Analysis of Leakage Flow characteristics in Bent Axis Motors

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***Abstract*— This article presents a practical approach to determine the characteristics of the leakage flow of the bent axis hydro-motor. In this study, a mathematical model is developed by considering the flow through the clearances between the piston and the barrel-hole (i.e. annular section) and the gap between the ball and socket joint at the spherical head of the piston. To develop the mathematical model for the leakage flow of the hydro-motor, the expressions for the leakage through the clearances are obtained using Navier-Stokes equation, where the change in thickness of the clearances with the differential pressure across the leakage path is also taken into consideration. The co-efficients of the polynomial expression obtained through the modeling that varies with the operating pressure are identified through the experimental investigation. From the characteristics of the leakage flow of the hydro-motor, it is observed that the flow loss of the hydro-motors increases with the increase in operating pressure, speed and displacements of the motor.**

Keywords*— *Mathematical model; Leakage flow; Bent-axis hydro-motor; Volumetric efficiency; Leakage resistance.

I. Introduction

The hydrostatic drives are widely used in the construction and heavy earth moving machineries. The bent axis hydro-motors are used in such drives due to their high power to weight ratio. The hydraulic components that are used in a system have flow and torque losses, the estimation of which are appropriate for the practical engineers. The flow loss reduces its volumetric efficiency, thereby resulting in overall power-loss of the system. The flow loss or the leakage flow may be internal, external or both. The internal leakage in the hydraulic pumps or the motors is usually the inter-port leakages. The inter-port leakage in the bent axis motor is the leakage between the valve plate and the cylindrical barrel, which is very difficult to segregate from the external leakage that is across the clearance passages. The external leakage is the flow losses through the different clearances provided for easier motion of the pistons and lubricating some mating surfaces.

The characteristics of the leakage flow of the hydraulic pumps and the motors have been analyzed by many authors. The linear model proposed by Wilson [1] was further refined by many others [2-5]. Based on the model proposed by McCandlish and Dorey [5], this article analyses the flow loss characteristics of the bent axis hydro-motors. The leakage flow characteristics of different types of pumps were studied by Inaguma [6, 7], where the effects of oil temperature on the friction torque characteristics in hydraulic pumps are focused.

In this paper, the mathematical model for the leakage characteristics in the clearances of the bent axis motor is developed, where the variation of thickness of the clearances with the differential pressure across the leakage path is also taken into consideration. The characteristic of the leakage flow that varies with the pressure difference across the hydro-motor is found to be non-linear in nature. The dependencies of the co-efficients of the non-linear equation on the operating parameters of the system and the geometrical parameters of the motor are identified from the test data. In this respect, an experimental set up was designed and fabricated. The characteristics of the leakage flow of the commercially available three different sizes of the bent axis hydro-motors were determined experimentally for a wide range of speed using standard procedure [8].

II. Mathematical modeling

A. Leakage path of the Bent Axis Motor

For any hydraulic pump or motor, there is an inlet and an outlet port. The increase or decrease in the volume of the pumping chambers results in suction or discharge respectively. In case of hydraulic motor, the inlet port is at high pressure compared to the discharge port which results inter-port flow loss from inlet to outlet port, known as internal leakage of the motor. Apart from this, the leakage flow arises due to the pressure difference between the pumping chambers and the case drain. In this section, the flow losses at different possible clearance zones are discussed.

The leakage paths for the bent axis hydraulic motors are shown in *“Fig. 1,”*.

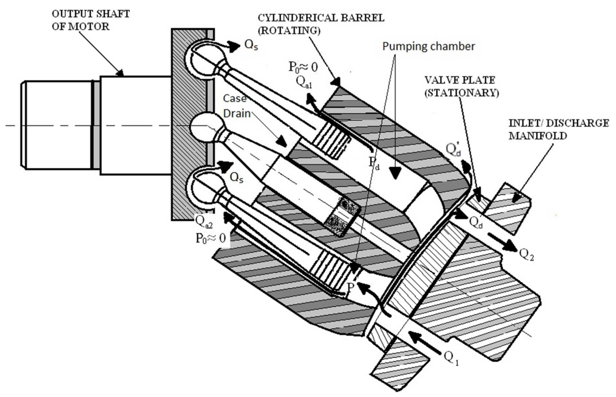

Fig. 1. Leakage paths for the Bent Axis Hydraulic Motor

The possible clearance zones through which the leakage flow may arise are:

a) Annular clearance between the piston and the barrel hole
b) Clearance between the ball and socket joint
c) Clearance between the valve plate and cylindrical barrel

However, the inter-port leakage in the clearance passage between the valve plate and the cylindrical barrel is much less as compared to the leakages in the remaining clearance passages and therefore, it has not been accounted for the present study. Therefore, the total flow loss is sum of the leakage in the clearance between the annular zone and that in between the ball and socket joint.

B. Expression for leakage

Referring to *“fig. 1,”*, the geometry of the clearances, discussed in the previous sub-section, is represented in *“fig. 2,”*.

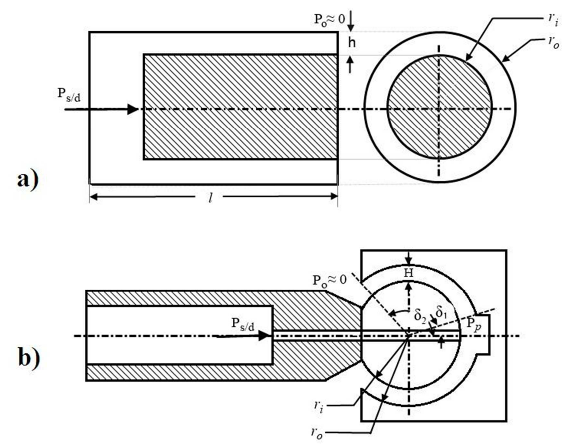

Fig. 2. Configuration of clearances in Bent-Axis motor

While modeling, the following assumptions are made:

a) The leakage flow in the above said clearances is assumed to be laminar and is thus proportional to the differential pressure across the leakage flow path. The laminar behaviour of the leakage flow is due to less flow and higher effective length of the leakage path as compared to the clearances.
b) The case drain pressure is negligible.
c) Leakage through the clearances between the valve plate and cylindrical barrel is negligible.
d) The flow loss due to compressibility of the fluid is insignificant.

Using Navier- Stokes (N-S) equation [9], the expressions for the leakage flow through the clearances shown in *“fig. 2,”* are developed. However, the clearances in the hydro-motor vary with the operating parameters of the system. Therefore, the leakage flow through the respective clearances, obtained from N-S equation, is accordingly modified, which is discussed in the successive sub-sections

B.1. Leakage flow through the annular zone ($\mathrm{Q_a}$)

Referring to *“fig. 2(a),”*, the pressure induced leakage flow through the clearance between the piston and barrel hole is expressed as:

$$\mathrm{Q_a} = \frac{\pi\, \mathrm{r}_0\, h_a^{\,3}\, \Delta p}{6 \mu l} \tag{1}$$

where, r_0 = Inner radius of barrel-hole,

h_a = Radial clearance barrel hole and piston,

Δp = Differential pressure across the leakage flow path ($= P_s - P_0$ or $= P_d - P_0$),

P_s = Suction pressure

P_d = Discharge pressure

P_0 = Pressure outside the clearance zone or at drain (≈ 0)

l = Effective length of assembly of piston and barrel-hole

μ = Absolute viscosity

This equation holds good if the piston and the barrel-hole are coaxial and the annular clearance (h_a) is constant.

Considering the variation of the clearance passage with respect to the differential pressure (ε_1), “(1),” can be expressed as:

$$Q_a^* = \frac{\pi r_0 [h_a (1+\varepsilon_1 \Delta p)]^3 \Delta p}{6 \mu l} \tag{2}$$

where, Q_a^* = Modified leakage flow through the clearance between the annular zone

ε_1 = Change in thickness of clearance with respect to differential pressure

B.2. Leakage flow through the clearance between ball and socket (Q_s)

Referring to *"fig. 2(b),"*, the pressure induced leakage flow through the clearance between the ball and socket joint is expressed as:

$$Q_s = \alpha P_p \frac{\pi H^3}{6\mu} \tag{3}$$

Where, P_p = Pressure in the clearance zone of ball and socket ($\approx P_s$ or P_d)

H = Clearance between ball and socket

$$\alpha = \frac{1}{\ln\left(\dfrac{\tan\left(\delta_2/2\right)}{\tan\left(\delta_1/2\right)}\right)}$$

δ_1 and δ_2 = Geometry parameters of the slipper as shown in *"fig. 2 (b),"*.

Similarly, considering the change of clearance passage in the ball and socket joint as ε_2, "(3)," can be expressed as:

$$Q_s^* = \alpha \frac{\pi [H(1+\varepsilon_2 \Delta p)]^3 \Delta p}{6\mu} \tag{4}$$

where, Q_s^* = Modified leakage flow through the clearance between the ball and socket joint.

ε_2 = Change of clearance in the ball and socket joint with the differential pressure of the hydro-motor

The total leakage flow (ΔQ) in the hydro-motor is the effect of leakages in the respective clearance passage and hence, it is the sum of the leakage flow as expressed in "(1)," and "(3),".

$$\Delta Q = \sum Q_a + \sum Q_s$$

$$\Delta Q = \frac{\pi r_0 h_a^3 \Delta p}{6\mu l} + \alpha P_p \frac{\pi H^3}{6\mu} \tag{5}$$

The "(5)," is the expression for the total leakage when the clearance thickness is assumed to be constant. The expression for the total leakage flow, which accounts the variation of the clearance passage, is obtained from "(2)," and "(4),". Therefore the total leakage of the bent axis hydrostatic motor is obtained by the addition the leakage flow at the two clearance zones.

$$\Delta Q^* = \sum Q_a^* + \sum Q_s^*$$

$$\Delta Q^* = \sum \frac{\pi r_0 [h_a (1+\varepsilon_1 \Delta p)]^3 \Delta p}{6\mu l} + \sum \alpha \frac{\pi [H(1+\varepsilon_2 \Delta p)]^3 \Delta p}{6\mu}$$

$$\Delta Q^* = C_1 \Delta p + C_2 \Delta p^2 + C_3 \Delta p^3 + C_4 \Delta p^4 \tag{6}$$

where,

$$C_1 = \frac{\pi}{6\mu}\left[\sum \frac{r_0 h_a^3}{l} + \sum \alpha H^3\right]$$

$$C_2 = \frac{\pi}{2\mu}\left[\sum \varepsilon_1 \frac{r_0 h_a^3}{l} + \sum \varepsilon_2 \alpha H^3\right]$$

$$C_3 = \frac{\pi}{2\mu}\left[\sum \varepsilon_1^2 \frac{r_0 h_a^3}{l} + \sum \varepsilon_2^2 \alpha H^3\right]$$

$$C_4 = \frac{\pi}{6\mu}\left[\sum \varepsilon_1^3 \frac{r_0 h_a^3}{l} + \sum \varepsilon_2^3 \alpha H^3\right]$$

The co-efficients of the polynomial "(6)," is non-linear expressions which depend on the some fixed geometric parameters like r_0, l, α, h_a *and* H. However, h_a *and* H are function of rate of change of clearances (ε), which again depends on the operating parameters like load (differential pressure) and speed. These co-efficients are obtained through experimental investigations of the leakage characteristics of the hydrostatic motors.

III. EXPERIMENTAL INVESTIGATION

For experimental investigation, the experimental set-up was designed and fabricated. The schematic hydraulic circuit diagram of the fabricated set-up is shown in "fig. 3,".

A. Description of the test set-up

Referring to *"fig. 3,"*, the test set up consists of the variable displacement swash plate pump (2) coupled with the electric motor (1) that gives variable flow to the bent axis hydraulic motor (5). The bent axis hydraulic motor is coupled with the loading unit, which consists of the loading pump (7) and the pressure relief valve (PRV) (8). The assembly was fitted with different sensors (item no. 3.1 to 3.3 and 4.1 to 4.2) for measuring the pressure at different nodes (P_{op} , P_{im} and P_{om}), inlet- outlet flow (Q_{im} and Q_{om}) and the speed (ω) of the hydro-motor. Temperature sensor (6) was used to measure the fluctuation of temperature of the oil at the outlet. To study the leakage characteristics of the hydro-motor, the commercially available different sizes of the bent axis motors were used. By varying the swash plate angle of the main pump and the loading pump, the flow, differential pressure and speed of the hydro-motor is controlled. Using the PRV (8), the load to the hydro-motor was varied and thus the operating parameter of the hydraulic test rig was varied.

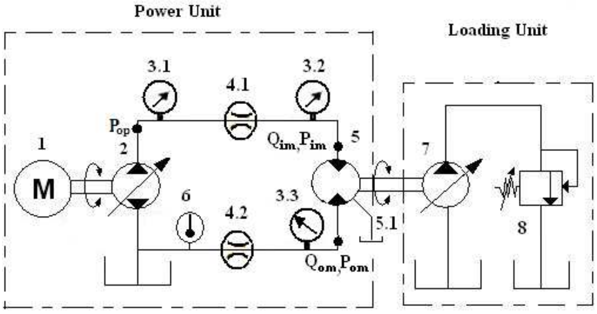

Fig. 3. Hydraulic circuit of the Test Set up

The detailed specifications of the components used in the test set-up are shown in the table I.

TABLE I. DETAILED SPECIFICATION OF THE COMPONENTS USED IN TEST SET-UP

Sl. No.	Name of the components	Specifications	
1.	Electric motor	Output power Rated speed	:15 kW :1500 rpm
2.	Axial piston variable displacement pump or main pump	Displacement Nominal pressure	:28 cc/rev :400 bar
3.	Pressure Sensor	Pressure Range	:0-150 bar
4.	Flow Control Valve or sensor	Max. range Linearity	:60 LPM :0.42 %
5.	Bent axis fixed displacement motor	Hydro- motor 1	: 16 cc/rev
		Hydro- motor 2	: 12 cc/rev
		Hydro- motor 3	: 5 cc/rev
6.	Temperature sensor	0-100 ^{0}C	
7.	Axial piston pump	Displacement Nominal pressure	:28 cc/rev :400 bar
8.	Pressure relief valve	Max. Set Pressure Max. flow	:350 bar :200 LPM

B. Experimental procedure

The experiments were conducted over a wide range of operating speed and pressure levels, following a standard procedure [8]. During the experiments, the oil temperature was maintained at 50±2^{0}C to keep its viscosity constant with reasonable accuracy. A stable source of power supply was provided to the electric motor driving the main pump that supplies the flow (Q_{im}) to the hydro-motor. The speed (ω) of the motor was varied by varying the swash plate angle of the main pump while the pressure difference (Δp) across the motor was controlled by adjusting the swash plate angle of the loading pump and the set pressure of the PRV at the loading unit. The average leakage flow rate ($\Delta Q^{'}$) of the hydraulic motor was measured by collecting the leakage oil for a time interval of 30 sec, at different pressure level of the hydro-motor. The estimated leakage flow has been verified from the inlet and outlet flow of the hydro- motor, obtained from the respective sensors (item no. 4.1 and 4.2 in *"fig. 3,"*, respectively). The effect of operating parameters like differential pressure and the speed on the leakage flow of the bent axis motor was studied.

Experiments were conducted several times to examine repeatability before collecting the data. Test data were collected at different levels of the differential pressure across hydro-motor.

C. Results and discussion

From the collected test data, the leakage flow characteristics of the hydro-motors at different pressure levels were plotted using the best fit lines to the data points.

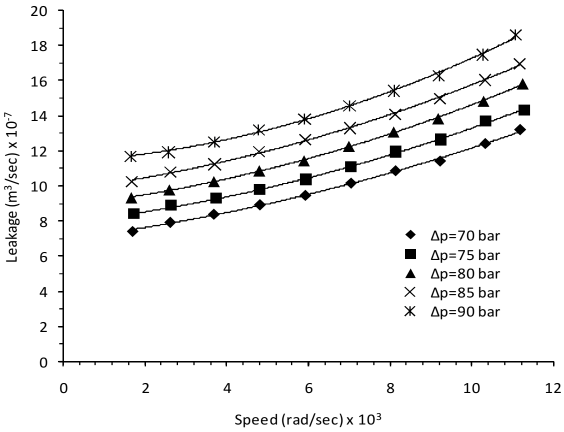

Fig. 4. Leakage characteristics of the hydro-motor 1

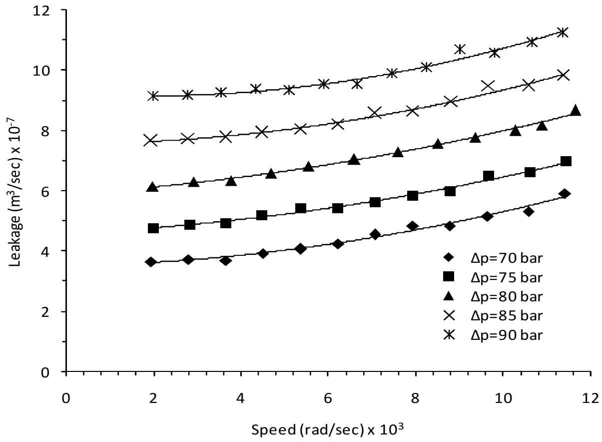

Fig. 5. Leakage characteristics of the hydro-motor 2

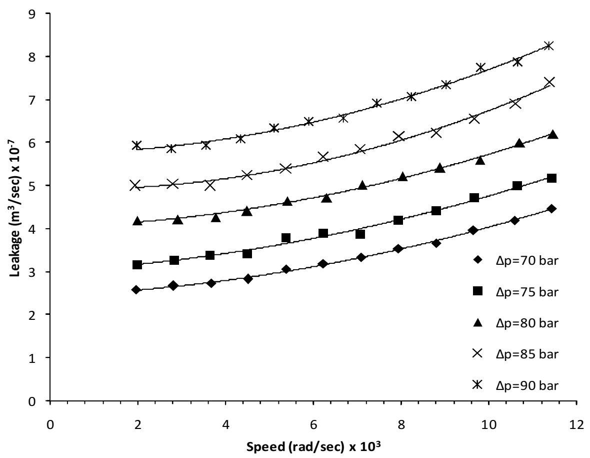

Fig. 6. Leakage characteristics of the hydro-motor 3

The expression of the leakage flow of the hydro-motors is found to be function of the speed and the differential pressure across it. The expressions for the leakage characteristics of the hydraulic motors of different sizes are given below:

$$\begin{aligned}\Delta Q'\big|_{Hydro-motor1} = &\left(-7\times10^{-20}\omega^2+7\times10^{-16}\omega-7\times10^{-12}\right)\Delta p^4\\ &+\left(2\times10^{-17}\omega^2-2\times10^{-13}\omega+2\times10^{-9}\right)\Delta p^3\\ &+\left(-3\times10^{-15}\omega^2+2\times10^{-11}\omega-2\times10^{-7}\right)\Delta p^2\\ &+\left(1\times10^{-13}\omega^2+1\times10^{-9}\omega+1\times10^{-5}\right)\Delta p\\ &+\left(-3\times10^{-12}\omega^2+2\times10^{-8}\omega+1\times10^{-5}\right)\end{aligned} \tag{7}$$

$$\begin{aligned}\Delta Q'\big|_{Hydro-motor2} = &\left(-3\times10^{-11}\right)\Delta p^4\\ &+\left(2\times10^{-13}\omega+9\times10^{-9}\right)\Delta p^3\\ &+\left(-2\times10^{-11}\omega-1\times10^{-6}\right)\Delta p^2\\ &+\left(1\times10^{-9}\omega+5\times10^{-5}\right)\Delta p\\ &+\left(-2\times10^{-8}\omega-1\times10^{-3}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}\Delta Q'\big|_{Hydro-motor3} = &\left(-2\times10^{-14}\omega\right)\Delta p^3\\ &+\left(2\times10^{-12}\omega\right)\Delta p^2\\ &+\left(-1\times10^{-10}\omega+2\times10^{-8}\right)\Delta p\\ &+\left(2\times10^{-9}\omega-1\times10^{-6}\right)\end{aligned} \tag{9}$$

The last term in “(7),” to “(9),” is independent of the differential pressure and is much less as compared to other terms. This term may be considered as the inter-port leakage, which is neglected for the present study. Hence, the last term in “(7),” to “(9),”, which is independent of differential pressure, is neglected.

$$\begin{aligned}\Delta Q'\big|_{Hydro-motor1} = &\left(-7\times10^{-20}\omega^2+7\times10^{-16}\omega-7\times10^{-12}\right)\Delta p^4\\ &+\left(2\times10^{-17}\omega^2-2\times10^{-13}\omega+2\times10^{-9}\right)\Delta p^3\\ &+\left(-3\times10^{-15}\omega^2+2\times10^{-11}\omega-2\times10^{-7}\right)\Delta p^2\\ &+\left(1\times10^{-13}\omega^2+1\times10^{-9}\omega+1\times10^{-5}\right)\Delta p\end{aligned} \tag{10}$$

$$\begin{aligned}\Delta Q'\big|_{Hydro-motor2} = &\left(-3\times10^{-11}\right)\Delta p^4\\ &+\left(2\times10^{-13}\omega+9\times10^{-9}\right)\Delta p^3\\ &+\left(-2\times10^{-11}\omega-1\times10^{-6}\right)\Delta p^2\\ &+\left(1\times10^{-9}\omega+5\times10^{-5}\right)\Delta p\end{aligned} \tag{11}$$

$$\begin{aligned}\Delta Q'\big|_{Hydro-motor3} = &\left(-2\times10^{-14}\omega\right)\Delta p^3\\ &+\left(2\times10^{-12}\omega\right)\Delta p^2\\ &+\left(-1\times10^{-10}\omega+2\times10^{-8}\right)\Delta p\end{aligned} \tag{12}$$

Comparing “(10),” to “(12),” with the theoretical expression of the leakage flow given in “(6),”, the co-efficients of the expression are obtained for different hydro-motors. The dependencies of the co-efficient on the system parameters and operating parameters are studied. It is observed that the co-efficients are found to be the non-linear function of the speed that effects the variation of clearances with respect to differential pressure (ε). From *“fig. 4,”to “fig. 6,”*, it is observed that the variation of leakage flow of the hydro-motor at different pressure levels with speed is non-linear in nature. However, the non-linear relation of the co-efficients with speed decreases as the displacement of the hydro-motor is decreased (*“fig. 7,”*).

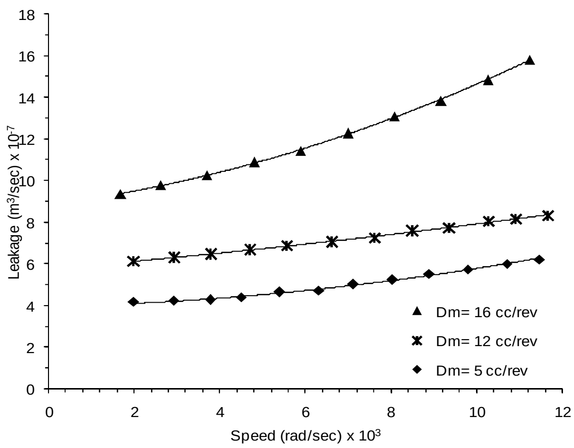

Fig. 7. Leakage characteristics of the hydro-motors of different displacements at different speed and constant differential pressure (80 bar)

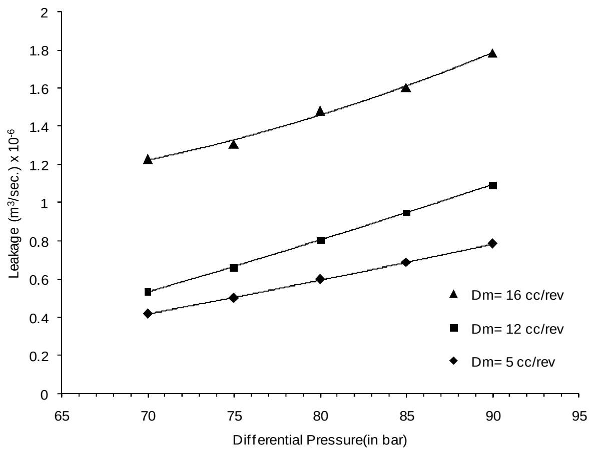

Fig. 8. Leakage characteristics of the hydro-motors of different displacements at different pressure and constant speed (10,500 rad/sec)

The leakage flow characteristic of the three bent axis motors of different displacement is studied from the test data. *“Fig. 8,”* shows the leakage behaviour of different hydro-motors at different pressure when all the hydro-motors were running at same speed of 10,500 rad/sec. It is observed that the effect of differential pressure on the leakage flow is non-linear. However, the non-linearity decreases for the hydro-motor having smaller displacement. The non-linear effect of pressure on the smaller displacement hydro-motor decreases because the effective length of the leakage flow path increases compared to the higher displacement motor. As a result of which the laminar flow of the viscous oil in the clearance passage becomes more dominant for the smaller sized hydro-motor. Therefore, “(5),” is followed as the displacement of the motor is decreased.

IV. CONCLUSION

In the present study, the leakage flow characteristics of the bent axis hydro-motors are analyzed. From the study, it is observed that the operating parameters like speed and differential pressure across the hydro-motor have significant effect on the leakage flow. In this respect, the mathematical expression for the leakage flow of the hydraulic motor is developed from the N-S equation. The dependencies of the co-efficients of the obtained expression on the operating parameters of the system are obtained through experiments with reasonable accuracy. From further experiments conducted on different sizes of the hydro-motors, it was found that the nature of variation of the leakage flow with the speed and the differential pressure vary with the displacement of the hydro-motor. It was observed that the non-linear dependency of the leakage flow on the differential pressure decreases with decrease in the displacement of the hydro-motor. However, due to limited data, the dependencies of the leakage flow on the size of the hydrostatic motor could not be established.

Further studies on the effect of displacements of hydro-motors and temperature of oil on the leakage characteristics can be made.

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