. 3. EHL lubricant film thickness in NCR gears

2) *Rolling Power Loss*: The instantaneous rolling power loss in kW is given by

$$(P_R)_i = 10^{-3}(F_R)_i(v_T)_i \quad (19)$$

The calculated rolling force and the power loss are shown in Figure 4

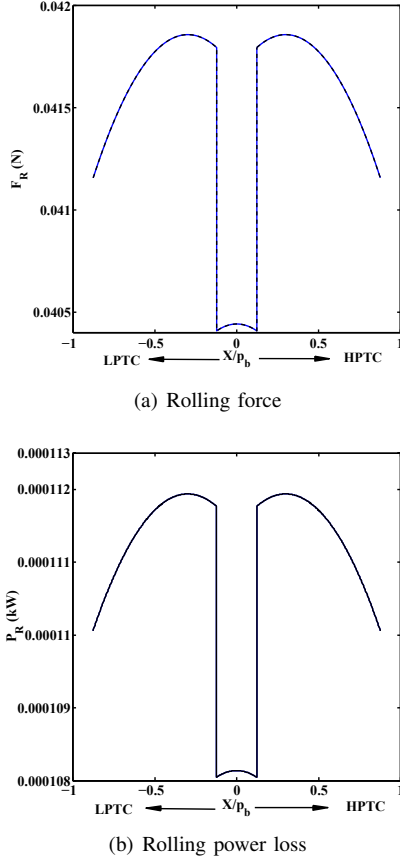


Fig. 4. Rolling force and power loss

### C. Total Mesh Power Losses

The total instantaneous tooth mesh loss at any contact point on the path of contact is the summation of the rolling and sliding power losses at that particular instant. As contact alternates between double pairs and single pair, in NCR gears, the contribution from both teeth pairs, which are in simultaneous contact, has to be considered for calculating the total mesh power losses along the path of contact. The total instantaneous power loss  $((TP_{int})_i)$  for a teeth pair is given as

$$(TP_{int})_i = (P_s)_i + (P_R)_i \quad (20)$$

Total power losses (TP) due to sliding and rolling for NCR gear for the simultaneous contact is given as

$$TP = \left( \begin{array}{l} 2 \sum_{A'}^{B'} (TP_{int})_i dx + \sum_{B'}^{C'} (TP_{int})_i dx + \\ 2 \sum_{C'}^{D'} (TP_{int})_i dx \end{array} \right) \quad (21)$$

The contact points A' to D' along the path contact are given in [24]

and same for HCR gear is given as

$$TP = \left( \begin{array}{l} 3 \sum_{A}^{B} (TP_{int})_i dx + 2 \sum_{B}^{C} (TP_{int})_i dx + \\ 3 \sum_{C}^{D} (TP_{int})_i dx + 2 \sum_{D}^{E} (TP_{int})_i dx + \\ 3 \sum_{E}^{F} (TP_{int})_i dx \end{array} \right) \quad (22)$$

The contact points A to F along the path contact are given in [24]

Thus, the average total power loss for a mesh cycle is given as

$$(TP)_{ave} = \frac{TP}{A'D'} \quad \text{for NCR gear} \quad (23)$$

$$(TP)_{ave} = \frac{TP}{AF} \quad \text{for HCR gear} \quad (24)$$

where A'D' and AF are the length of path of contact of NCR and HCR gear respectively

The efficiency is given by

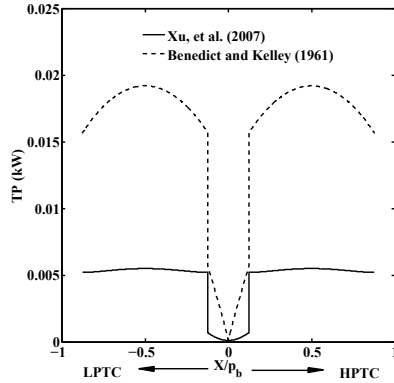
$$\eta = 100 \left( \frac{P_{in} - (TP)_{ave}}{P_{in}} \right) \quad (25)$$

The instantaneous total power loss and efficiency calculated at every discrete point along the path of contact are shown in Figure 5. It is observed that the Benedict and Kelly [13] model predicts more power loss when compared to that of Xu et al.[1]. The Xu et al.[1] model has been used in this work for further parametric analysis on HCR gear drives.

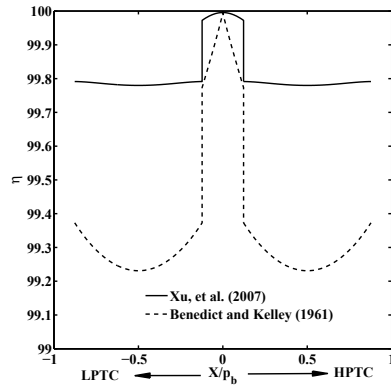
### III. EFFECT OF ADDENDUM

The instantaneous values of  $\mu$  and  $h_L$  obtained for two different values of addendum (1m for NCR gear and 1.29m of HCR gear) are shown in Figure 6. Simulated results show higher  $\mu$  at the tip and root for HCR gear pair because of higher sliding velocities at these points due to enlarged addendum. There is a sudden rise in the value of  $\mu$  at highest point of single tooth contact (HPSTC) and lowest point of single tooth contact (LPSTC) due to single pair contact for NCR gear pair. The  $h_L$  is less in single pair contact region due to high load acting at this region for NCR gear. Whereas,  $h_L$  is more near the pitch point region due to the reduced shared load caused by triple pair contact for HCR gear pair.

The sliding and rolling forces(Figure 7(a) to 7(b)), respective power losses(Figure 8(a) to 8(b)), total power loss and efficiency(Figure 9 to 10) are compared between NCR and HCR gears. It is observed that the total power loss is higher in HCR gear drive compared to that of NCR gear. This is attributed to the increase in mesh duration for HCR gear drive compared to NCR gear drive that ultimately increases the power loss. The total power loss is the area under the total power loss curve and this has been calculated using Simpson's one third rule. The average total power losses calculated for all the six cases (addendum =1m,1.093m,1.125m,1.158m,1.223m,1.291m) are shown in Figure 11. From the results, it is observed that an increase in



(a) Total mesh power loss



(b) Efficiency

Fig. 5. Instantaneous total mesh power loss and efficiency.

the addendum always increases both the sliding and rolling power losses, but the amount of increase is more in sliding power loss when compared to rolling power loss.

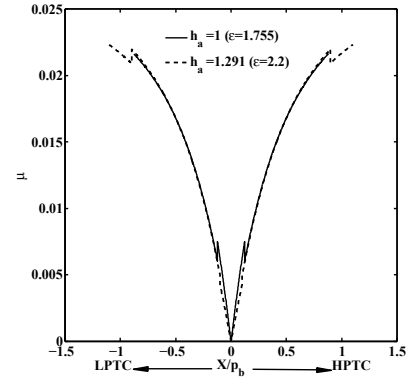
#### IV. CONCLUSIONS

Analytical study on instantaneous power loss in NCR and HCR gear drives and the effects of gear parameters on load dependent total mesh power loss on HCR gear drives have been studied and discussed in this work. The simulated results reveal that an increase in addendum to increase the contact ratio always increases the power loss in a similar way a decrease in the pressure angle to increase the contact ratio also increases the power loss.

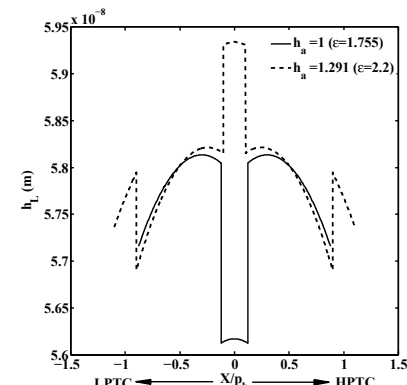
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(a) Coefficient of friction



(b) EHL film thickness

Fig. 6. Coefficient of friction and film thickness for NCR and HCR gear drive

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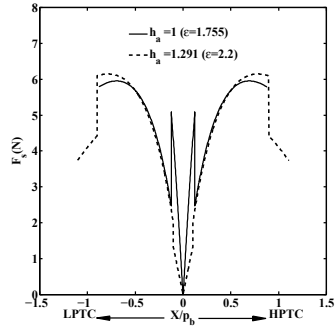
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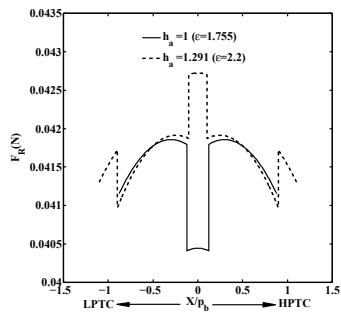
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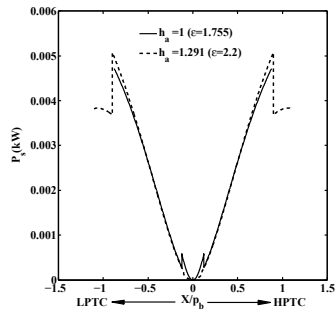


(a) Sliding frictional force

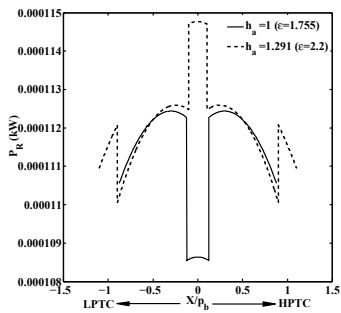


(b) Rolling force

Fig. 7. Sliding and Rolling force



(a) Sliding power loss



(b) Rolling power loss

Fig. 8. Sliding and Rolling power loss

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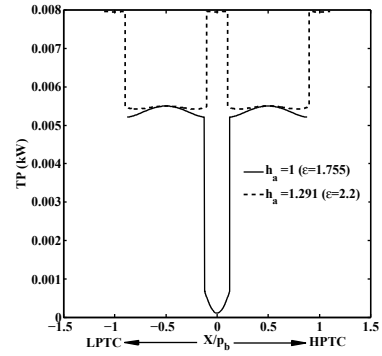


Fig. 9. Total mesh power loss

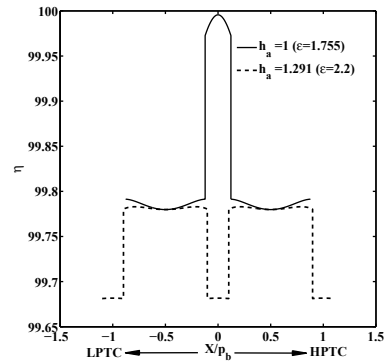


Fig. 10. Efficiency

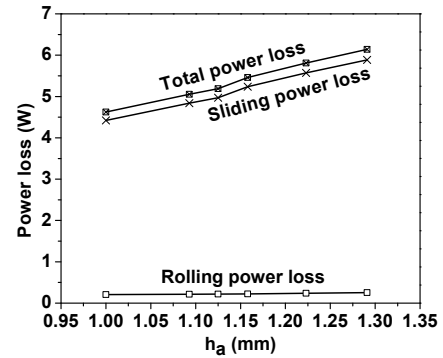


Fig. 11. Power loss with respect to addendum.

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