Dynamics of a Spatial Multibody System using Equimomental System of Point-masses

Vinay Gupta  
Dept. of Mechanical Engg.  
Indian Institute of Technology Delhi  
New Delhi, India  
vinayguptaec@gmail.com

Himanshu Chaudhary  
Dept. of Mechanical Engg.  
MNIT  
Jaipur, India  
hchaudhary@mnit.ac.in

Subir K. Saha  
Dept. of Mechanical Engg.  
Indian Institute of Technology Delhi  
New Delhi, India  
saha@mech.iitd.ac.in

Abstract—This paper presents dynamic modelling of a spatial multibody system using the equimomental system of point-masses which has several advantages, e.g., one needs to write only the translational equations of motion without the necessity of writing the rotational equations as the point masses have no dimension. For this, a rigid body is represented as a set of rigidly connected seven point-masses. Accordingly, the velocities of the point-masses are derived as a linear transformation of the joint-rates resulting into a set of three decoupled matrices called the Decoupled Natural Orthogonal Complement (DeNOC) matrices for the point-masses. The matrices are then used to form a minimum set of constrained equations of motion from the uncoupled Newton’s equations of linear motion. The methodology is illustrated using a spatial solid pendulum.

Keywords—Dynamics, spatial multibody system, equimomental system, DeNOC.

I. INTRODUCTION

Dealing with the dynamics of a spatial multibody system is always a challenging task. Dynamic equations of motion of the multibody system can be written using various coordinate systems. Absolute coordinate system represents the configuration of a body relative to the global reference frame. This results in a large number of equations of motion [1]. In joint coordinate system, the generalized coordinates are the relative coordinates between the bodies connected by a kinematic pair or joint [2]. On the other hand, an equimomental system of point masses is the one which consists of several point masses whose total mass, overall mass-center location and the moment of inertia are same as that of the original rigid body [3-4]. Such representation has several advantages. For example, one needs to deal with the translational equations of motion only, as the point masses have no dimension. Besides, it is more convenient to perform optimum design of the shapes of the bodies forming the overall multibody system, and its balancing. These advantages have motivated the researchers to use the concept of “Natural” coordinates”, as presented in [5] and others. In [6] point coordinates replaced the rigid body and the equations of motion were derived in joint coordinates using the velocity transformation matrix. In [7-8] point coordinates is used to represent the rigid body and linear and angular momentum concept applied for dynamical formulation. The point mass representation was also used in [9-11], where balancing of several mechanisms was carried out.

This paper presents modelling of a spatial multibody system using the concept of equimomental system of point-masses and the corresponding Decoupled Natural Orthogonal Complement (DeNOC) matrices. A set of seven point-masses, as proposed in [11], was used to define a rigid body moving in the three-dimensional Cartesian space. The preference of seven point-masses over typically used four point-masses [3] is mainly due to the presence of linear set of algebraic equations to find the point-masses [11]. The point-mass velocities were then expressed as a linear transformation of the joint-rates. The associated matrix is called the DeNOC matrices for the point-mass system, similar to the one presented in [12-13] for a serial-chain system. Further the equations of motion were derived using the Newton’s equations of linear motion only. The methodology is shown by applying it to a spatial solid pendulum. A similar methodology was presented in [14] for a planar system.

The paper is organized as follows: Section II introduces the concept of point-mass system for a rigid-body in a spatial motion, whereas Section III derives the DeNOC matrices for the point-mass system. Section IV presents dynamic modeling and Section V illustrates the modeling using a spatial pendulum. Finally, Section 6 concludes the paper.

II. EQUIMOMENTAL SYSTEM OF SEVEN POINT-MASSES

Consider a rigid body or link, denoted as \( i^{th} \) link, moving in three-dimensional Cartesian space. The rigid link is dynamically represented as a set of rigidly connected seven point-masses, as proposed in [10-11]. It is shown in the Fig.1. To avoid the coincidence of any two points, the point-masses \( m_j \) are located at the vertices of a parallelepiped, whose center is located at the origin \( O_{i+1} \) which has sides \( 2h_{x}, 2h_{y}, 2h_{z} \) that are parallel to the axes of the body-fixed frame \( O_{i+1}X_{i+1}Y_{i+1}Z_{i+1} \). The two systems of rigid-body and seven point-mass system are equimomental if the following conditions are satisfied:

\[
m_{1} + m_{2} + m_{3} + m_{4} + m_{5} + m_{6} + m_{7} = m_i \quad (1)
\]
is mass, \( \mathbf{x}_{i} \) is the 21-dimensional vector of point-mass \( i \), i.e., \( \mathbf{x}_{i} = m_{i} \mathbf{v}_{i} \). For a rigid body \( \mathbf{h}_{i} \), \( \mathbf{h}_{i} \), and \( \mathbf{h}_{c} \) were found separately using (8-10). For a system of seven point-masses, \( \mathbf{h}_{i} \), \( \mathbf{h}_{i} \), and \( \mathbf{h}_{c} \) will never have imaginary values. Then (2-7) becomes linear in \( m_{1}, \ldots, m_{7} \) and solve for them. This will be carried out in Section V.

### III. DEOC MATRICES FOR THE POINT-MASS SYSTEM

Consider the \( i^{th} \) rigid link in spatial motion. The link is represented as a set of rigidly connected seven point-masses shown in Fig. 1. The velocities of the point-masses of the \( i^{th} \) link is derived in general form from the velocity of the origin \( O_{i} \) of the link as

\[
\mathbf{v}_{i} = \mathbf{v}_{i} + \mathbf{\omega}_{i} \times \mathbf{d}_{i}
\]

\[\vdots\]

\[
\mathbf{v}_{7} = \mathbf{v}_{7} + \mathbf{\omega}_{7} \times \mathbf{d}_{7}
\]

where \( \mathbf{\omega}_{i} \) and \( \mathbf{v}_{i} \) are the angular velocity and linear velocity of the origin \( O_{i} \) of the \( i^{th} \) link, respectively, whereas the vector \( \mathbf{d}_{i} = a_{i} + r_{j} \), for \( j = 1, \ldots, 7 \), denotes the position of the \( j^{th} \) point-mass of the \( i^{th} \) body from its origin \( O_{i} \). Vector \( \mathbf{a}_{i} \) is the link length vector of the \( i^{th} \) body, i.e., the vector representing the position of \( O_{i,j} \) from \( O_{i} \), whereas vector \( \mathbf{r}_{j} \), \( j = 1, \ldots, 7 \), is the position of the \( j^{th} \) point-mass from \( O_{i,j} \). For a system of seven point-masses, their velocities are given by (14-20), which can be expressed in a compact form as

\[
\mathbf{\hat{v}}_{i} = \mathbf{D}_{i} \mathbf{t}_{j}
\]

where \( \mathbf{\hat{v}}_{i} \) is the 21-dimensional vector of point-mass velocities, and \( \mathbf{D}_{i} \) is the 21x6 point-mass matrix defined as

\[
\mathbf{D}_{i} = \begin{bmatrix}
-d_{i,1} \times 1 & 1 \\
\vdots & \vdots \\
-d_{i,6} \times 1 & 1
\end{bmatrix}
\]

In (22), \( \mathbf{d}_{i,j} \times 1 \), for \( j = 1, \ldots, 7 \), is the 3x3 cross-product tensor associated with the vector \( \mathbf{d}_{i,j} \), i.e., \( (\mathbf{d}_{i,j} \times 1) \mathbf{x} = \mathbf{d}_{i,j} \times \mathbf{x} \), for any 3-dimensional Cartesian vector \( \mathbf{x} \). Moreover, the term \( 1 \) is the 3x3 identity matrix, and \( \mathbf{t}_{j} \) is defined as the twist of the \( i^{th} \) body [11, 13]. For a rigid
body moving in the three-dimensional Cartesian space, the 6-dimensional twist is defined as

$$ t_i = \begin{bmatrix} \omega_i \\ v_i \end{bmatrix} \tag{23} $$

where \( \omega_i \) and \( v_i \) are the 3-dimensional vectors of angular velocity and linear velocity of the origin of the \( i \)th body, i.e., \( O_i \), respectively. If the rigid bodies are coupled with the help of a single-degree-of-freedom (DoF) joints, say, a revolute and prismatic, the twists of all rigid bodies in the system represented as a generalized twist, i.e.,

$$ t = \begin{bmatrix} t_1' \ldots t_n' \end{bmatrix} $$

can be written as

$$ t = N \theta, \quad \theta = \begin{bmatrix} \dot{\theta}_1 \ldots \dot{\theta}_n \end{bmatrix}^T \tag{24} $$

and the matrix \( N \) is referred to as the Natural Orthogonal Complement or NOC of the velocity constraint matrix [13]. However, if the bodies or the links are connected with a joint having more than one-DoF joint, say, a 3-DoF spherical joint, then the scalar \( \dot{\theta} \) associated with the \( i \)th joint needs to be replaced with the 3-DoF joint-rate vector. Now, if a multibody system has serial-chain with \( n \) rigid bodies, the velocities of its equimomentals point-masses can be expressed as

$$ \tilde{v} = Dt \tag{25} $$

where \( \tilde{v} \) is the 21\( n \)-dimensional vector, \( t \) is the 6\( n \)-dimensional vector of generalized twist, and \( D \) is 21\( n \times 6 \) matrix defined as

$$ D \equiv diag[D_1, \ldots, D_n] \tag{26} $$

Combining (24-26), one can write the point-mass velocities \( \tilde{v} \), in terms of the joint rates, i.e., \( \theta \), as

$$ \tilde{v} = N \theta, \quad \theta = \begin{bmatrix} \dot{\theta}_1 \ldots \dot{\theta}_n \end{bmatrix} $$

The matrices \( N_i \) and \( N_d \) for one-DOF joints coupling the bodies are given in [13], which are given by

$$ N_i = \begin{bmatrix} 1 & O & \cdots & O \\ A_{21} & 1 & \ldots & O \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \ldots & 1 \end{bmatrix} ; \quad N_d = \begin{bmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_n \end{bmatrix} \tag{28} $$

Where \( O \) is the matrix of zeros whose dimension is compatible according to the matrix where it appears. For example, \( O \) of \( N_i \) is the 6\( x \)6 matrix of zeros. Similarly, in \( N_d \) of (28), \( \theta \) represents the 6-dimensional vector of zeros. Moreover, \( I \) denotes the 3x3 identity matrix, and the 6x6 matrices of \( N_i \), i.e., \( A_{i,i-1} \), called twist propagation matrix. Finally, the vectors of \( N_d \), i.e., \( p \), are the joint-motion propagation vector. The expressions for the 6\( x \)6 matrix, \( A_{i,i-1} \), and the 6-dimensional vector \( p \) are given by

$$ A_{i,i-1} \equiv \begin{bmatrix} 1 & O \\ a_{i,i-1} & 1 \end{bmatrix} ; \quad p_i = \begin{bmatrix} e \\ 0 \end{bmatrix} \tag{29} $$

in which \( a_{i,i-1} = -a_{i,i-1} \) is the vector associated to the skew-symmetric matrix \( a_{i,i-1} \times 1 \). Note here that if the joints are having more DoF, the joint-motion propagation vector becomes a matrix whose columns are corresponding to the DoF of the joint at hand. For a 3-DoF spherical joint, illustration will be given in Section V.

IV. DYNAMIC MODELLING

The equations of motion for the point-mass system were derived using Newton’s equations of linear motion and the corresponding DeNOC matrices derived in Section III. The unconstrained Newton’s equations of motion for the \( j \)th point-mass of the \( i \)th body are given by

$$ m_j \ddot{v}_j = f_j, \quad j = 1, \ldots, 7 \tag{30} $$

where \( m_j \) is the mass of the \( j \)th point-mass in the \( i \)th body, whereas, \( \ddot{v}_j \) is the acceleration of point-mass, and \( f_j \) is the force including gravity acting on the \( j \)th point mass of the \( i \)th body. Equation in (30) can be written for an \( n \)-link system as

$$ \ddot{\tilde{v}} = \tilde{f} \tag{31} $$

In (31), 21\( n \times 21 n \) matrix \( \ddot{\tilde{M}} \) and the 21\( n \)-dimensional vectors \( \ddot{\tilde{v}} \) and \( \tilde{f} \) are defined as follows:

$$ \ddot{\tilde{M}} = \begin{bmatrix} m_1, \ldots, m_n, 1, \ldots, m_n, 1 \end{bmatrix} \tag{32a} $$

$$ \ddot{\tilde{v}} = \begin{bmatrix} \ddot{v}_{i1}, \ldots, \ddot{v}_{in}, \ldots, \ddot{v}_{i1}, \ldots, \ddot{v}_{in} \end{bmatrix} \tag{32b} $$

$$ \ddot{\tilde{f}} = \begin{bmatrix} f_{i1}, \ldots, f_{i1}, \ldots, f_{i1}, \ldots, f_{i1} \end{bmatrix} \tag{32c} $$

Now, upon pre-multiplication of the transpose of \( \ddot{\tilde{N}} \) of (27) to (31) gives the minimal set of constrained equations of motion, i.e.,

$$ \ddot{\tilde{N}}^T \ddot{\tilde{M}} \ddot{\tilde{v}} = \ddot{\tilde{N}}^T \tilde{f} \tag{33} $$

where \( \dot{\tilde{v}} = \tilde{N} \theta + \tilde{N} \ddot{\theta} \). Equations in (33) lead to the set of independent equations of motion in generalized coordinates by

$$ \ddot{\tilde{N}}^T \ddot{\tilde{M}} \ddot{\tilde{v}} = \ddot{\tilde{N}}^T \tilde{f} \tag{34} $$

in which the \( I \) is \( n n \) matrices, is called the Generalized Inertia Matrix (GIM) and \( \tilde{C} \) is the matrix containing convective inertia terms. Moreover, \( \tau \) is \( n \)-dimensional vector of torques. The expressions of \( I, \tilde{C} \) and \( \tau \) are given by
Note that the expressions in (35) provide the same set of final expressions obtained using the Newton-Euler uncoupled equations of motion and the corresponding DeNOC matrices, as derived in [11,13].

V. ILLUSTRATION: SPATIAL PENDULUM

Consider a spatial pendulum shown in Fig. 2. The rigid body of the pendulum is coupled to the fixed-base by a spherical joint which has 3-DoF. The pendulum is represented by the set of rigidly connected seven point-masses. The point-masses are located at the vertices of a parallelepiped whose center is located at origin $O_2$ of the body-fixed frame, as shown in Fig. 2. For the joint motion of the 3-DOF spherical joint, Euler Angles (EA) can be used. Out of 12 possible EA sets to represent a spherical joint, the ZYZ set was chosen for the purpose of dynamic modelling of the spatial pendulum using the system of point-masses. For the ZYZ set, the rotation matrix required for several coordinate transformation of matrices and vector is given from [13] as

$$Q = \begin{bmatrix}
C\theta C\phi & -S\theta C\phi & -S\phi & C\phi \\
S\theta C\phi & C\theta S\phi & -S\phi & -C\phi \\
S\theta & S\phi & 0 & C\theta \\
-S\theta C\phi & C\theta S\phi & C\phi & 0
\end{bmatrix}$$

where $S$ and $C$ stand for the “Sine and “Cosine”, whereas $\phi, \theta, \phi$ are the ZYZ Euler angle set. Fig. 3 shows the pendulum position for zero EA set. The coordinates of the point-masses or the component of vector $r_j$, for $j=1,...,7$, calculated using (11-13) in the body-fixed frame are given in Table 1.

The mass of the point-masses were calculated using (1-7) for the following input parameters of the rigid body: $m = 2.512$ kg, $a = 1$m, diameter of link, $d = 0.02$m, mass center location, $\bar{x} = -a/2$, $\bar{y} = 0$, $\bar{z} = 0$, and the moment of inertia and the product of inertia, $I_{xx} = ma^2/8$, $I_{yy} = I_{zz} = (md^2/16) + (md^2/3)$, $I_{xy} = I_{yz} = I_{zx} = 0$ kg-m$^2$. Note that the mass-center and the inertia tensor were referred in the body-fixed frame $O_2X_2Y_2Z_2$. The point-masses were then calculated as,

$$m_1 = 0.6288$ kg; $m_2 = -0.543$ kg; $m_3 = 1.171$ kg; $m_4 = 0$ kg; $m_5 = 1.171$ kg; $m_6 = -0.543$ kg; $m_7 = 0.6288$ kg.

A. General Inertia Matrix

The general inertia matrix was derived using the expression given in (35) as, $I = \tilde{N}^T \tilde{M} \tilde{N}$, where the $21 \times 21$ mass matrix for the seven point-mass system is given as, $\tilde{M} = \text{diag}[m_1, ..., m_7]$ and $\tilde{N} = DN_d$, where the

<table>
<thead>
<tr>
<th>TABLE I. COMPONENT OF VECTOR $r_{ij}$ IN BODY-FIXED FRAME, i.e., $O_2X_2Y_2Z_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{11}$ &amp; $r_{12}$ &amp; $r_{13}$ &amp; $r_{14}$ &amp; $r_{15}$ &amp; $r_{16}$ &amp; $r_{17}$</td>
</tr>
<tr>
<td>Along $X_2$ &amp; $h_x$ &amp; $h_x$ &amp; $-h_x$ &amp; $-h_x$ &amp; $-h_x$ &amp; $h_x$ &amp; $h_x$</td>
</tr>
<tr>
<td>Along $Y_2$ &amp; $h_y$ &amp; $h_y$ &amp; $h_y$ &amp; $h_y$ &amp; $-h_y$ &amp; $-h_y$ &amp; $h_y$</td>
</tr>
<tr>
<td>Along $Z_2$ &amp; $h_z$ &amp; $-h_z$ &amp; $-h_z$ &amp; $h_z$ &amp; $h_z$ &amp; $h_z$</td>
</tr>
</tbody>
</table>
21×6 point-mass matrix \( \mathbf{D} \), the 6×6 matrix \( \mathbf{N}_f \), and the 6×3 matrix \( \mathbf{N}_d \) for the 3-DoF spherical joint, are given by

\[
\mathbf{D} = \begin{bmatrix}
-d_{11} \times \mathbf{1} & \mathbf{1} \\
\vdots & \vdots \\
d_{17} \times \mathbf{1} & \mathbf{1}
\end{bmatrix};
\mathbf{N}_f = \begin{bmatrix} \mathbf{1} & \mathbf{O} \\
\mathbf{O} & \mathbf{1} \end{bmatrix};
\mathbf{N}_d = \begin{bmatrix} \mathbf{L}_1 \\
\mathbf{O} \end{bmatrix}
\tag{37}
\]

where the 3-dimensional vectors \( \mathbf{[d_{1j}]_2} \), for \( j = 1, \ldots, 7 \) in body-fixed frame is given as

\[
[d_{1j}]_2 = \begin{bmatrix} a_i + h_i & a_j + h_j & a_k \end{bmatrix}^T;
\quad[d_{12}]_2 = \begin{bmatrix} a_i + h_i & a_j + h_j & a_k - h_k \end{bmatrix}^T;
\quad[d_{13}]_2 = \begin{bmatrix} a_i - h_i & a_j + h_j & a_k - h_k \end{bmatrix}^T;
\quad[d_{14}]_2 = \begin{bmatrix} a_i - h_i & a_j + h_j & a_k + h_k \end{bmatrix}^T;
\quad[d_{15}]_2 = \begin{bmatrix} a_i - h_i & a_j - h_j & a_k + h_k \end{bmatrix}^T;
\quad[d_{16}]_2 = \begin{bmatrix} a_i + h_i & a_j - h_j & a_k + h_k \end{bmatrix}^T;
\quad[d_{17}]_2 = \begin{bmatrix} a_i + h_i & a_j - h_j & a_k - h_k \end{bmatrix}^T.
\]

In case these vectors need to be represented in base-frame, they can be calculated as \( \mathbf{[d_{1j}]_1} \equiv \mathbf{Q}[d_{1j}]_2 \), where the matrix \( \mathbf{Q} \) is given by (36). The matrices \( \mathbf{1} \) and \( \mathbf{O} \) in (37) are the 3×3 identity and zero matrices, whereas matrix \( \mathbf{L}_1 \) is 3×3 matrix associated with the angular velocity of the spatial pendulum represented as \( \mathbf{O}_1 \) with the rates of the ZYZ Euler angles represented as \( \dot{\mathbf{O}}_1 \equiv [\dot{\phi} \ \dot{\theta} \ \dot{\phi}]^T \), i.e.,

\[
\mathbf{O}_1 = \mathbf{L}_1 \dot{\mathbf{O}}_1
\tag{38a}
\]

where the 3×3 matrix \( \mathbf{L}_1 \) has the following representation [13]:

\[
\mathbf{L}_1 = \begin{bmatrix} 0 & -S\phi & S\theta C\phi \\
0 & C\phi & S\theta S\phi \\
0 & 0 & C\theta \end{bmatrix}
\tag{38b}
\]

B. Convective Inertia terms

The convective inertia terms were calculated using (35) as,

\[
\mathbf{C} \equiv \mathbf{N}^T \mathbf{M} \dot{\mathbf{N}}, \quad \dot{\mathbf{N}} = \mathbf{D} \mathbf{N}_d + \mathbf{D} \mathbf{N}_d + \mathbf{D} \mathbf{N}_d
\]

in which the elements of \( \mathbf{D} \) were calculated as \( \mathbf{[d_{1j}]_1} \equiv \mathbf{Q}[d_{1j}]_2 \), and \( \mathbf{N}_f \) is the 6×6 matrix of zeros. The matrix \( \mathbf{N}_d \) has the following expression:

\[
\mathbf{N}_d = \begin{bmatrix} \mathbf{L}_1 \\
\mathbf{O} \end{bmatrix}
\tag{39}
\]

C. Generalized forces

The generalized forces were calculated using (35) as \( \mathbf{f} = \dot{\mathbf{N}}^T \mathbf{F} \), where \( \mathbf{F} \) includes externally applied forces and those due to gravity acting at each point-mass. The expression of force is given by \( \mathbf{f} = \mathbf{f}^E + \mathbf{f}^G \) in which the 21-dimensional vector \( \mathbf{f}^E \) and \( \mathbf{f}^G \) are as follows:

\[
\mathbf{f}^E = \begin{bmatrix} \mathbf{0}, \ldots, \mathbf{0} \end{bmatrix}^T \quad \text{and} \quad \mathbf{f}^G = \begin{bmatrix} m_1 \mathbf{g}, \ldots, m_7 \mathbf{g} \end{bmatrix}^T
\tag{40}
\]

(a) Joint angles
(b) Joint rates

Fig. 4. Simulation corresponding to planar behavior
where, \( g = [0 \ 0 \ -g]^T \); \( \theta = [0 \ 0 \ 0]^T \) Hence, \( \tau = \tilde{N} \dot{f}^E + \tilde{N} \dot{f}^G \equiv \tau_1 + \tau^G \).

D. Simulation

Simulation was performed using (35) and the corresponding expressions derived in Section V. The input parameters in terms of the point-masses were calculated in the beginning of Section V. Since the free-fall simulation was obtained, no input torque, i.e., \( \tau_1 = 0 \), was considered. The acceleration due gravity is \( g = 9.81 \, \text{m/s}^2 \). The initial joint angles in terms of the ZYZ Euler angles were taken as \( \phi = 90 \), \( \theta = 90 \), \( \varphi = 90 \), whereas joint rates were taken as zeros, i.e., \( \dot{\theta}_i (0) = 0 \). Fig. 4 shows the simulation results. Since the configuration corresponds to a planar solid pendulum, as analyzed in [13], the results show exact match, thus, apparently validating the algorithm based on the point-mass system. To study the spatial behavior, another set of initial conditions were considered, which are: \( \phi = 90 \), \( \theta = 45 \), \( \varphi = 45 \), whereas joint rates were taken as zeros, i.e., \( \dot{\theta}_i (0) = 1 \, \text{m/s} \). The resulting plots are shown in Fig. 5 indicating clearly spatial behavior of the pendulum.

VI. Conclusions

In this paper, dynamic modelling methodology for spatial multibody systems is presented using the equimomental system of point-masses, where each body is represented as a set of rigidly connected seven point-masses. The corresponding point-mass matrix and the DeNOC matrices were derived from the velocities of the point-masses. A numerical example is provided with a spatial pendulum where the joint motion has 3-DoF. The derived matrices were used to form the minimal set of constrained equations of motion from the unconstrained Newton’s equations of linear motion. Simulation of the spatial pendulum was performed using MATLAB’s “ode45” function.

Fig.5. Simulation for spatial behavior

ACKNOWLEDGMENT

The financial support given to the first author from the project entitled “Design and Development of Mathematical Model based 6-DoF Electrical Motion Platform” sponsored by the Simulator Development Division, Secunderabad is duly acknowledged.

REFERENCES


