Method of Defining of Intervals of Joints' Initial Coordinates for Kinematic Synthesis of Planar Lever Mechanisms

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***Abstract* – In this paper is considered the approach to solve the problem of defining of initial conditions intervals. Shown, that the choosing of intervals is the self-contained problem. The problem is solved by the optimization of a certain objective function. This function is formed on the basis of established properties of dyad's transfer functions.**

The proposed method permits to scan a multidimensional coordinate space of basis points and to define intervals there their initial values may be located. The method can do it in acceptable time span.

Keywords – initial coordinates, joints, kinematic synthesis, planar lever mechanism

I. INTRODUCTION

In [1, 2] it is shown that any planar lever mechanism can be formed as a mechanical series chain of dyads and input links (the number of such links is equal to degree of motion of the mechanism). As it known, a dyad is the simplest mechanical lever system consists of two levers that form a kihematic pair. In Assur group dyads form feedbacks and feedforwards between themselves.

The sample (Fig.1) shows the mechanism that is formed by the chain of dyads and the lever 8-9. Any dyad is the simplest transforming mechanical device with two inputs (external kinematic pairs) and one output (internal kinematic pair). In the scheme of mechanism (Fig.1a) joints a marked with numbers, and links are marked with two numbers in compliance with numbers of joints, the link is formed by.

Position of the input link (3-4-10) is defined by generalized coordinate – the angle ($\varphi+\varphi_0$). It is required to find out lengths of all the links and angles of initial positions of input (φ_0) and output (θ_0) links. It should be done in such a way that the point 0 will move along the given trajectory, which describes by the function $S=S(\varphi)$. And the output link (7-8-9) will move rotationally in compliance with the required low of motion $\theta=\theta(\varphi)$, if $\varphi_0 \le \varphi \le \varphi_{max}$.

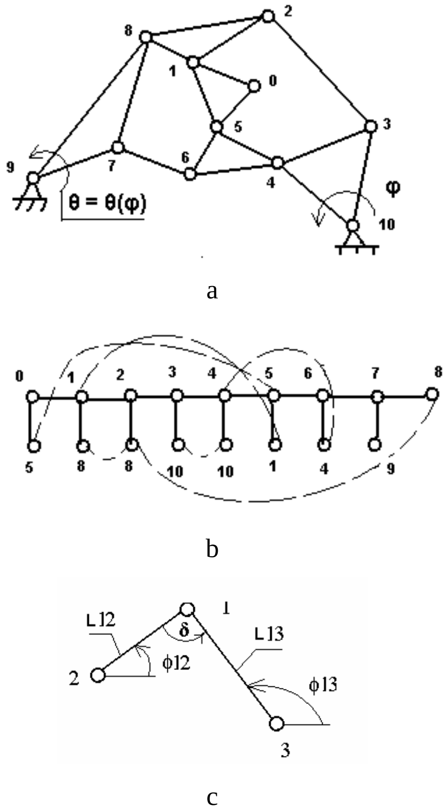

Fig. 1. The structure of lever mechanism

In [3] it is given the general method of optimization synthesis of planar lever mechanisms. The method based on a solving of ODE system that describes a transfer function of a mechanism by the transfer functions of dyads which the mechanism consists of. To solve the problem of mechanism synthesis it is necessary to choose initial conditions for ODE integration. These conditions are coordinates of kinematic pairs in mechanism's initial position. Initial conditions are finding in the course of optimizing synthesis of the mechanism. At the beginning of the synthesis it is necessary to define the permissible intervals of initial conditions changing. The accuracy of choosing these intervals has an effect on rate and quality of the synthesis. This is the known problem of initial conditions. At the time there is no any common and reasonable approach to solve this problem.

In the paper is considered the method of defining of intervals of initial values of coordinates. The choosing

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of intervals is self-contained problem. It is solved by scanning optimum values of an objective function. Objective function is formed on the basis of defined properties of the dyads' transfer function.

II. PROPERTY OF A DYADS' TRANSFER FUNCTION

For the dyad (Fig.1c) there are equations of mechanical links between the points 1, 2 and 1, 3

$$\left.\begin{aligned}(x1-x2)^2+(y1-y2)^2 &= l12^2\\ (x1-x3)^2+(y1-y3)^2 &= l13^2\end{aligned}\right\} \quad (1)$$

here xi, yi - coordinates of corresponding points (joints) in a given Cartesian coordinate system, which are functions of generalized coordinate.

lij - length of corresponding link (lever).

After differentiate the equations (1) with respect to generalized coordinate we obtain the following system:

$$\left.\begin{aligned}(x1-x2)\cdot(x1'-x2')+(y1-y2)\cdot(y1'-y2') &= 0\\ (x1-x3)\cdot(x1'-x3')+(y1-y3)\cdot(y1'-y3') &= 0\end{aligned}\right\} \quad (2)$$

Solve the system (2) for output parameters (projections of points' velocities) $x1', y1'$. Write down the equation as a matrix:

$$\begin{bmatrix} x1' \\ y1' \end{bmatrix} = [J2]\times\begin{bmatrix} x2' \\ y2' \end{bmatrix} + [J3]\times\begin{bmatrix} x3' \\ y3' \end{bmatrix}, \quad (3)$$

here $[J2]=\begin{bmatrix} i1 & i2 \\ i3 & i4 \end{bmatrix}$, $[J3]=\begin{bmatrix} i4 & -i2 \\ -i3 & i1 \end{bmatrix}$ - matrixes of inverse transfer function of dyad from the point 2 to the point 1, and from the point 3 to the point 1.

The components of the matrixes may be presented both by coordinates of points or by angles of links positions:

$$i1=\frac{\tan\phi 31}{\tan\phi 31-\tan\phi 21}, \quad i2=\frac{\tan\phi 21\cdot\tan\phi 31}{\tan\phi 31-\tan\phi 21},$$
$$i3=\frac{-1}{\tan\phi 31-\tan\phi 21}, \quad i4=\frac{-\tan\phi 21}{\tan\phi 31-\tan\phi 21}; \quad (4)$$

System (2) may be solved also for output parameters (coordinates of points) $x1, y1$:

$$\begin{bmatrix} x1 \\ y1 \end{bmatrix} = [J2']\times\begin{bmatrix} x2 \\ y2 \end{bmatrix} + [J3']\times\begin{bmatrix} x3 \\ y3 \end{bmatrix} \quad (5)$$

here $[J2']=\begin{bmatrix} i1' & i2' \\ i3' & i4' \end{bmatrix}$, $[J3']=\begin{bmatrix} i4' & -i2' \\ -i3' & i1' \end{bmatrix}$ - matrixes of inverse transfer function of dyad from the point 2 to the point 1, and from the point 3 to the point 1.

Components of the matrixes may be presented both by velocities of the points or by angles of links positions:

$$i1'=\frac{\cot an\phi 31}{\cot an\phi 31-\cot an\phi 21}, \; i2'=\frac{\cot an\phi 21\cdot\cot an\phi 31}{\cot an\phi 31-\cot an\phi 21},$$
$$i3'=\frac{-1}{\cot an\phi 31-\cot an\phi 21}, \; i4'=\frac{-\cot an\phi 21}{\cot an\phi 31-\cot an\phi 21} \quad (6)$$

In contrast to (4), there functions $\tan\phi 21, \tan\phi 31$ are present, in (6) there are functions $\cot an\phi 21, \cot an\phi 31$. It is because vectors of relative velocities are turned on $\pm 90^0$ relatively to corresponding links. So, the coefficients (5) (coefficients of initial matrix) and the coefficients (6) (calculated by obtained solutions) are connected to each other as:

$$|i1'|=|i4|, \; |i2'|=|i3|, \; |i3'|=|i2|, \; |i4'|=|i1| \quad (7)$$

So, coefficients of the matrix (3) or (5) may be redefined by already obtained solutions. I.e. if solution of the system is correct than values of coefficients, both initial and obtained from solution must be equal. This property of coefficients of transfer function makes it possible to form an objective function for optimization search coefficients of transfer function. Finally, it gives the possibility to find out coordinates of base points of a mechanism in its initial position.

III. INTERVALS OF CHANGING OF COEFFICIENTS OF DYAD'S TRANSFER FUNCTION

In Fig.1c it is shown the scheme of the dyad with marked angles of links positions. It is convenient to represent these angles as the follows: $\Phi 12 = \alpha$, $\Phi 13 = \alpha + \delta$.. The criterion of the dyad work quality is an angle of pressure. In this particular case it is the angle δ. As practice show, for regular work of dyad the angle of pressure should be: $10^0 < \delta < 170^0$ and $190^0 < \delta < 350^0$ if $0^0 < \alpha < 360^0$. By scanning the values of coefficients of transfer function using (4) was obtained minimum and maximum values of coefficients of dyad's transfer function for given intervals of angles of links positions. The extreme values of coefficients are:

$$\begin{aligned} &-2.379 < i1 < 3.379,\\ &-5.715 < i2 < 5.715,\\ &-5.715 < i3 < 5.715, \quad (8)\\ &-2.379 < i4 < 3.379 \end{aligned}$$

Out of equations (4) we can make a conclusion that coefficients of dyad's transfer function are interchangeable. These relationships may be described as:

$$i4 = i1-1, \quad i2=\frac{i1\cdot(i1-1)}{i3} \quad (9)$$

IV. ALGORITHM OF FINDING VELOCITIES OF MECHANISM'S JOINTS

Mechanism joints' velocities are connected to each other by ODE system which is at ones the system of linear equations of these velocities [3]:

$$
\left.\begin{aligned}
\begin{pmatrix} x7' \\ y7' \end{pmatrix} &= J17 \cdot \begin{pmatrix} x8' \\ y8' \end{pmatrix} \\
\begin{pmatrix} x6' \\ y6' \end{pmatrix} &= J8 \cdot \begin{pmatrix} x7' \\ y7' \end{pmatrix} + J15 \cdot \begin{pmatrix} x4' \\ y4' \end{pmatrix} \\
\begin{pmatrix} x5' \\ y5' \end{pmatrix} &= J7 \cdot \begin{pmatrix} x6' \\ y6' \end{pmatrix} + J14 \cdot \begin{pmatrix} x1' \\ y1' \end{pmatrix} \\
\begin{pmatrix} x4' \\ y4' \end{pmatrix} &= J6 \cdot \begin{pmatrix} x5' \\ y5' \end{pmatrix} \\
\begin{pmatrix} x3' \\ y3' \end{pmatrix} &= J5 \cdot \begin{pmatrix} x4' \\ y4' \end{pmatrix} \\
\begin{pmatrix} x2' \\ y2' \end{pmatrix} &= J4 \cdot \begin{pmatrix} x3' \\ y3' \end{pmatrix} + J11 \cdot \begin{pmatrix} x8' \\ y8' \end{pmatrix} \\
\begin{pmatrix} x1' \\ y1' \end{pmatrix} &= J3 \cdot \begin{pmatrix} x2' \\ y2' \end{pmatrix} + J10 \cdot \begin{pmatrix} x8' \\ y8' \end{pmatrix} \\
\begin{pmatrix} x0' \\ y0' \end{pmatrix} &= J2 \cdot \begin{pmatrix} x1' \\ y1' \end{pmatrix} + J9 \cdot \begin{pmatrix} x5' \\ y5' \end{pmatrix}
\end{aligned}\right\} \quad (10)
$$

The algorithm of solving this system is the following.

1) Choose the values of projections of velocities of joint 0. These values are known as initial data for mechanism synthesis.

2) Choose for each dyad the values of coefficients i1 and i3 from the intervals (8).

3) Calculate the coefficients i4 and i2 for each dyad.

4) Find out matrixes of transfer functions Ji for each dyad when mechanism is in initial position.

5) Solve system (10) – calculate projections of velocities in the initial position for all joints of mechanism.

V. Algorithm of Calculating of Coordinates of Mechanism's Joints

Coordinates of mechanism's joints are connected to each other by a system of linear equation, which is formed on the basis equations of dyad (5). For the chain of dyads of the mechanism (Fig.1) the system of equation will be:

$$
\left.\begin{aligned}
\begin{pmatrix} x7 \\ y7 \end{pmatrix} &= J17' \cdot \begin{pmatrix} x8 \\ y8 \end{pmatrix} + J16' \cdot \begin{pmatrix} x9 \\ y9 \end{pmatrix} \\
\begin{pmatrix} x6 \\ y6 \end{pmatrix} &= J8' \cdot \begin{pmatrix} x7 \\ y7 \end{pmatrix} + J15' \cdot \begin{pmatrix} x4 \\ y4 \end{pmatrix} \\
\begin{pmatrix} x5 \\ y5 \end{pmatrix} &= J7' \cdot \begin{pmatrix} x6 \\ y6 \end{pmatrix} + J14' \cdot \begin{pmatrix} x1 \\ y1 \end{pmatrix} \\
\begin{pmatrix} x4 \\ y4 \end{pmatrix} &= J6' \cdot \begin{pmatrix} x5 \\ y5 \end{pmatrix} + J13' \cdot \begin{pmatrix} x10 \\ y10 \end{pmatrix} \\
\begin{pmatrix} x3 \\ y3 \end{pmatrix} &= J5' \cdot \begin{pmatrix} x4 \\ y4 \end{pmatrix} + J12' \cdot \begin{pmatrix} x10 \\ y10 \end{pmatrix} \\
\begin{pmatrix} x2 \\ y2 \end{pmatrix} &= J4' \cdot \begin{pmatrix} x3 \\ y3 \end{pmatrix} + J11' \cdot \begin{pmatrix} x8 \\ y8 \end{pmatrix} \\
\begin{pmatrix} x1 \\ y1 \end{pmatrix} &= J3' \cdot \begin{pmatrix} x2 \\ y2 \end{pmatrix} + J10' \cdot \begin{pmatrix} x8 \\ y8 \end{pmatrix} \\
\begin{pmatrix} x0 \\ y0 \end{pmatrix} &= J2' \cdot \begin{pmatrix} x1 \\ y1 \end{pmatrix} + J9' \cdot \begin{pmatrix} x5 \\ y5 \end{pmatrix}
\end{aligned}\right\} \quad (11)
$$

For the known velocities of joints (Paragraph 3) the system of equations (11) makes it possible to define unknown coordinates of joints, if it will be supplemented with equation for levers 2-10, 3-10, 7-9, 8-9. These equations are kinematic and connect coordinates and velocities of levers' points between themselves.

$$
\left.\begin{aligned}
x2 - x10 &= \frac{y2'}{\omega 1} \\
y2 - y10 &= \frac{-x2'}{\omega 1} \\
x3 - x10 &= \frac{y3'}{\omega 1} \\
y3 - y10 &= \frac{-x3'}{\omega 1} \\
x7 - x9 &= \frac{y7'}{\omega 7} \\
y7 - y9 &= \frac{-x7'}{\omega 7} \\
x8 - x9 &= \frac{y8'}{\omega 7} \\
y8 - y9 &= \frac{-x8'}{\omega 7}
\end{aligned}\right\} \quad (12)
$$

here ω1, ω7 – given analogs of angular velocities of links 2-3-10 and 7-8-9.

The equations (11) and (12) form a overdetermined system of linear equation relatively to coordinates of all joints of the mechanism. For the studied mechanism the system of equation consists of 24 equations and 20 unknowns.

The algorithm of solving of the system (for coordinates) is the following:

1) Choose values of projections of joint's 0 velocity. These values are known as initial data for synthesis of a mechanism.

2) Find out values of right parts of equations (12), using already calculated xi' and yi'.

3) Calculate coefficients $i1'$, $i2'$, $i3'$, $i4'$ for each dyad, using already calculated (Paragraph 3) xi' and yi'.

4) Define matrixes of transfer functions Ji' for all dyads of a mechanism in its initial position, using velocities of joints.

5) Solve the systems (11, 12) and find out coordinates of all mechanism's joints in initial position.

VI. Objective Function for Scanning the Field of Allowable Values of Coordinates of Mechanism's Joints

A value of an objective function may be defined by the algorithm A.

1) Calculate velocities of mechanism's joints (Paragraph 3).

2) Calculate coordinates of mechanism's joints (Paragraph 4).

3) Calculate coefficients of transfer functions of dyads, using the known velocities of mechanism's joints (6).

4) Calculate (for all dyads) deferens between coefficients of dyads' transfer function, using equations (7)

$$\Delta i4_j = |i4_j| - |i1'_j|,\ \Delta i3_j = |i3_j| - |i2'_j|,\quad \Delta i2_j = |i2_j| - |i3'_j|,$$
$$\Delta i1_j = |i1_j| - |i4'_j|.$$

5) Calculate sum of standard deviations of dyads' transfer functions.

$$\Delta = \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i1_j)^2}{8}} + \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i2_j)^2}{8}} + \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i3_j)^2}{8}} + \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i4_j)^2}{8}} \quad (13)$$

A value of an objective function may be defined by the algorithm B.

1) Execute pp. 1-3 of the algorithm A.

2) Calculate coefficients of transfer functions of dyads, using known coordinates of mechanism's joints (4) $i1^*_j$, $i2^*_j$, $i3^*_j$, $i4^*_j$.

3) Calculate difference of coefficients' values of transfer functions of all dyads.

$$\Delta i4_j = |i4_j| - |i4^*_j|,\quad \Delta i3_j = |i3_j| - |i3^*_j|,\quad \Delta i2_j = |i2_j| - |i2^*_J|,$$
$$\Delta i1_j = |i1_j| - |i1^*_J|.$$

4) Calculate sum of differences of dyads' transfer function

$$\Delta = \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i1_j)^2}{8}} + \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i2_j)^2}{8}} + \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i3_j)^2}{8}} + \sqrt{\frac{\sum_{j=1}^{j=8}(\Delta i4_j)^2}{8}} \quad (14)$$

VII. EXAMPLE OF METHOD USING

For the given mechanism (Fig.1) it is required to render the following lows of motion (Table 1).

Using the initial data the linear interpolation was carried out and numerical relationships as a functions was found out: θ = θ(φ), x0 = x0(φ) и y0 = y0(φ). Also was found out the derivatives of these functions: $\theta' = \theta'(\varphi)$, $x0' = x0'(\varphi)$ и $y0' = y0'(\varphi)$.

TABLE I. GIVEN LOWS OF MOTION OF MECHANISM'S ELEMENTS

Angular position of the input link φ, deg.	0	45	55	60	65	69	73
Angular position of the output link θ, deg.	0	19	33	44	52	58	62
Coordinate X of the point O	2.18	2.14	1.98	1.80	1.60	1.42	1.27
Coordinate Y of the point O	0.84	1.07	1.21	1.37	1.52	1.64	1.72

To find the minimum of the objective function (14) was used the genetic algorithm of global optimization and its MATHCAD implementation [4]. To find the minimum of the objective function for all dyads were chosen the intervals of coefficients (8). The objective function was calculated 20 times. As it proved the objective function has many minimums, which correspond to necessary conditions but not sufficient conditions of mechanism existence. The necessary condition is constant lengths of levers. It is achieved as a result of mechanism synthesis. The minimums of the objective functions were achieved for different values of coefficients of transfer function. For these values were found out the coordinates of joints in initial position of the mechanism. Then for all the joints were found out the minimum and the maximum coordinates among the 20 values. In such a way the intervals of initial values of joints' coordinates in initial position were found out under fulfillment of conditions (8). The values of intervals of initial coordinates are given in table 2.

TABLE II. INTERVALS OF INITIAL COORDINATES OF MECHANISM'S JOINTS

coor	**x1**	**y1**	**x2**	**y2**	**x3**	**y3**	**x4**	**y4**	**x5**	**y5**
min	-0,9	-1,5	-5,7	-1,1	-1,4	-5,3	-0,7	-4,1	-1,4	-1,3
max	5,1	3,7	2,5	4,6	3,1	2,4	3,6	2,1	4,6	3,2
coor	**x6**	**y6**	**x7**	**y7**	**x8**	**y8**	**x9**	**y9**	**x10**	**y10**
min	-1,7	-2,5	-1,0	-1,7	-1,1	-1,3	-1,4	-2,1	-0,2	-5,1
max	5,7	1,7	4,6	4,2	1,9	2,0	2,1	2,4	2,2	1,9

The intervals from the table 2 were used in synthesis of the mechanism [3]. Formerly [3] great time was needed to empirical fitting of initial condition that is by no means obvious and very complicated problem. For calculated intervals of coordinates the immediate time of synthesis decreased approximately of 25% when accuracy was kept. The initial conditions (coordinates of joints) of mechanism motion, which was find out by synthesis are given in table 3.

TABLE 3. INITIAL COORDINATES OF POINTS OF THE MECHANISM

Point No	**0**	**1**	**2**	**3**	**4**	**5**
X	2,179	0,712	0,595	-0,259	-0,363	-0,698
Y	0,843	-0,255	-0,218	-0,986	-1,280	-0,928
Point No	**6**	**7**	**8**	**9**	**10**	
X	-0,878	-0,260	0,177	-0,164	-0,415	
Y	-1,104	0,937	0,252	0,679	-0,878	

VIII. CONCLUSIONS, PROSPECTS OF METHOD ENHANCEMENT

The proposed method of finding of initial conditions for synthesis of lever mechanisms is directly connected to the method of synthesis itself and based on structural and kinematic properties of planar lever mechanism. The method makes it possible to find out intervals of initial coordinates of joints on the basis of kinematic condition of a synthesis and structural scheme of a mechanism. The certain limitation of the method is the finite number of iteration of calculations of objective function what it is scanned. It because of an accuracy of defining of intervals bounds depends on number of its derivatives when objective function is scanned. This limitation may be overcome on the basis of interval analysis. By now methods of solution of interval systems of linear algebraic equations are developed. The methods make it possible to define in a short time solution region and applicable domain of such systems. And this region may be multiply connected or disconnected. This corresponds to different variants of the mechanism, which mechanisms realize the same low of motion.

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