Determination of Stress Intensity Factor and Interaction Behaviour of Opposite Surface Cracks in a Rectangular Bar

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Abstract*— Multiple cracks emanating from fasteners and rectangular plates represent one of the most common fracture sources in aircraft structures and automobiles. Due to complexities in analysis of multiple cracks, many researchers have studied the damage tolerance analysis of single crack in rectangular plate, notched and un-notched round bar and thus the effect of crack interaction has been neglected. This paper present the SIF and interaction behavior of multiple cracks which are located in opposite direction in a rectangular bar subjected to far filed tension loading. Two surface cracks of same dimensions were introduced in the opposite sides of a rectangular bar. The crack depth ratio (a/t) ranging between 0.1 and 0.4 was considered with two different crack aspect ratios (a/c = 0.6 and 1.0). The crack tip was meshed with iso-parametric singular elements to determine the SIF values. The SIF values of single and multiple cracks are compared to determine the interaction behaviour of opposite cracks on SIF. Higher SIF values are observed at the crack surface region for a circular crack (a/c=1.0) at lower crack depths ((a/t) < 0.2 whereas the SIF values are higher at the crack middle region for an elliptic crack (a/c = 0.6) irrespective of the crack depth ratios. It is observed that, SIF values are not symmetric with crack middle region at higher depths due to additional effect of mode II and mode III fracture. It is also noted that the interaction effect of the growing cracks is more significant at higher crack depths. The effect is more significant at the crack middle region compared to crack surface region. The SIF values at the middle region of an elliptic crack are higher than circular crack irrespective of crack depth ratios considered in the present study.

***Keywords*— Iso-parametric element, interaction effect, crack aspect ratio**

I. INTRODUCTION

In general engineering structures such as, pressure vessels, aerospace components and the petro-chemical industries, various kinds of cracks occur during service life because of fatigue loading, welding process, corrosion, or built in material defects. Most of these cracks propagate into three-dimensional cracks and it may be simplified into 3-D planar cracks for the purpose of design and analysis. Usually, for convenience, these 3D cracks are simplified into semi circular (a/c = 1.0), semi elliptical (a/c = 0.2) and straight cracks. The crack tip stresses are determined by Stress Intensity Factor (SIF). Because of the complexity of elastic 3D cracked bodies, analytical solution is possible only for single embedded circular and elliptical cracks subjected to simple far field tensile loading in the elastic range. For a single and surface crack no closed-form solution is available, though many semi-analytical solutions have emerged.

A number of SIF solutions have been developed for both semi elliptical (a/c = 0.2; where ‘a’ crack length and ‘c’ semi major axis of the ellipse) crack front and semi-circular (a/c = 1) crack front in notched and un-notched round bars subjected to both tension and bending loading [1-3]. Generally, a surface crack has been approximated by a semi-circular, semi-elliptical or straight edge crack in a bar. The geometry correction factor (Y) used in SIF calculations ($Y = K/\sigma\sqrt{\pi a}$) depends on the crack depth ratio (a/d), crack aspect ratio (a/c) and the location of the point considered along the crack front (P/P_0). SIF for three dimensional cracked bodies have been evaluated by theoretical, numerical and experimental methods. While majority of the studies consider through-the-thickness cracks, the SIF problem becomes complex, when there is a surface crack. Raju and Newman [4] developed the SIF equation for a semi elliptical surface crack in a plate from 3-D finite element analysis and investigated the variation of SIF at the surface region (P/P_0= -1,+1) of the crack. Shih and Chen [5] studied the SIF for a semi-elliptic surface crack in a round bar and observed that, the SIF increased with an increase in crack depth ratio (a/d), and the SIF values decreased with increase of aspect ratio (a/c). A three parameter (‘a’, ‘c’ and‘d’ and location of point along crack profile) relationship for SIF was obtained by numerical curve fitting method. It is noted that the relationship is limited to mode-I fracture. Cai and Shin [6] obtained negative SIF values when they used the above three parameter approach, possibly due to errors in selection of order of polynomials. Couroneau and Royer [7] and Carpinteri [8] determined the SIF for a surface crack in a round bar subjected to constant amplitude tension cyclic load to understand the changeover in crack profile during fatigue loading and observed a significant influence of crack aspect ratio on crack front evolution. Several numerical simulations have been carried out to estimate SIF values for a notched round bar. Toribio et al. [9] numerically estimated the mode-I SIF for a cracked bolt subjected to tension and bending loads. Noda and Takase [10] estimated the SIF values for a V-notched round bar for various shaped notches. The estimation is independent of geometric parameters such as, stress concentration factor. Guo et al. [11] determined the SIF values for a semi elliptical surface crack in a notched tensile round bar for different notch shapes and stress concentration factors. Their analysis showed that the SIF values are strongly dependent on the stress concentration factor, and not on the shape of the notch geometry (U, V etc). Several researchers have determined SIF for a crack located in a notched and un-notched round bar. Available SIF solution of multiple cracks in a rectangular bar is rare and many of the researchers have considered the single crack model to estimate the SIF values. The prime objectives of the present paper are

1. Determination of SIF for a single and multiple cracks in a rectangular bar located in opposite direction and subjected to far field tensile loading
2. Analyze the effect of crack depth ratio (a/d) and aspect ratios (a/c) on SIF of multiple cracks in rectangular bar.
3. Determination of interaction behaviour of opposite cracks located in a rectangular bar.

2.0 DESCRIPTION OF THE FINITE ELEMENT MODEL

The numerical determination of SIF for cracks located in opposite direction in a rectangular bar subjected to far field loading condition was carried out using ABAQUS Finite Element software. A 3-D rectangular bar with surface cracks (Fig. 1a) of same size located in opposite direction was considered for the analysis. The dimensions of the rectangular bar are listed below. Two aspect ratios (a/c) and four crack depth ratios (a/d) were considered in the present work to understand the effect of these variables on SIF. The meshed model of the rectangular bar is shown in Fig.1b. Semi minor and major axis ‘a’ and ‘c’ correspond to the crack depth and transverse length of the crack.

Length of the bar : 20 mm
Width of the bar : 5 mm
Thickness of the bar : 5 mm
Aspect ratio (a/c) : 0.6, and 1.0
Crack depth ratio (a/t) : 0.1, 0.2, 0.3, and 0.4

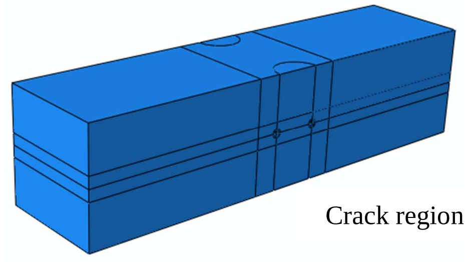


Fig. 1a. FE model of the multiple cracked rectangular bar

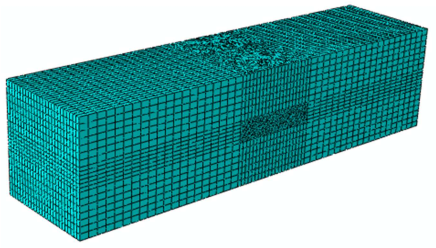


Fig. 1b. Meshed model of the rectangular bar with multiple cracks

2.1 Crack modeling

3-D meshed model of the rectangular bar with multiple cracks is shown in Fig.1b. The region around the crack front was partitioned from the rectangular bar to apply fine mesh around it (Fig.2a). In the present work, the “Contour integral evaluation” approach was used to determine the SIF values around the crack region. The onset of cracking in quasi-static problems can be studied using contour integral evaluation method; however, contour integral estimation does not predict the crack growth. Each contour is a ring of elements surrounding the crack front. Many contour integral evaluations are possible at each location along the crack front whereas, in the present work three contours were created around the crack front as shown in Fig.2b.

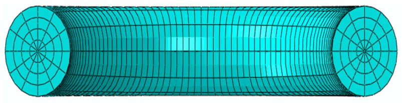


Fig. 2a. Meshed crack region

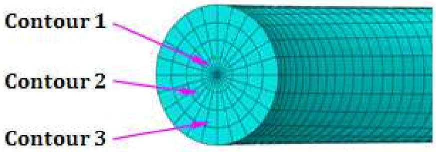


Fig. 2b. Contours around crack region

2.2 Specification of crack front and crack extension direction

Figure 3a shows the rectangular bar subjected to multiple cracks located in opposite direction. The crack front is the region which defines the first contour. The crack front in the rectangular bar can be considered to be equivalent to the crack line in three dimensional problems. The direction of the virtual crack extension can be specified at each node along the crack front by specifying either the normal to the crack plane (n) or by specifying the crack extension direction. Since the present problem is 3-D in nature the crack propagation direction can not be predicted and thus, normal to the crack plane approach was used to define the crack extension direction. In most of the fracture problems, the singularity at the crack tip should be considered for small strain analysis since it improves the accuracy of SIF values. To obtain a square root singularity, collapsed second order elements are used. To create a 3D crack tip singularity, 20-node brick and 27-node brick elements can be used with a collapsed face. If all the midface nodes are moved to their quarter points closest to the crack line, $\frac{1}{\sqrt{r}}$ singularity can be modeled. In the present work several contours have been considered around the crack front region and the top surface of the bar is constrained for all degrees of freedom and a tensile load is applied at the bottom surface as shown in Fig. 3b.

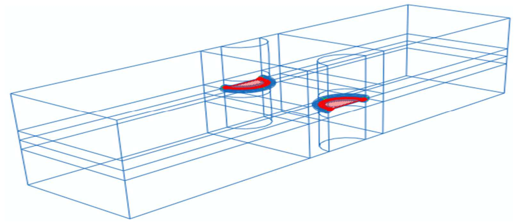


Fig. 3a. FE model with multiple cracks

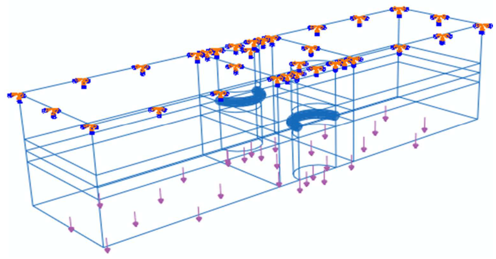


Fig. 3b. FE model with boundary conditions

3.0 RESULTS AND DISCUSSION

SIF estimation of a rectangular bar subjected to multiple cracks located in opposite direction has been carried out using 3-D finite element analysis. Figure 4 shows a good agreement between present SIF values with results of Raju and Newman [4]. The available SIF solution of Raju and Newman is limited to single crack model subjected to far field tension loading condition whereas, in the present work SIF solution multiple cracks located in opposite direction has been considered.

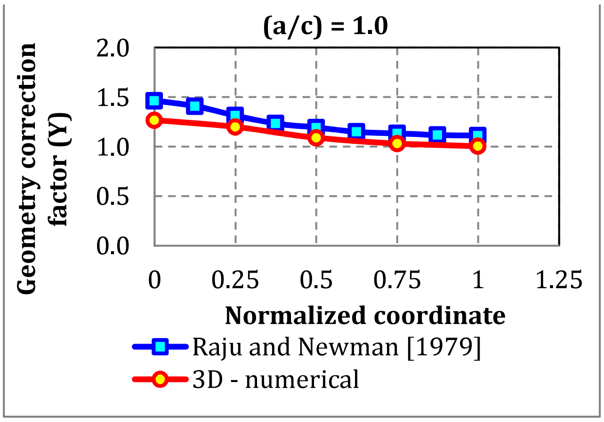
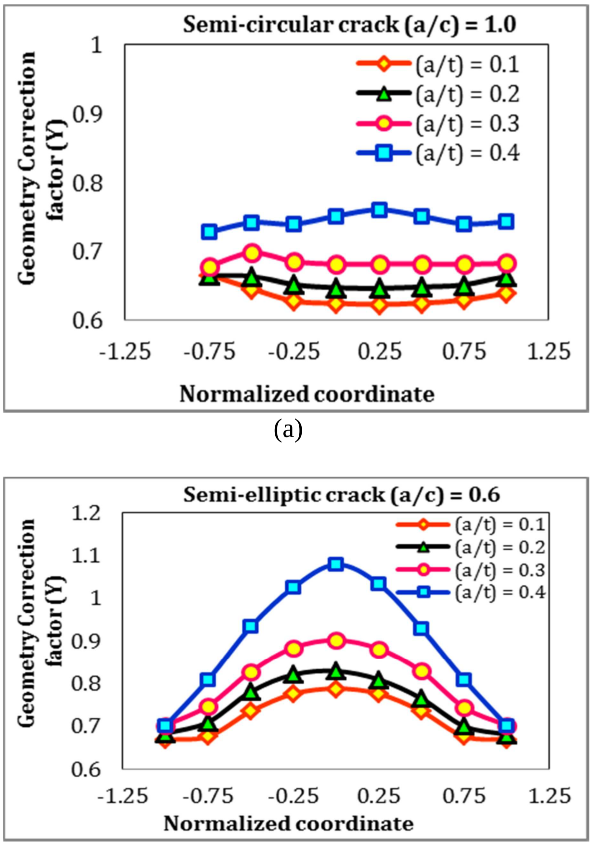


Fig. 4. Variation of geometry correction factor

3.1 Effect of Crack Depth Ratio (a/d)

Figure 5 shows the variation of SIF for a semi-circular crack (Fig.5a) and semi-elliptic (Fig.5b) for different crack depth ratios (a/t) ranging between 0.6 and 1.0. The SIF values of circular crack are higher at the crack surface region compared to middle region at short crack depths (a/t < 0.3) as shown in Fig. 5a. Normal coordinate system is used to represent the points along the crack front. (P/P_0 = -1) refers to the left edge of the crack front whereas (P/P_0 = +1) refers to right edge of the crack. (P/P_0 = 0) refers to middle region of the crack. Higher SIF values were observed at the crack front middle region for an elliptic crack (Fig. 5b) irrespective of the crack depth ratios considered. Thus one can expect higher crack growth rate at the middle region as the crack depth ratio increases. Non symmetric SIF distribution is observed for a semi-circular crack. The non-symmetric distribution is due to the interaction effect of the growing crack with the crack depth ratio which increases the additional effect of mode-II and mode –III fracture.



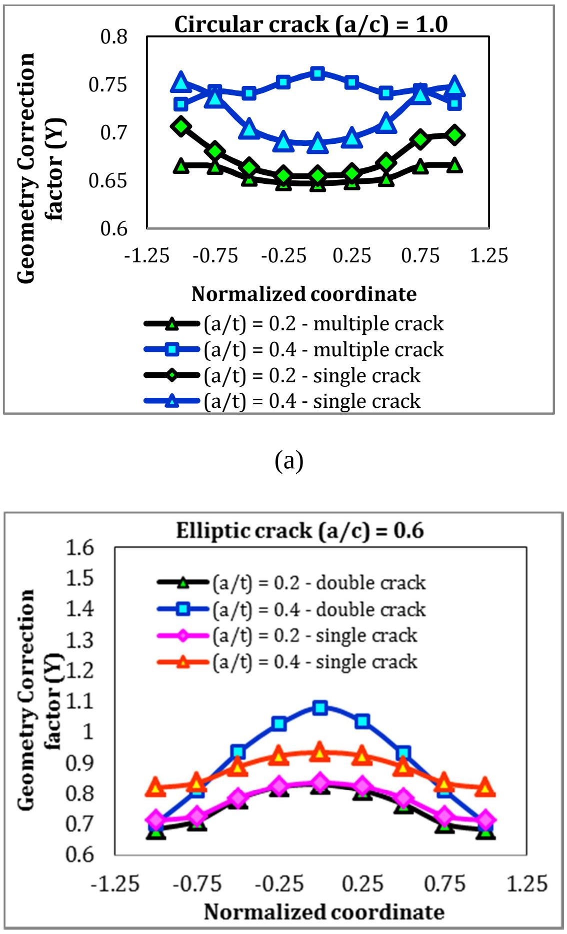
(a)

(b)

Fig.5. Effect of crack depth ratio on SIF

3.2 Effect of multiple cracks

Figure 6 shows the variation of SIF for multiple semi-circular (Fig. 6a) and semi-elliptic (Fig. 6b) cracks located in opposite direction in a rectangular bar. Two different crack depth ratios were considered in the range of 0.6 and 1.0. It is observed that the effect of additional crack is more pronounced at the middle region of the crack compared to surface region. As the crack depth ratio increases, SIF values are higher at the middle region at higher crack depths. At lower crack depths, the effect of additional crack on SIF is higher for circular crack geometry. Thus one can expect higher crack growth rate at the middle region of a multiple circular crack.



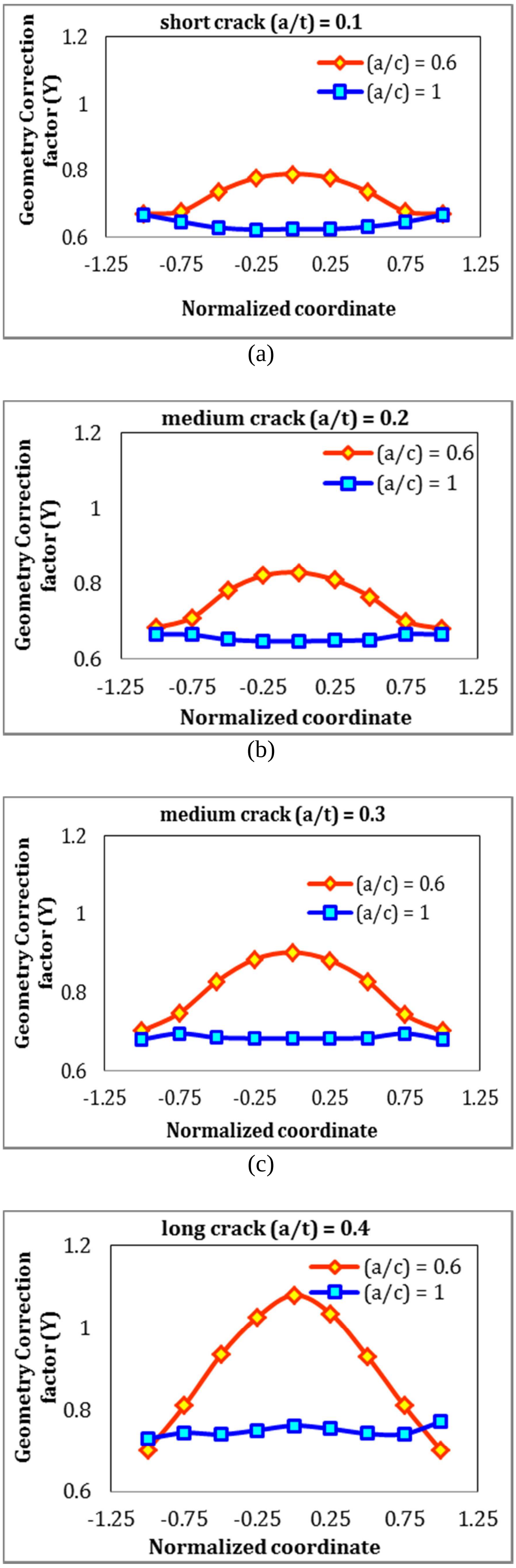
(a)

(b)

Fig.6. SIF variation for multiple cracks

3.3 Effect of crack aspect ratio

Figure7 shows the effect of crack aspect ratio (a/c) on SIF for a rectangular bar subjected to far-field loading condition. Aspect ratios of 0.6 and 1.0 were considered in the present work and the SIF values were measured along the crack front as shown in the Fig. 7. SIF values of elliptic crack are higher than circular crack (Fig. 7a to Fig. 7d) for the range of crack depth and aspect ratios considered in the present work. The effect of aspect ratio is higher at the middle region of the crack front irrespective of the crack depth ratios considered and the effect increases with increasing crack depth ratios. Higher values of SIF for an elliptic crack suggest higher crack growth at the middle region of the crack.



(a)

(b)

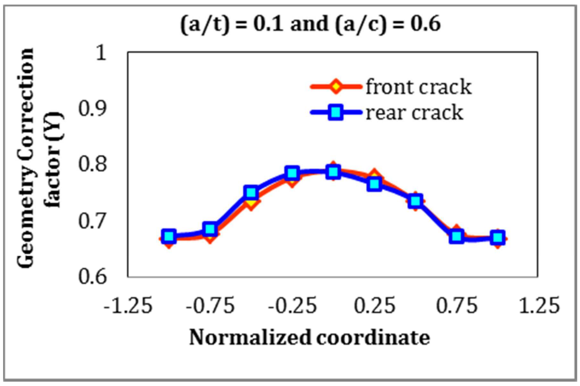
(c)

(d)

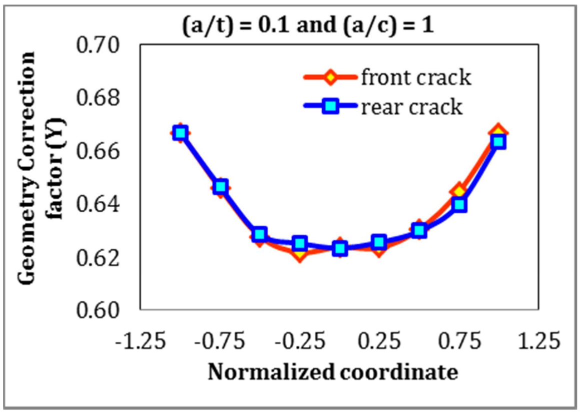
Fig.7. Effect of crack aspect ratio on SIF

3.4 Effect of crack location in a rectangular bar

Figure 8 shows the comparison of SIF values for the front and rear sides of the multiple surface cracks in a rectangular bar. It is observed that SIF values at the middle region are independent of number of cracks as shown in Figures 8a and 8b. The interaction of surface crack increases with crack depth ratio at the surface region. Higher SIF values are observed for multiple cracks in a rectangular bar compared to single cracked bar. As the number of cracks in a bar reduces the stiffness, SIF values are observed to be higher for a member with multiple cracks.



(a)



(b)

Fig. 8. SIF variation along the crack front

3.5 Effect of crack interaction on SIF

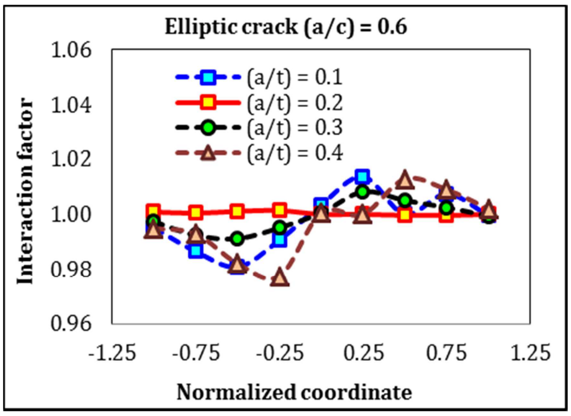
Figure 9 shows the interaction effect of opposite cracks in a rectangular plate subjected to semi- circular (Fig.9a) and semi-elliptic cracks (Fig.9b) for various crack depth ratios. The interaction factor is calculated by the relation

$$\text{Interaction factor} = K_{1,1} / K_{1,2} \qquad \text{----------------} \ (1)$$

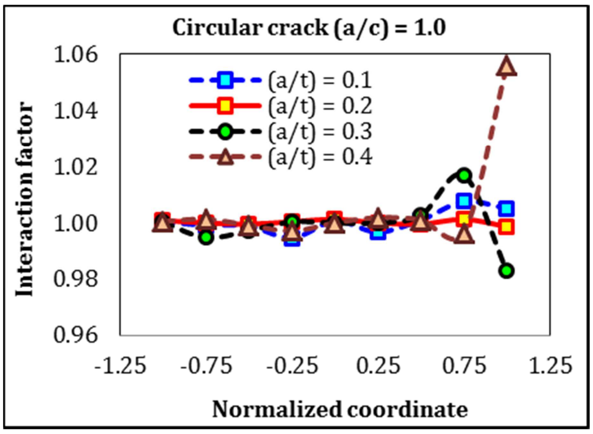
Where $K_{1,1}$ – mode I SIF for the front face crack

$K_{1,2}$ _ mode I SIF for the rear face crack

It is observed that the interaction effect is more significant at higher crack depth ratios.



(a)



(b)

Fig. 9. Variation of crack interaction factor

The interaction factors of semi- elliptic cracks for various crack depths are listed in Table.1. Table.2 provides the interaction factors for a semi-circular crack. Thus one can use the interaction table while determining the SIF for a multiple cracked bodies.

Table 1. Interaction factor for elliptic crack (a/c = 0.6)

Location (P/P_0)	(a/t)			
	0.1	0.2	0.3	0.4
-1	0.9951	1.0008	0.9974	0.9949
-0.75	0.9866	1.0004	0.9924	0.9929
-0.5	0.9809	1.0011	0.9912	0.9822
-0.25	0.9906	1.0014	0.9951	0.9774
0	1.0035	1.0000	1.0005	1.0006
0.25	1.0137	1.0000	1.0082	1.0000
0.5	1.0000	0.9996	1.0050	1.0128
0.75	1.0074	0.9996	1.0025	1.0089
1	0.9995	1.0000	0.9993	1.0020

Table 2. Interaction factor for an elliptic crack (a/c = 0.6)

Location (P/P_0)	(a/t)			
	0.1	0.2	0.3	0.4
-1	1.0004	1.0008	1.0003	1.0005
-0.75	0.9991	1.0000	0.9947	1.0013
-0.5	0.9989	0.9996	0.9973	0.9992
-0.25	0.9946	1.0004	1.0003	0.9971
0	1.0007	1.0013	1.0000	1.0000
0.25	0.9967	1.0000	1.0000	1.0019
0.5	1.0010	0.9996	1.0027	1.0011
0.75	1.0075	1.0013	1.0168	0.9965
1	1.0052	0.9987	0.9830	1.0560

4.0 CONCLUSIONS

SIF of multiple cracks located in opposite direction in a rectangular plate subjected to far field loading condition has been determined numerically. The comparison of SIF values for a single and multiple cracks reveals the following conclusions.

1. SIF values are higher for semi-elliptic crack than semi-circular crack irrespective of the crack depth ratios considered. Thus one can expect higher crack growth rate at the middle region of a semi-elliptic crack compared with semi-circular crack
2. As the aspect ratio increases SIF values decreases considerably along the crack front. This is due to curvature effect of the surface cracks with increasing aspect ratios
3. The interaction effect of opposite cracks is marginal at the middle region of the crack front whereas at the crack surface region the interaction effect is more significant.
4. The effect of interaction effect increases with crack depth ratio and it is higher for the deep cracks.

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