Estimation of States of Seeker System of a Missile using Sliding Mode Observer and Kalman filter approaches - A Comparative Study

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***Abstract*— This paper presents a comprehensive comparison of estimation of states for seeker system of a missile using Sliding Mode Observer and Kalman filter approaches. This estimation accounts for the angular position and displacement rates of the seeker pitch and yaw gimbals. These seeker body rates are required to estimate the line of sight (LOS) rate to determine target position. Both estimation approaches are simulated and a comparative analysis is presented. The peculiar advantages of the two methodologies; i.e. parametric robustness of SMO and noise damping of Kalman filter, have been validated in simulation.**

Keywords – Seeker System; Sliding Mode Observer; Extended Kalman Filter

I. Introduction

The seeker system of a missile works to detect the target by performing a scan of the area in the final phase of missile's launch and hit trajectory. This operation is practically realized with a two axis gimbal structure, which forms the skeleton of this seeker system. The sensor or the energy sensing device is the payload of this gimbal structure. The sensor pointing control is achieved with two DC motors which provide the actuating torque to the two channels of the gimbal structure, yaw and pitch. Yaw channel is the outer gimbal which provides movement in horizontal (azimuth) plane and pitch channel is the inner gimbal providing vertical movement (elevation) to the sensor [1]. The seeker has to generate a spiral scan movement to detect the target.

The performance of most modern control systems depends on assumption of near perfect feedback signals which are not available or are noisy signals. The system used for the purpose of providing an accurate estimate of state vector is called an observer. The observer theory was invented during 1960's by D.G.Luenberger. The observer also called Luenberger observer, was proposed to be a dynamic system which generates an approximate estimate of the state vector using the plant information i.e. the dynamics and inputs, outputs. Observers based on sliding mode approach were first developed by V. Utkin in 1978. Initially this concept was introduced for linear systems. Major work in the area of Sliding Mode Observer (SMO) appeared in 1980s by Slotine et al, Walcott et al etc. [2] [3] [4]. The SMO based on assumption of bounded uncertainties or non linearities, does not require its knowledge in designing the observer dynamics. Using this design concept, many researchers have implemented SMO for various systems [5] [6]. The SMO gain is designed by using pole placement approach to ensure stability of the observer system and asymptotic estimation error convergence. These observers provide the advantage of robustness, numerical stability and can be implemented for various plants.

In 1960, R.E. Kalman proposed the filter design for prediction, estimation problem, now popularly known as the Kalman filter [7]. A Kalman filter can be defined as a optimal recursive data processing algorithm. Kalman filter is characterized by accurate estimation of state variables under noisy condition, which makes it suitable for drives, robotic manipulators and other industrial applications. The algorithm is formulated in two steps which involve; prediction and updation. The complete Kalman filter dynamics can be found to be similar to a Luenberger observer, however the gain matrix is found via an optimal recursive algorithm [8]. The Kalman filter is known to be extremely robust to process noise and output noise but introduces additional computational complexity.

These Kalman filter and Sliding Mode Observer techniques have been implemented for the state estimation of seeker system for decoupled yaw and pitch channels. The mathematical modeling of the seeker system is done by classical first principle approach.

The mathematical modeling of the seeker system is described in Section II. A brief theoretical overview of the Kalman filter and Sliding Mode Observer is presented in Section III. Simulation results are presented in Section IV and Section V concludes this comparative study.

II. Modeling of Seeker System

The seeker system is mounted at the tip of the missile and consists of a two axes gimbal system as its core structure. The outer gimbal or the yaw, provides azimuth rotation of the payload or the sensing device. The inner gimbal or the pitch provides elevation. DC motors are used to provide actuating torque to yaw and pitch gimbals

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configuring the seeker. The gimbal control loop works in scan and track modes. In the scan mode, the seeker searches for target within the gimbals field of regard with a spiral search pattern. Once the target is detected and identified (through code matching), seeker locks on the target and tracks the target. In the track mode, seeker is driven by detection of exact location of target from scan mode, and current position of missile body. The error in the positions is continuously reduced to zero till the target is destroyed. The schematic block diagram for the seeker system is shown in Fig. 1

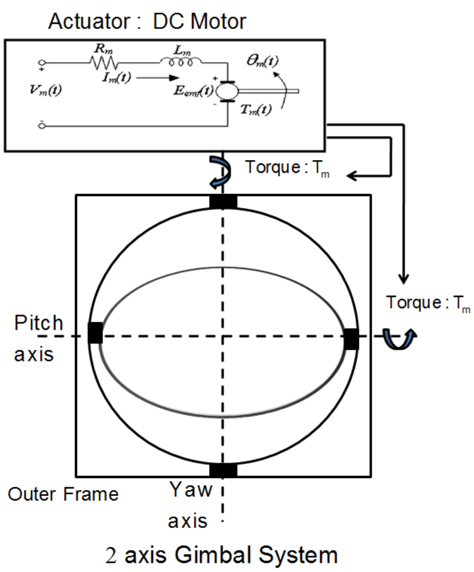

Fig. 1: Block Diagram Schematic

The model is represented by decoupled pitch and yaw channels, with interconnection via disturbance torques due to body rates and cross coupling. Here
V_m = DC input voltage in Volts,
I_m = DC current in Amps,
R_m and L_m = Equivalent resistance (Ω) and inductance (henry) of motor,
T_m = Equivalent motor torque in Nm,
θ_m = Motor angular displacement in degrees,
The roll, pitch and yaw components of angular velocities in body, pitch and yaw frames are given as:

$$\omega = \begin{bmatrix} \mathrm{p} \\ \mathrm{q} \\ \mathrm{r} \end{bmatrix}; \omega_y = \begin{bmatrix} \mathrm{p_k} \\ \mathrm{q_k} \\ \mathrm{r_k} \end{bmatrix}; \omega_p = \begin{bmatrix} \mathrm{p_a} \\ \mathrm{q_a} \\ \mathrm{r_a} \end{bmatrix} \tag{1}$$

Here p, q and r represent the roll, pitch and yaw components respectively. The inertia matrix of pitch gimbal is considered for roll, pitch and yaw rotations as,

$$J_A = \begin{bmatrix} J_{ax} & D_{xy} & D_{xz} \\ D_{xy} & J_{ay} & D_{yz} \\ D_{xz} & D_{yz} & J_{az} \end{bmatrix}. \tag{2}$$

The moments of inertia are denoted by J and products of inertia by D. The inertia matrix of yaw gimbal can be given as:

$$J_K = \begin{bmatrix} J_{kx} & d_{xy} & d_{xz} \\ d_{xy} & J_{ky} & d_{yz} \\ d_{xz} & d_{yz} & J_{kz} \end{bmatrix}. \tag{3}$$

The modeling of seeker system has been completely shown in [6] [9]. The pitch channel dynamics can be written as;

$$\ddot{v}_2 = 12.26V_m - 6.513\dot{v}_2 + 95.78(T_D - 3\times 10^{-4}\dot{q}_k), \tag{4}$$

where v_2 is the pitch channel angular displacement, T_D is the disturbance in pitch channel due to cross coupling with yaw channel and body rotations of missile system.

$$\begin{aligned} T_D = {} & (J_{az} - J_{ax})p_a r_a + D_{xz}(p_a^2 - r_a^2) \\ & -\mathrm{D}_{yz}(\dot{r}_a - p_a q_a) - D_{xy}(\dot{p}_a + q_a r_a). \end{aligned} \tag{5}$$

The parameters in the T_D expression are the components of inertia matrix and angular velocity corresponding to yaw and pitch gimbals [6]. The entire term $95.78(T_D - 3\times 10^{-4}\dot{q}_k)$ is defined as the disturbance torque in pitch channel T_{Dp}. Therefore, we get the pitch dynamics as,

$$\ddot{v}_2 = 12.26V_m - 6.513\dot{v}_2 + T_{Dp}. \tag{6}$$

Similarly, the yaw channel dynamics can be written as,

$$\begin{aligned} \ddot{v}_1 = {} & 12.03V_m - 6.415\dot{v}_2 + 93.98(T_d - (J_{ax}\sin^2 v_2 \\ & -J_{az}\cos^2 v_2 + D_{xz}\sin(2v_2))\dot{r}_k - 5\times 10^{-4}\dot{r}). \end{aligned} \tag{7}$$

where, $T_d = T_{d1} + T_{d2} + T_{d3}$,
$T_{d1} = [J_{kx} + J_{ax}\cos^2 v_2 + J_{az}\sin^2 v_2 + D_{xz}\sin(2v_2) - (J_{ky} + J_{ay})]p_k q_k$,
$T_{d2} = -[d_{xz} + (J_{az} - J_{ax})\sin v_2 \cos v_2 + D_{xz}\cos(2v_2)] \times (\dot{\mathrm{p}}_k - q_k r_k) - (d_{yz} + D_{yz}\cos v_2 - D_{xy}\sin v_2)(\dot{q}_k + p_k r_k) - (d_{xy} + D_{xy}\cos v_2 + D_{yz}\sin v_2)(p_k^2 - q_k^2)$
$T_{d3} = \ddot{v}_2(D_{xy}\sin v_2 - D_{yz}\cos v_2) + \dot{v}_2[(J_{ax} - J_{az})(p_k\cos(2v_2) - r_k\sin(2v_2)) + 2D_{xz}(p_k\sin(2v_2) + r_k\cos(2v_2)) + (D_{yz}\sin v_2 + D_{xy}\cos v_2)(q_a + q_k) - J_{ay}p_k]$.
Substituting disturbance as T_{Dy},

$$\ddot{v}_1 = 12.03V_m - 6.415\dot{v}_1 + T_{Dy}, \tag{8}$$

where v_1 is the yaw channel angular displacement. It is assumed that the nominal functions and disturbance terms are Lebesgue measurable and uniformly bounded.

Application of control techniques like State Feedback control, Sliding Mode Control etc. require the entire state vector which can be made available by use of estimation methodologies. These algorithms partially remove the necessity of sensors. The estimation techniques have particular features for robustness to disturbances and external noise. Two of such estimation approaches, Kalman filter and Sliding Mode observer, each with its peculiarities are applied for seeker system state estimation and their comparative study is presented. Such a comparison with these two methodologies is shown in [10] for an Induction Machine.

III. Estimation Methodologies

A brief overview of the Kalman filter and Sliding Mode Observer is presented here along with the design particulars for seeker system.

A. Sliding Mode Observer

Sliding Mode Observers (SMO) evolved as an application of Sliding Mode Concept for estimation purpose, imparting its inherent robustness properties. As with the control technique, the SMO has two design steps: stable surface consideration and design of observer gain. The signum function with appropriate gain ensures sliding on the manifold.

The state available for measurement in both channels is the angular displacement of the gimbals. The plant dynamics as described in (6) and (8), respectively for pitch and yaw, can be written in state space form as;

$$\left.\begin{array}{l}\dot{\mathbf{x}}_i = A_i\mathbf{x}_i + \mathbf{b}_i u_i + \mathbf{e}T_{Di}\\ y_i = C_i\mathbf{x}_i.\end{array}\right\} \tag{9}$$

Here $\mathbf{x}_i$ is the state vector $\in \Re^2$, A_i is state matrix $\in \Re^{2\times 2}$, $\mathbf{b}_i$ is the input matrix $\in \Re^{2\times 1}$ and C_i is the output matrix $\in \Re^{1\times 2}$. $e = [0 \ \ 1]^T$, which represents the disturbance term in the input channel. Suffix i (i= p and y) denotes both pitch and yaw dynamics respectively; both having similar configuration. Input $u_i \in \Re^m$ and output $y_i \in \Re^p$, where m is the number of inputs and p is the number of outputs, both are 1. The state vector has angular displacement and its velocity as its states.

The system is transformed using $z \to T_c\mathbf{x}_i$, so as to get $C_i = [0 \ \ I_p]$, where I_p is the identity matrix of order p. The system in regular form is given as,

$$\left.\begin{array}{l}\dot{z_1}(t) = A_{11}z_1(t) + A_{12}z_2(t) + b_1u_i(t) + e_1h\\ \dot{z_2}(t) = A_{21}z_1(t) + A_{22}z_2(t) + b_2u_i(t) + e_2h.\end{array}\right\} \tag{10}$$

h represents the lumped disturbance. Note that due to the transformation $y_i = z_2$. The state vector z is partitioned as $[z_1 \ \ z_2]^T$ such that $z_1 \in \Re^{n-p}$ and $z_2 \in \Re^p$. Sliding mode observer (SMO) dynamics [3] for the pitch and yaw channels separately can be written as,

$$\left.\begin{array}{l}\dot{\hat{z}}_1(t) = A_{11}\hat{z}_1(t) + A_{12}\hat{y}_i(t) + b_1u_i(t) + L_ik_i\text{sign}(s)\\ \dot{\hat{y}}_i(t) = A_{21}\hat{z}_1(t) + A_{22}\hat{y}(t) + b_2u_i(t) + k_i\text{sign}(s).\end{array}\right\} \tag{11}$$

where $(\hat{z_1},\hat{y_i})$ are the estimates of (z_1,y_i). Sliding surface s is defined as

$$s = y_i - \hat{y}_i = e_y. \tag{12}$$

The observer gain matrix $L_i \in \Re^{2\times 1}$. The positive scalar k_i is the tuning parameter and is chosen to ensure the existence of sliding. If the error between the estimates and the true states are written as e_{z1} and e_y then from (10) and (11) the following error dynamics are obtained,

$$\left.\begin{array}{l}\dot{e}_{z1}(t) = A_{11}e_{z1}(t) + A_{12}e_y(t) + e_1h - L_i\nu_i\\ \dot{e}_y(t) = A_{21}e_{z1}(t) + A_{22}e_y(t) - \nu_i.\end{array}\right\} \tag{13}$$

Discontinuous term $k_i\text{sign}(s)$ is denoted by ν_i. During sliding, $s = \dot{s} = 0 \Rightarrow e_y = \dot{e_y} = 0$, hence (13) becomes;

$$\dot{e}_{z1}(t) = (A_{11} - L_iA_{21})e_{z1}(t) + e_1h. \tag{14}$$

L_i is designed to ensure $(A_{11} - L_iA_{21})$ to be stable.
Stability Analysis
Consider a Lyapunov candidate function as

$$\upsilon(s) = \frac{1}{2}e_y^2 \tag{15}$$

To ensure sliding on the error manifold, $e_y\dot{e}_y \leq 0$;
$\Rightarrow e_y\dot{e}_y \leq -\eta|e_y|$
where η is a small positive constant.

$$\begin{array}{rcl} e_y\dot{e}_y & = & e_y(A_{21}e_{z1}(t) + A_{22}e_y(t) - k_i\text{sign}(s))\\ & \leq & -|e_y|(k_i - |A_{21}e_{z1}(t) + A_{22}e_y(t)|)\end{array}$$

If k_i is chosen such that $k_i > |A_{21}e_{z1}(t) + A_{22}e_y(t)| + \eta$; then it is ensured that $\Rightarrow e_y\dot{e}_y \leq -\eta|e_y|$.

B. Extended Kalman Filter

The Kalman filter is essentially a mathematical algorithm that implements a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance when some presumed conditions are met. The Kalman filter is named after Rudolph E. Kalman, who in 1960 published his paper describing a recursive solution to the discrete data linear filtering problem. In its original formulation, the state has been estimated at discrete points of time. Kalman filter is known to estimate the state vector with noisy process and sensor performances.

The Extended Kalman Filter (EKF) is a direct extension of standard Kalman filter to nonlinear systems. The Kalman gain is computed by linearizing the system dynamics. For the seeker plant under study, the EKF estimation approach is used for dynamic model in discrete form. As in EKF the nonlinearities can be considered; the system model is written in the following form;

$$\left.\begin{array}{l}\dot{\mathbf{x}}_i = A_i\mathbf{x}_i + \mathbf{b}_i u_i + \mathbf{e}f(x(t))\\ y_i = C_i\mathbf{x}_i.\end{array}\right\} \tag{16}$$

A_i is the system matrix in linearized form and f(x(k)) includes the nonlinear terms due to cross coupling and body rotation as written in (5) and (7). The system matrices are same as used for the SMO however in discrete form with the addition of the nonlinearities. The system can be written in discrete form as;

$$\left.\begin{array}{l}\mathbf{x(k+1)}_i = A_i\mathbf{x(k)}_i + \mathbf{b}_iu(k)_i + \mathbf{e}f(x(k))\\ y(k)_i = C_i\mathbf{x(k)}_i + \text{v}(k).\end{array}\right\} \tag{17}$$

State vector $x(k) = [v_2(k) \ \ v_2(\dot{k}+1)]^T$ and $[v_1(k) \ \ v_1(\dot{k}+1)]^T$, for pitch and yaw channels respectively. v(k) is the zero-mean Gaussian random vectors representing measurement noise. The system uncertainties can be considered as process noise. In this paper, EKF is designed for this non linear model for coupled pitch and yaw channels separately. The EKF algorithm is given in Appendix A.

IV. Simulation Results

The seeker system as modeled in (6) and (8) is controlled using a Sliding Mode Controller as described in [9] to obtain the desired spiral scan. To achieve this, the pitch and yaw channels of the gimbal are given sine and cosine signals of varying amplitude. This controlled system is analyzed for estimation using the two approaches - SMO and EKF. The controller is provided with the estimated state vector in both the methods. These estimation approaches are compared with respect to system conditions like parametric uncertainties, matched disturbances and sensor noise. The SMO gain matrix is computed using pole placement methodology [6]. The Kalman gain though computed recursively at each instant, requires accurate values of Q and R matrices. The numerical values of these matrices were chosen as per the reasonable level of process and measurement noise magnitude.

A. Parametric Uncertainties

The system parameters are subjected to $\pm 5\%$ uncertainties. The SMO being inherently robust to system uncertainties is expected to give good performance and the uncertainties in system are considered in process noise for EKF. Thus the Q matrix is adjusted to give optimum performance.

$$Q = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}; \quad R = 0.5$$

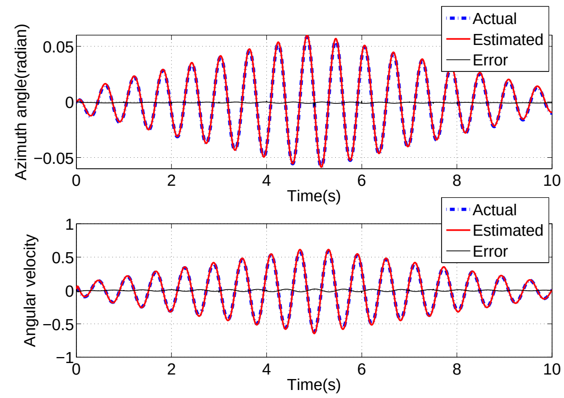

Fig. 3: EKF: Parametric Uncertainties; Estimation results for yaw channel

Fig. 2 and 3 depict the EKF estimations with parametric uncertain conditions. It can be seen that the estimation error does not converge to zero. The simulation results using SMO are shown in Fig. 4 and 5.

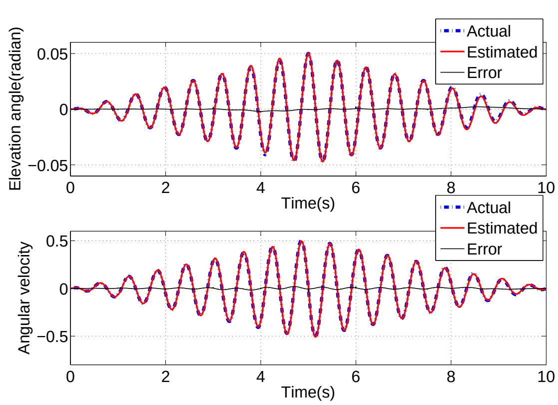

Fig. 2: EKF: Parametric Uncertainties; Estimation results for pitch channel

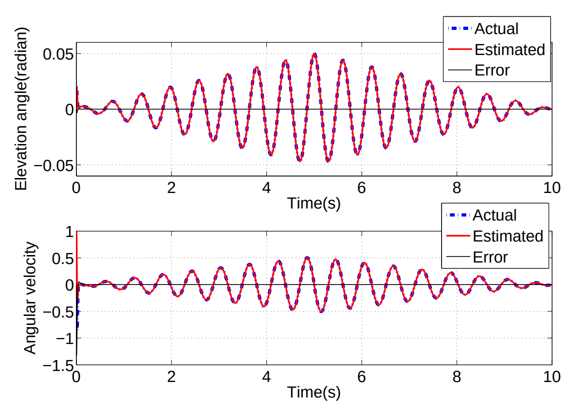

Fig. 4: SMO: Parametric Uncertainties; Estimation results for pitch channel

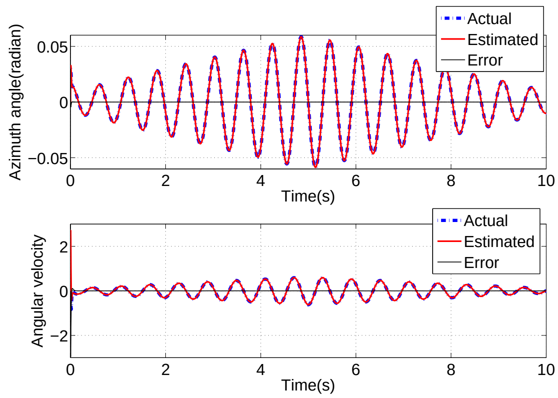

Fig. 5: SMO: Parametric Uncertainties; Estimation results for yaw channel

It can be observed from these figures, that the output estimation error converges to zero in finite time. This proves to be advantageous over Kalman filter results. It was also observed that on changing initial conditions of plant, the Kalman filter had its estimated signal shifted by the value of that initial condition. Hence the results of Kalman filter have been shown with initial conditions zero. Thus it can be seen that both estimation methods give satisfactory results with parametric uncertainties.

B. Matched Disturbance in Input Channel

$d = 0.1sin(2\pi 1.5)t$, a smooth, matched sinusoidal disturbance is added in the input channel of plant to analyze the performance of the estimation methodologies. To cater to the disturbance, the values of Q matrix in EKF is increased ($Q_{11} = Q_{22} = 80$). Figures 6 and 7 show the estimation performance for EKF and Fig. 8 and 9 for SMO.

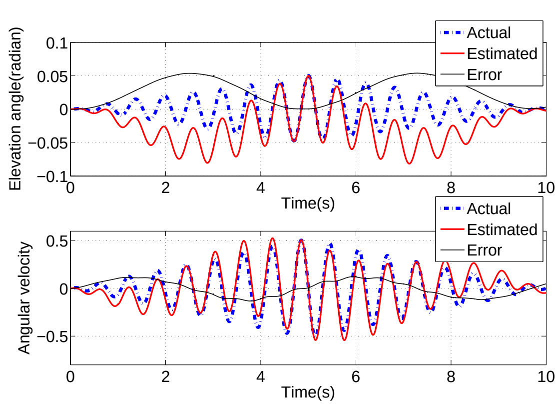

Fig. 6: EKF: Sinusoidal Disturbance; Estimation results for pitch channel

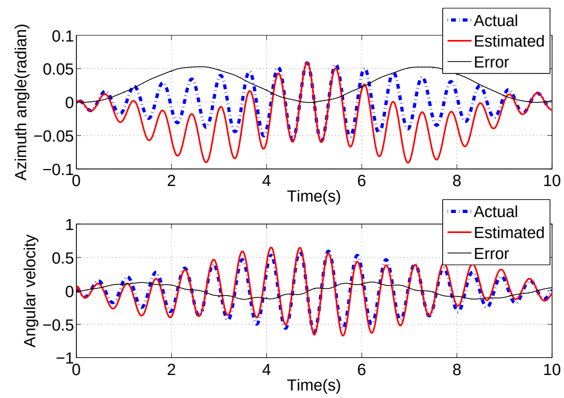

Fig. 7: EKF: Sinusoidal Disturbance; Estimation results for yaw channel

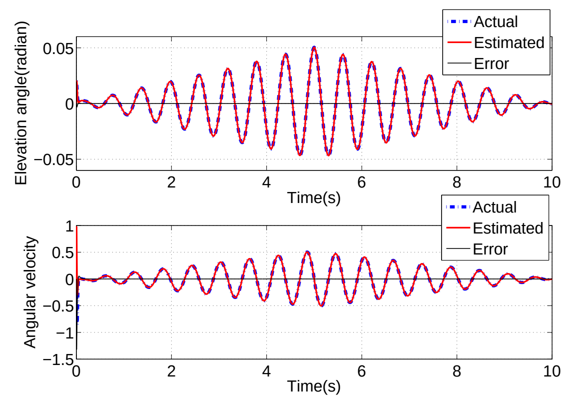

Fig. 8: SMO: Sinusoidal Disturbance; Estimation results for pitch channel

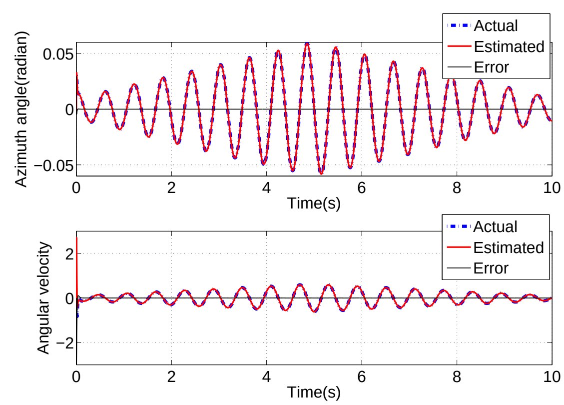

Fig. 9: SMO: Sinusoidal Disturbance; Estimation results for yaw channel

It can be seen that the performance of SMO is unaffected, thus proving its robustness properties against matched disturbances. The Kalman filter performance is however highly affected proving its incapability to disturbances in plant input channel.

C. *Sensor Noise*

A random noise signal with zero mean and variance 3, is added to the output signal y, that is provided to the observer/ Kalman filter. Correspondingly, the R matrix of EKF is increased to 3 and Q matrix has parameters $Q_{11} = Q_{22} = 1e^{-6}$. The results are shown in Figures 10, 11, 12 and 13.

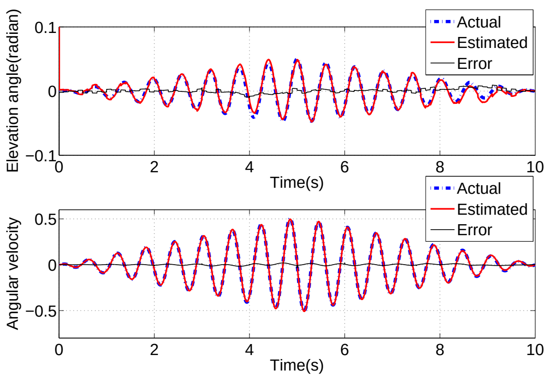

Fig. 10: EKF: Sensor Noise; Estimation results for pitch channel

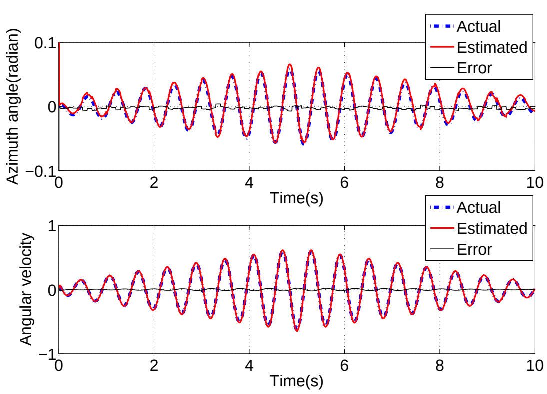

Fig. 11: EKF: Sensor Noise; Estimation results for yaw channel

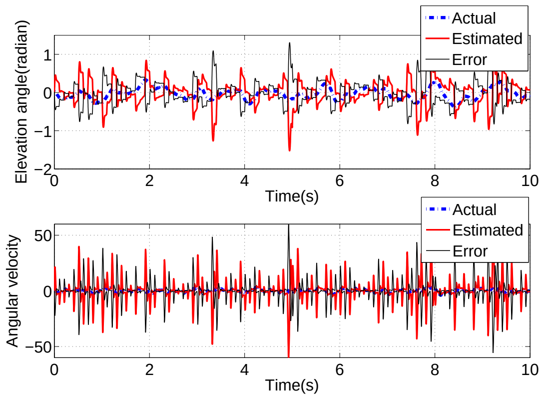

Fig. 12: SMO: Sensor Noise; Estimation results for pitch channel

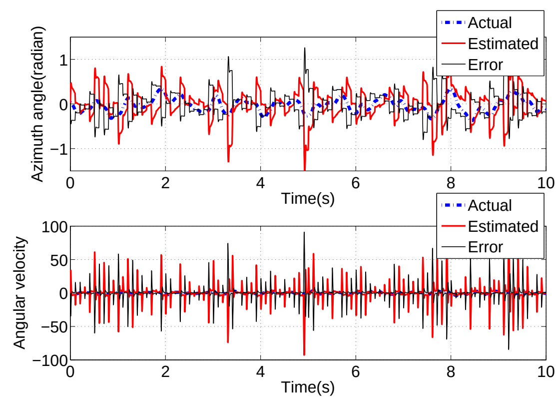

Fig. 13: SMO: Sensor Noise; Estimation results for yaw channel

The variance of noise signal is reduced to 0.1 for SMO. It can be seen that the SMO estimation is highly corrupted by the sensor noise whose signal with higher variance is easily surpassed by Kalman filter to give exact estimations.

V. Conclusion

The estimation strategies- Extended Kalman filter and Sliding Mode Observer are compared in this paper with respect to their application to a two axis gimbal system of seeker system of a missile. Few conclusions can be enumerated from this comparative study;

1) The robustness of the Sliding Mode Observer to parametric uncertainties and matched input disturbances is guaranteed. The Kalman filter fails to cater to such matched disturbances in the input to plant.

2) The Extended Kalman Filter gives approximate estimations with total insensitivity to output noise. Thus proper tuning of the covariance matrices imparts robustness to noise signals. SMO estimations get corrupted on introduction of noise in output. For practical implementations, it is recommended that corrupted SMO results are to be passed through an appropriately designed filter.
3) In terms of complexity of implementation; the tuning of covariance matrices of EKF is a rigorous task. SMO gains once tuned can be maintained constant for various matched uncertainties.

Thus, SMO proves to be a robust, simple to implement estimation methodology and no knowledge of the noise statistics is required. However, the performance of Kalman is superior when issue of sensor noise arises.

APPENDIX A
EKF ALGORITHM

A discrete system is represented as;

$$\left.\begin{array}{l} x(k+1) = f(x(k), u(k)) + \mathrm{w}(k) \\ y(k) = g(x(k)) + \mathrm{v}(k). \end{array}\right\} \quad (18)$$

Process noise w(k)$\sim N(0,Q)$ and measurement noise v(k)$\sim N(0,R)$, is assumed to be Gaussian. In practice, the process noise covariance and measurement noise covariance matrices (Q and R respectively) might change with each time step or measurement, however here we assume it to be constant.

The Kalman filter works in two steps: prediction and updation [11]. A priori state estimate (prediction) is denoted as $\hat{x}(k)(-)$ and posteriori state estimate as $\hat{x}(k)(+)$. We can define a priori and a posteriori estimate errors as

$$\begin{array}{rcl} e_k^- &=& x(k) - \hat{x}(k)(-) \\ e_k^+ &=& x(k) - \hat{x}(k)(+) \end{array}$$

The a priori estimate error covariance is then $P(k)(-) = E[e_k^- e_k^{-T}]$ and the a posteriori estimate error covariance is $P(k)(+) = E[e_k^+ e_k^{+T}]$. The goal is finding an equation that computes posteriori state estimate as a linear combination of an a priori estimate and a weighted difference between an actual measurement and a measurement prediction. The algorithm can be summarized as follows:

1) The priori state estimate is computed from system dynamics with nominal values used in the system function.

$$\hat{x}(k+1)(-) = f(\hat{x}(k), u(k)) \quad (19)$$

2) The priori estimate error covariance matrix is given as

$$P(k+1)(-) = \phi(k)P(k)(+)\phi(k)^T + Q \quad (20)$$

where $\phi(k) = \frac{\partial f}{\partial x}|_{x(k)(+)}$.
3) The measurement equation gives the a priori output estimation

$$\hat{y}(k+1)(-) = g(\hat{x}(k+1)(-)), \quad (21)$$

4) The $n \times m$ matrix K, is chosen to be the gain or blending factor that minimizes the a posteriori error covariance equation. It is called the Kalman Gain.

$$K(k+1) = P(k+1)(-)C^T(k+1) [C(k+1)P(k+1)(-)C^T(k+1) + R]^{-1} \quad (22)$$

where $C(k+1) = \frac{\partial g}{\partial x}|_{\hat{x}(k+1)(-)}$.
5) Thus the posterior state estimate is computed as

$$\hat{x}(k+1)(+) = \hat{x}(k+1)(-) + K(k+1) [y(k+1) - \hat{y}(k+1)(-)] \quad (23)$$

REFERENCES

[1] B. Ekstrand, "Tracking filters and models for seeker applications," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 37, no. 3, pp. 965–977, July 2001.

[2] J. Slotine, J. K. Hedrick, and E. A. Misawa, "Nonlinear state estimation using sliding observers," *25th Conference on Decision and Control, Athens, Greece*, pp. 332–339, December 1986.

[3] V. Utkin and S. Drakwnov, "Sliding mode observers tutorial," *34th Conference on Decision and Control, New Orleans LA*, pp. 3376–3378, December 1995.

[4] B. L. Walcott and S. H. Zak, "State observation of nonlinear uncertain dynamical systems," *IEEE transactions on Automatic Control*, vol. AC-32, no. 2, pp. 166–170, February 1987.

[5] B. Bandyopadhyay, P. S. Gandhi, and S. Kurode, "Sliding mode observer based sliding mode controller for slosh-free motion through pid scheme," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3432–3442, September 2009.

[6] B. Tamhane, S. Kurode, P. Parkhi, and Dharmveer, "Simultaneous state and disturbance estimation for seeker system using sliding modes," *12th International Conference on Control, Automation and Systems (ICCAS)*, pp. 1358–1363, October 2012.

[7] R. E. Kalman, "A new approach to linear filtering and prediction problems," *ASME J. Basic Eng*, 1960.

[8] D. Simon, "Kalman filtering with state constraints: a survey of linear and nonlinear algorithms," *IET Control Theory Appl.*, vol. 4, no. 8, p. 13031318, 2010.

[9] Dharmveer, S. Kurode, P. Parkhi, and B. Tamhane, "Robust control for seeker scan loop using sliding modes," *12th International Conference on Control, Automation and Systems (ICCAS)*, pp. 49–54, October 2012.

[10] F. Chen and M. Dunnigan, "Comparative study of a sliding-mode observer and kalman filters for full state estimation in an induction machine," *IEE Proc. Electr. Power Appl*, vol. 149, no. 1, January 2002.

[11] G. Welch and G. Bishop, *An Introduction to the Kalman Filter*.