Minimization of Shaking Force and Shaking Moment in Multiloop Planar Mechanisms

Kailash Chaudhary
Department of Mechanical Engineering
Malaviya National Institute of Techanology Jaipur
Jaipur, India
k.chaudhary.mech@gmail.com

Himanshu Chaudhary
Department of Mechanical Engineering
Malaviya National Institute of Techanology Jaipur
Jaipur, India
himanshumnitj@gmail.com

***Abstract*—This paper presents an optimization method to dynamically balance the complex multiloop planar mechanisms. The shaking force and shaking moment transmitted to the ground are balanced through optimization to improve the mechanism's dynamic performance. The pareto optimal conditions are proposed considering the shaking force and shaking moment as two objective functions. First, the force balancing of the mechanism is obtained numerically. The shaking moment was found increased for a force balanced mechanism. To reduce both the shaking force and shaking moment, a multi-objective optimization problem is formulated and solved using conventional and genetic algorithm (GA) techniques. The genetic algorithm produces several optimum solutions (pareto optimal points) and the best solution can be chosen from this set of optimum solutions. The optimization method presented in this paper is general and can be applied to any mechanism whereas the analytical solutions already available are for the specific mechanisms. The effectiveness of the proposed method is shown considering a Stephenson six-bar mechanism.**

Keywords—Dynamic balancing; multiloop mechanism; shaking force; shaking moment; optimization; genetic algorithm

I. INTRODUCTION

Any mechanism in motion, if not properly balanced, transmits forces and moments to its ground body known as shaking force and shaking moment. These forces and moments cause vibration and fatigue in the mechanism, and affect its performance. Several techniques have been proposed in the literature to reduce these shaking forces and shaking moments. The method for complete force balancing is based on the principle of making the total center of mass of the mechanism stationary [1]. The position equation of the total center of mass is written in the terms of the positions of the links of the mechanism first and then the time dependent terms are set equal to zero. These analytically derived conditions state the mass distribution of the links to eliminate the shaking force completely in simple mechanisms. This method was further extended by considering the force balance restrictions arising from the use of prismatic joints within a mechanism and minimum number of counterweight needed for full force balance in single and multi-degree of freedom planar mechanisms [2-4]. Using the same approach, an another method uses ordinary vectors to found the conditions of the full force balance as against the complex vectors used in previous methods [5].

The complete force balancing alone is not sufficient as it will increase shaking moment and/or driving torques. The complete balancing of shaking force and shaking moment is not possible without adding duplicate mechanisms [6]. Use of the duplicate mechanism and/or adding links which neutralizes inertia is not recommended due to practical aspects and complexity. However, it could be used where it needed for the purpose as in slider-crank mechanism in multi-cylinder engines [7]. In a fully force-balanced in-line planar four-bar mechanism, the root-mean-square (RMS) value of the shaking moment was reduced through optimization in which the feasible limits of the link parameters can also be considered [8-10]. To minimize different dynamic quantities simultaneously, a trade-off method was developed in which the maximum values of the bearing force, the input moment, and the shaking moment of a constant input speed four-bar mechanism were simultaneously minimized, while obtaining a prescribed maximum value of the shaking force [11].

The link masses can be optimally redistributed for simultaneous reduction in shaking force and shaking moment. These dynamic forces and moments depend on link masses, their locations of CGs and moment of inertias. Hence, the optimization problem formulation can be simplified if links are modelled as point-masses connected firmly [12]. A dynamically equivalent system of point-masses, referred to *equimomental* system, is used to represent the inertial properties of the mechanism [13]. After dynamically representing the links by the system of equimomental point-masses, the optimum distribution of the link masses can be found out for dynamically balance the mechanisms [14-17]. Using two and four point-mass models [18-21], dynamic quantities like driving torque, shaking force and shaking moment can be optimally balanced. In another method, the sets of three and seven equimomental point-masses for each link in planar and spatial mechanisms, respectively, were proposed [22]. The balancing problem of planar rigid-body mechanisms can also be formulated as a convex optimization problem [23-25] to determine the optimal shape, position and mass of the counterweights. The evolutionary optimization techniques like particle swarm optimization (PSO) and genetic algorithm (GA) are useful for the purpose of minimizing the multi-objective functions [26].

In this paper, the force balancing is achieved for a Stephenson six-bar mechanism by optimizing the point-mass parameters of the counterweights. Next, the multi-objective optimization problem is formulated to simultaneously minimize the shaking force and shaking moment. The genetic algorithm is applied for this multi-objective optimization problem produces a number of optimum solutions known as "pareto optimal points". The best solution can be chosen from this set of optimum solutions and thus the global optimum solution of the problem can be found very quickly and efficiently.

The structure of this paper is as follows. Section 2 presents the equations of motion for rigid body and point-masses. The formulation of the optimization problem is shown in section 3. Section 4 presents a example of Stephenson six bar mechanism which is solved using the proposed method. Results are presented and pareto optimal solutions are found using genetic algorithm in section 5. Finally, conclusions are given in section 6.

II. EQUIMOEMENTAL SYSTEM OF POINT-MASSES

This section discusses the principle of equimomental system of point-masses and the dynamic equations of motion for the rigid body are rewritten in terms of the point-masses parameters.

2.1 Equations of Motion of Rigid Body

Consider a rigid body moving in X-Y plane. The frame X_i-Y_i is fixed to the body at O_i. The link length is defined by the distance from the joints, O_i to O_{i+1}. A body fixed ground X_i-Y_i is defined such that origin is at O_i and axis X_i is aligned from O_i to O_{i+1}. The location of the mass center, C_i, is defined by polar coordinates, d_i and θ_i, as shown in Fig. 1 where the bold-faced $\mathbf{a}_i$, and $\mathbf{d}_i$ denote the vectors. For dynamic analysis, the Newton-Euler (NE) equations of motion for the ith rigid link in planar motion with respect to the origin, O_i, are written as [27]:

$$\mathbf{M}_i\dot{\mathbf{t}}_i + \mathbf{C}_i\mathbf{t}_i = \mathbf{w}_i \tag{1}$$

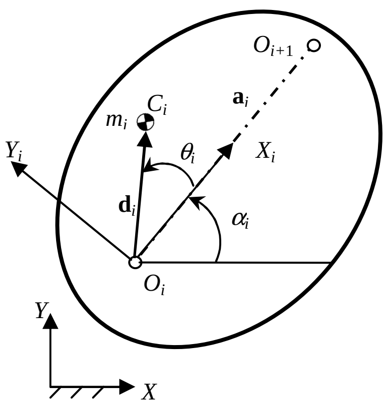

Fig. 1. Planar rigid body

In (1), $\mathbf{t}_i$, $\dot{\mathbf{t}}_i$, and $\mathbf{w}_i$, are 3-dimesional vectors, defined as the twist, twist-rate, and wrench of the ith link with respect to O_i, respectively, i.e.,

$$\mathbf{t}_i = \begin{bmatrix} \omega_i \\ \mathbf{v}_i \end{bmatrix};\ \dot{\mathbf{t}}_i = \begin{bmatrix} \dot{\omega}_i \\ \dot{\mathbf{v}}_i \end{bmatrix} \text{ and } \mathbf{w}_i = \begin{bmatrix} n_i \\ \mathbf{f}_i \end{bmatrix} \tag{2}$$

In (2), ω_i and $\mathbf{v}_i$ are the scalar angular velocity about the axis perpendicular to the plane of motion, and the 2-dimensinal vector of linear velocity of the origin of the ith link, O_i, respectively. Accordingly, $\dot{\omega}_i$ and $\dot{\mathbf{v}}_i$ are the time derivatives of ω_i and $\mathbf{v}_i$, respectively. Also, the scalar, n_i, and the 2-dimesional vector, $\mathbf{f}_i$, are the resultant moment about O_i and the resultant force at O_i, respectively. The 3×3 matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$ are defined as:

$$\mathbf{M}_i = \begin{bmatrix} I_i & -m_i\mathbf{d}_i^{\mathrm{T}}\mathbf{E} \\ m_i\mathbf{E}\mathbf{d}_i & m_i\mathbf{1} \end{bmatrix} \text{ and } \mathbf{C}_i = \begin{bmatrix} 0 & \mathbf{0}^{\mathrm{T}} \\ -m_i\omega_i\mathbf{d}_i & \mathbf{O} \end{bmatrix} \tag{3}$$

where $\mathbf{1}$ and $\mathbf{O}$ are the 2×2 identity and zero matrices, respectively, and $\mathbf{0}$ is the 2-dimesional vector of zeros, whereas the 2×2 matrix $\mathbf{E}$ is defined as:

$$\mathbf{E} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

2.2 Modified Equations of Motion for Equimomental System of Point-masses

The links of a mechanism can be modeled as rigid bodies for kinematic and dynamic analysis. The mass and moment of inertias of the links govern shaking force and shaking moment transmitted to the ground. The determination and optimization of moment of inertias is useful in minimizing the shaking force and shaking moment. This problem can be simplified by modelling the rigid links as point-masses connected firmly. The essential requirements for the dynamic equivalence between rigid link and the system of point-masses are (1) same mass, (2) same center of mass and (3) same inertia tensor with respect to same coordinate frame. Such dynamically equivalent system is called equimomental system of point-masses [22]. As shown in Fig. 2, a set of dynamically equivalent system of rigidly connected k point-masses, m_{ik}, located at l_{ik}, θ_{ik} can be found using the equimomental conditions.

The conditions used to find the equivalent point-masses for a rigid body are:

$$\sum_k m_{ik} = m_i \tag{5}$$

$$\sum_k m_{ik} l_{ik} \cos(\theta_{ik} + \alpha_i) = m_i d_i \cos(\theta_i + \alpha_i) \tag{6}$$

$$\sum_k m_{ik} l_{ik} \sin(\theta_{ik} + \alpha_i) = m_i d_i \sin(\theta_i + \alpha_i) \tag{7}$$

$$\sum_k m_{ik} l_{ik}^2 = I_i \tag{8}$$

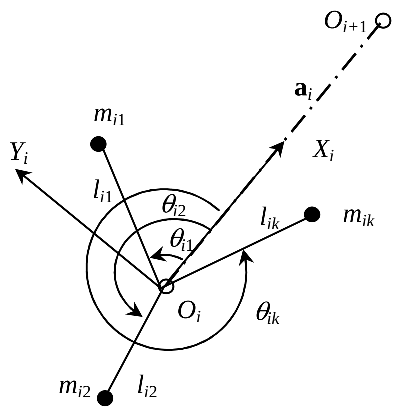

Fig. 2. Equimomental system of point-masses of the ith link

where m_i and I_i are the mass of the ith link and its mass moment of inertia about O_i. The subscripts i and k are used to represent the link number and the point-mass, respectively. It is important to note here that all the vectors are represented in the fixed inertial frame, *OXY*. The NE equations of motion, (1), are now rewritten for the equimomental system of point-masses of the ith link. Equation (1) does not change except the elements of matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$, which are given as:

$$\mathbf{M}_i = \begin{bmatrix} \sum_k m_{ik} l_{ik}^2 & -\sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i) & \sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i) \\ -\sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i) & \sum_k m_{ik} & 0 \\ \sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i) & 0 & \sum_k m_{ik} \end{bmatrix};$$

$$\mathbf{C}_i = \begin{bmatrix} 0 & 0 & 0 \\ -\omega_i \sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i) & 0 & 0 \\ -\omega_i \sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i) & 0 & 0 \end{bmatrix} \qquad (9)$$

where C and S are abbreviations for cosine and sine functions, respectively. There are total 3k parameters, m_{ik}, θ_{ik}, l_{ik}, for k=1, 2,…,k if k point-masses are defined for the ith link. For a mechanism of n moving links, there will be total 3kn point-mass parameters. All or some of these can be taken as the design variables.

III. Formulation of Optimization Problem

For balancing the mechanism using counterweights, the point-mass parameters of the counterweights are considered as the design variables in the optimization problem. The matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$, defined in (9) can be modified for the counterweight addition as:

$$\mathbf{M}_i = \begin{bmatrix} \mathrm{M}_{11} & -\mathrm{M}_{12} & \mathrm{M}_{13} \\ -\mathrm{M}_{21} & \mathrm{M}_{22} & \mathrm{M}_{23} \\ \mathrm{M}_{31} & \mathrm{M}_{32} & \mathrm{M}_{33} \end{bmatrix};$$

$$\mathbf{C}_i = \begin{bmatrix} 0 & 0 & 0 \\ -\omega_i (\sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i)_{\mathrm{o}} + \sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i)_{\mathrm{c}}) & 0 & 0 \\ -\omega_i (\sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i)_{\mathrm{o}} + \sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i)_{\mathrm{c}}) & 0 & 0 \end{bmatrix} \qquad (10)$$

The elements of this matrix $\mathbf{M}_i$ are:

$$\mathrm{M}_{11} = (\sum_k m_{ik} l_{ik}^2)_o + (\sum_k m_{ik} l_{ik}^2)_c; \ \mathrm{M}_{23} = \mathrm{M}_{32} = 0;$$

$$\mathrm{M}_{22} = \mathrm{M}_{33} = (\sum_k m_{ik})_o + (\sum_k m_{ik})_c;$$

$$\mathrm{M}_{12} = \mathrm{M}_{21} = ((\sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i))_o + (\sum_k m_{ik} l_{ik} S(\theta_{ik}+\alpha_i))_c)$$

$$\mathrm{M}_{13} = \mathrm{M}_{31} = ((\sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i))_o + (\sum_k m_{ik} l_{ik} C(\theta_{ik}+\alpha_i))_c)$$

In (10), the subscripts o and c are used for the original links and the counterweights, respectively. Equation (1) with these modified terms is used to evaluate the shaking force and shaking moment transmitted to the ground.

3.1 Design Variables

The counterweights added to a mechanism having n moving links, can be represented by a system of k equimomental point–masses per counterweight per link. The 3k-vector of design variables for counterweight for ith link is defined as:

$$\mathrm{DV}_1 = [m_{i1} l_{i1} \theta_{i1} \quad m_{i2} l_{i2} \theta_{i2} \quad \ldots \quad m_{ik} l_{ik} \theta_{ik}]^{\mathrm{T}} \qquad (11)$$

Similarly, the 3nk-vector of design variables, DV, for the mechanism can be defined as:

$$\mathrm{DV} = [\mathrm{DV}_1^{\mathrm{T}} \quad \mathrm{DV}_2^{\mathrm{T}} \quad \ldots. \quad \mathrm{DV}_{\mathrm{n}}^{\mathrm{T}}] \qquad (12)$$

3.2 Objective Function and Constraints

The shaking force is defined as the vector summation of all the inertia forces of the mechanism while the summation of moments of the inertia forces and the inertia couples about a joint is defined as the shaking moment about that joint [8, 19]. As shaking force and shaking moment are of different units, they are made dimensionless using the parameters of the first link to define the combined objective for the optimization problem. Considering the RMS values of the shaking force and shaking moment, the optimality criterion for simultaneous minimization of them is proposed here as

$$\textit{Minimize} \quad z = w_1 f_{sh,rms} + w_2 n_{sh,rms} \qquad (13)$$

$$\textit{Subject to} \quad m_i^o \le m_i \le 5 m_i^o$$
$$d_i \le a_i^o \qquad (14)$$
$$m_i d_i^2 \le I_i$$

here w_1 and w_2 are used as the weighting factors for giving weightage to the shaking force and shaking moment as per the requirements in the different cases. The values of these weighting factors vary between 0 and 1. The parameters m_i^o and a_i^o are representing the original mass and length of the ith link, respectively.

IV. Example: Stephenson Six-bar Mechanism

The proposed method is applied for the balancing of Stephenson six bar mechanism as shown in Fig. 3.

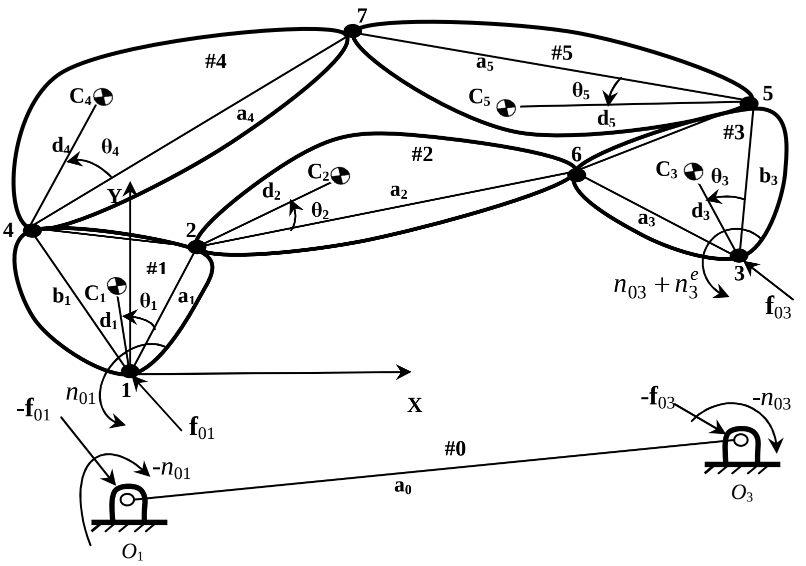

Fig. 3. Stephenson six-bar mechanism detached from the frame

Link #1 and #3 are tertiary links and links #2, #4, #5 are binary links whereas link #0 is fixed. These links are connected by the revolute joints. For the considered mechanism, the shaking force and shaking moment at and about the joint, 3, between ground and third link are given as:

$$\mathbf{f}_{sh} = -(\mathbf{f}_{01} + \mathbf{f}_{03}) \text{ and } n_{sh} = -(n_{01} + n_{03} + \mathbf{a_0} \text{ x } \mathbf{f_{01}} + n_3^e) \quad (15)$$

In (15), $\mathbf{f}_{01}$ and $\mathbf{f}_{03}$ are the reaction forces of the ground on the links 1 and 3, respectively. Similarly, n_{01} and n_{03} are the reactions of resultant inertia couple about the joint 1 and 3, respectively while n_3^e is the driving torque applied at joint 3. As all joints in the mechanism are revolute, $n_{01} = n_{03} = 0$ .Considering three point-masses per link, design variables are considered as:

$$\text{DV} = [m_{i1} l_{i1} \theta_{i1} \;\; m_{i2} l_{i2} \theta_{i2} \;\; m_{i3} l_{i3} \theta_{i3}]^{\text{T}} \text{ for } i = 1,2,\ldots,5 \quad (16)$$

Now, by putting suitable limits on the link masses and inertias, the optimization problem to reduce shaking force and shaking moment for this mechanism is formulated as:

$$\textit{Minimize} \qquad z = w_1 f_{sh,rms} + w_2 n_{sh,rms} \quad (17)$$

$$\textit{Subject to} \qquad m_i^o \le \sum_{k=1}^{3} m_{ik} \le 5 m_i^o$$

$$d_i \le a_i^o \quad (18)$$

$$m_i d_i^2 \le I_i \quad \textit{for}\ i = 1,\ldots,5$$

The parameters for the unbalanced Stephenson six bar mechanism are taken from [1] and given in Table 1. The kinematic simulation was carried out using the MotionView and MotionSolve of Altair HyperWorks 11.0 software [28]. For given example, link 3 rotates with a constant speed of 2π radians/second. The "*fmincon*" function in *Optimization Toolbox* of MATLAB [29], was used to solve this non-linear optimization problem subjected to constraints as defined in (17) and (18). This function finds a constrained minimum of a scalar function of several variables starting at an initial estimate. As explained in section 3, the values of weighting factors, w_1 and w_2, may vary depending on the particular requirement. For the problem considered, the standard and optimized values of the shaking force and shaking moment for different combinations of weighting factor values are shown in Table 2. The corresponding design vectors are given in Table 3.

TABLE I. PARAMETERS OF UNBALANCED STEPHENSON SIX-BAR MECHANISM [1]

Link *i*	**1**	**2**	**3**	**4**	**5**
a_i (m)	0.056	0.121	0.003	0.140	0.044
b_i (m)	0.058	---	0.003	0.152	---
γ_i (deg)	6.0	---	16.0	40.4	---
θ_i (deg)	3.0	0.0	5.0	19.0	0.0
d_i (m)	0.029	0.063	0.003	0.084	0.020
m_i (kg)	0.061	0.083	0.076	0.173	0.040

TABLE II. RMS VALUES OF SHAKING FORCE AND SHAKING MOMENT

	Value of w_1,w_2	**Shaking force (N)**	**Shaking moment# (N-m)**
Original Value		0.0450	3.7332
1. Only shaking force balance	1.0,0.0	0.0164 (-63.9)	10.8610 (+190.1)
2. Both shaking force and shaking moment balance	0.5,0.5	0.0192 (-57.7)	2.2651 (-39.3)
3. Only shaking moment balance	0.0,1.0	0.1031 (+127.1)	2.2516 (-39.7)

#The shaking moment is taken about the joint, 3;
The values in parenthesis denote percentage increment/decrement over the corresponding values of the original mechanism

TABLE III. DESIGN VECTORS FOR BALANCED STEPHENSON SIX-BAR MECHANISM

Case	**Design vector**
Case (1) w_1=1.0;w_2=0.0	0.0309 4.0270 1.7062 0.1197 3.9740 1.7338 0.1537 3.8813 4.8418 0.0365 2.9959 1.7388 -0.0102 3.1573 0.9680 0.0277 3.2215 5.1332 -0.0318 4.9628 1.9128 0.0319 4.9519 1.9128 0.0001 4.9174 5.0231 0.0603 3.4509 1.7738 -0.0503 3.3583 1.8414 0.0123 3.4443 4.5739 0.0349 2.1382 1.9216 -0.0043 2.4381 2.0465 0.0286 2.2983 4.9177
Case (2) w_1=0.5;w_2=0.5	0.4806 1.5366 1.6328 -0.3290 1.9244 1.5450 0.1526 0.7439 5.1208 -0.0199 0.9701 1.9528 0.0582 0.7075 2.0403 0.0177 1.2373 5.2512 -0.0144 4.3380 2.5218 0.0141 3.5602 1.9263 0.0136 2.5912 3.4444 -1.1293 6.6533 0.0172 0.8427 6.2160 0.2306 0.3973 6.6235 5.8622 0.0008 1.5700 2.2340 -0.0004 2.1455 2.0325 0.0077 0.1109 3.7604
Case (3) w_1=0.0;w_2=1.0	-0.4071 1.1193 1.1352 0.0106 2.5043 -0.6537 0.4743 0.9671 7.4688 0.0035 3.5774 1.7455 0.0020 3.5776 1.7470 0.0056 3.4643 4.8851 0.0974 0.8426 3.6560 0.0826 2.3066 2.2343 -0.0911 2.3636 2.6041 0.7422 1.2073 0.0586 -1.0356 2.6574 1.1808 0.6861 3.5710 7.7891 -0.1317-0.1939 1.7103 0.2981 -0.1594 1.9258 0.0001 -1.5667 4.7663

For only the shaking force balancing, weighting factor values, w_1 and w_2, are taken as 1 and 0, respectively in case (1). This results in 63.87 percent reduction in the shaking force but the shaking moment value in increased by 190.93 percent. This shows that force balancing increases the shaking moment in this case. In case (3), the weighting factor values, w_1 and w_2, are taken as 0 and 1, respectively for only the shaking moment balancing. As shown in Table 2, it results in 39 percent reduction in the shaking moment value while the shaking force is increased by 127 percent. These two cases support the fact that the reduction in one dynamic quantity increases the other. Thus a trade-off is necessary to reduce both the shaking force and shaking moment and both the quantities are assigned equal weighting factor value, i.e., 0.5 in the objective function (Case 2). The result shows 57.7 and 39 percent reduction in the values of the shaking force and shaking moment, respectively. The variations in the values of shaking force and shaking moment over entire cycle are shown in Fig. 4 and Fig. 5.

The point-mass parameters of the counterweights are obtained as the optimum design variables. Using the equimomental conditions presented in (5)-(8), these point-masses can be converted into the rigid body. The optimized counterweight parameters obtained are given in Table 4. Thus, the *fmincon* function in MATLAB returns the values of optimized shaking forces and shaking moments as well as design variables for the different combinations of weighting factors. Using the conditions of the equimomental point-mass system, the properties of the balanced links and the counterweights attached with them can be found very easily.

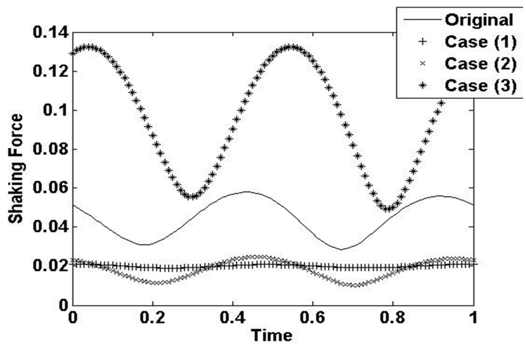

Fig. 4. Variation of shaking force with time

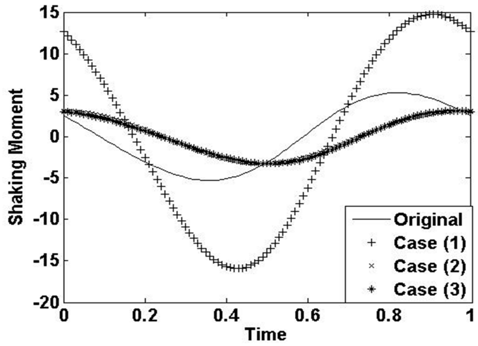

Fig. 5. Variation of shaking moment with time

TABLE IV. OPTIMUM COUNTERWEIGHT PARAMETERS

		Case 1	**Case 2**	**Case 3**
Counterweight for link 1	Mass (kg)	0.304	0.304	0.078
	d (m)	0.056	0.056	0.056
	θ (deg)	176.3	180.1	105.4
Counterweight for link 2	Mass (kg)	0.022	0.111	0.393
	d (m)	0.126	0.121	0.123
	θ (deg)	199.1	213.5	98.4
Counterweight for link 3	Mass (kg)	0.054	0.056	0.011
	d (m)	0.003	0.003	0.025
	θ (deg)	241.7	177.1	110.2
Counterweight for link 4	Mass (kg)	0.059	0.008	0.166
	d (m)	0.140	0.138	0.139
	θ (deg)	205.8	194.1	124.1
Counterweight for link 5	Mass (kg)	0.002	0.013	0.089
	d (m)	1.716	0.028	0.043
	θ (deg)	107.0	134.4	187.7

This conversion of optimized point-masses to rigid body helps in finding the mass, mass center location and moment of inertia of the links and counterweights of the balanced mechanism.

V. PARETO OPTIMAL SOLUTIONS USING GENETIC ALGORITHM

Genetic algorithms are a class of probabilistic optimization algorithms inspired by biological evolution process. It uses the concept of natural genetics and natural selection. The advantage with this algorithm is that multiple optimal solutions can be captured easily, thereby reducing the effort to use the same algorithm many times. In multiobjective optimization problems, there exist a number of optimum solutions which constitute a Pareto-optimal front [30]. The problem of balancing of Stephenson six bar mechanism is solved using "*gamultiobj*" function in *Genetic Algorithm and Direct Search Toolbox* of MATLAB. This function finds the minima using genetic algorithm and creates a set of Pareto optima for a multiobjective minimization. One can specify the initial population, bounds and linear constraints for variables. The general difficulties faced by users are: (1) requirement of large amount of calculation and (2) no unique and guaranteed optimum solution. The parallel computers and longer runs of algorithm help in overcoming these difficulties [31]. For the problem considered, the objective function is defined as

$$\text{Minimize} \qquad z = [f_{sh,rms}, n_{sh,rms}]^T \qquad (19)$$

This objective function is taken as the fitness function for the genetic algorithm. The optimum design variables obtained for case 2 in previous section is taken as the initial population and the algorithm was run for 100 generations. For the given fitness function and the initial population, the genetic algorithm generates a curve having multiple optimal solutions as shown in the Fig. 6. This plot shows the trade off between two objective functions, i.e. the shaking force and shaking moment. Thus it is advantageous to use GA for finding multiple optimal solutions without running the conventional algorithm many times.

The original and optimized values found using *fmincon* function are also shown in Fig. 6. The GA results are found better than the results obtained using conventional optimization algorithm as shown in this figure. The values of the shaking force and shaking moment corresponding to the best solution among available pareto optimal solutions are given in Table 5. The counterweight parameters for optimum design variables are calculated using the equimomental conditions and are presented in Table 6.

TABLE V. RESULTS USING GENETIC ALGORITHM

	Shaking force (N)	Shaking moment (N-m)
Original Value	0.0450	3.7332
Optimized Value	0.0069 (-84.0)	1.1260 (-69.8)

The values in parenthesis denote percentage increment/decrement over the corresponding values of the original mechanism

TABLE VI. OPTIMUM COUNTERWEIGHT PARAMETERS USING GA

Counter-weights	CW 1	CW 2	CW 3	CW 4	CW 5
Mass (kg)	0.305	0.112	0.057	0.012	0.014
d (m)	0.075	0.232	0.013	0.334	0.221
θ (deg)	200.5	250.1	136.0	144.7	214.2

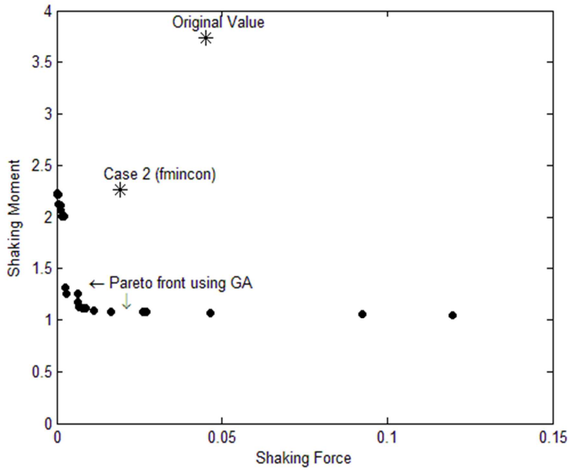

Fig. 6. Pareto optimal solutions set

VI. CONCLUSIONS

The minimization of shaking force and shaking moment in complex planar mechanisms is presented here as the single and multiobjective optimization problems. The proposed optimization method is efficient and simple as compared to the available analytical methods. The rigid links of the mechanism are represented by the equimomental point-masses and Newton-Euler equations of motion are used to evaluate the shaking force and shaking moment. Putting constraints on point-masses parameters, the objective function including both the shaking force and shaking moment is optimized using *fmincon* and *gamultiobj* functions of MATLAB software. In the shaking force balance problem, 64 percent reduction in shaking force is obtained, while the reduction of about 58 percent and 39 percent in shaking force and shaking moment is found using the conventional optimization algorithm. The simultaneous reduction in both these dynamic quantities is improved using genetic algorithm that is about 84 percent and 70 percent, respectively. Pareto optimal solution set is also found in which each point represents the optimum value of the multiobjective function. This helps in choosing the best solution amongst the available optimum solution as per the requirement.

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