Study Kinematic Chains and Distinct Mechanisms

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***Abstract*— A new method is proposed to identify the distinct mechanisms (DMs) of a given kinematic chain in this paper. The kinematic chains (KCs) and DMs are shown in the shape of a [JJ] matrix. Two constant numbers calculated with the help of polynomials of the [JJ] matrix are the sum of the absolute values of the polynomial coefficients (SCPC) and maximum absolute value of the polynomial coefficient (MCPC). These constant numbers have been used as the identification code of a KC and DMs and used to determine all DMs of KC of 1-DOF, 8-bar and 10-bars as well as 2-DOF, 9-bar simple kinematic paired KC. This study will facilitate and help the design engineer to choose the best DM to do a specified task in the conceptual stage of design. The suggested technique needs not any test for isomorphism separately.**

***Keywords*— *KC, DM; [JJ] matrix; SCPC; MCPC*.**

I. INTRODUCTION

In any mechanism design, the systematic basic steps are shown in Figure (a). the first and most important step in structural design is to determine all-possible KC and their DMs from given number and type of bars, number and type of kinematic pairs and DOF, so that the design engineer has a choice to choose the best and economical DM as per the functional requirements. In designing of KC and DMs, duplication is possible. To avoid this duplication, many techniques already have been suggested by a number of researchers. Most of the already suggested techniques in the literature are based on the adjacency matrix based on the number of bars [1] and the distance matrix [2]. For generating DMs of a KC, the bar disposition method [3], the flow matrix method [4] and the row sum of extended distance matrix methods [5] are generally used. Minimum code [6], characteristic polynomial of matrix [7], identification code [8], bar path code [9], summation polynomial [10] etc. are used to characterize the KC. The author feels in connection with these methods that most of the techniques already suggested in the literature are time consuming, unreliable and difficult to apply practically. For example, determination of all DMs from an n-bar KC using flow matrix [4] is a very lengthy and time-consuming process due to the requirement of n-flow matrices. Similarly, row sum of extended adjacency matrix method [5] determines 10 DMs from 6-bar, 1-DOF kinematic chains and 69 DMs from the family of 8-bar, 1-DOF KC instead of the 71, reported by most of the other researchers. Hence, there is a need to develop a computationally efficient method for determining the DM of a KC.

In the present work, a new method is proposed to determine the DMs of a KC. The study of kinematic chain and DMs has shown that the types of bars affect the performance of the kinematic pairs. For this purpose, a new Joint-Joint [JJ] matrix has been defined in this paper. From the [JJ] matrix, the two identification numbers SCPC and MCPC are determined based on the polynomial coefficients of the [JJ] matrix using MAT LAB. These identification numbers are the same for identical mechanisms and different for DMs. These identification numbers are also used to detect isomorphism in the KC.The method is explained with the help of examples of planner KC having all simple kinematic pairs and the results of all the DMs calculated from of 1-DOF, 8-bar and 10-bars KC as well as 2-DOF, 9-bar simple kinematic paired KC are summarized in Table -1.

II. DEFINITION OF TERMINOLOGY

Various types of bars used are shown in Figure (a).

III. THE [JJ] MATRIX

This matrix is defined by (1). Here *n* is the number of kinematic pairs in a KC.

$$[JJ] = \left\{ L_{ij} \right\}_{n \times n} \qquad \text{----------} \ (1)$$

Where

$$L_{ij} \begin{cases} = \text{Type of link between } i^{th} \text{ and } j^{th} \text{ joint} \\ \text{those are directly connected} \\ =0, \text{ if joint i is not directly connected} \end{cases}$$

Thus the form of [JJ] matrix will be:

$$[JJ] = \begin{pmatrix} 0 & L_{12} & L_{13} & - & - & - & L_{1n} \\ L_{21} & 0 & L_{23} & - & - & - & L_{2n} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ L_{n1} & L_{n2} & L_{n3} & - & - & - & 0 \end{pmatrix}$$

IV. CHARACTERISTIC POLYNOMIAL OF [JJ] MATRIX

The characteristic polynomial of [JJ] matrix is given by D (λ). The polynomial of degree n is given by (2).

$$|(JJ-\lambda I)| = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \text{-------}\ a_{n-1}\lambda + a_n \quad \text{----(2)}$$

Where; n = number of kinematic pairs in a KC and

1, a_1, a_2, a_{n-1}, a_n are the polynomial coefficients.

The two important properties of the polynomials are

1. SCPC is constant of a [JJ] matrix.

   $|1| + |a_1| + |a_2| + \text{-----} + |a_{n-1}| + |a_n| = \text{constant}$

2. MCPC is also constant of a [JJ] matrix.

A. Identification Numbers 'SCPC' and 'MCPC'

These identification numbers are quite unique for a [JJ] matrix and have been used to check the isomorphism among KC and DMs.

B. Isomorphism among Kinematic Chains.

Theorem: Two similar square symmetric matrices have the same characteristic polynomial. [16].

Proof:

When two KC are shown by the two identical matrices C and D such that $D = P^{-1}CP$, taking into account that the matrix λI commutes with the matrix P and $|P^{-1}| = |P|^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have

$|D-\lambda I| = |P^{-1}CP-\lambda I| = |P^{-1}(C-\lambda I)P| = |P^{-1}||(C-\lambda I)||P| = C-\lambda I|$

Hence, D (λ) of matrix 'C' = D (λ) of matrix 'D'

D (λ) =polynomial of the matrix.

It suggests that if D (λ) of two [JJ] matrices of two KC is same, their identification numbers 'SCPC' and 'MCPC' should be same and the two KC should isomorphic otherwise non-isomorphic KC.

V. CHECKING OF EQUIVALENT BARS AND DMs

A KC is shown by the [JJ] matrix. If in the [JJ] matrix, the diagonal elements of the corresponding kinematic pairs of the fixed bar 'a' are exchanged from 0 to 1, it will be the first mechanism with fixed bar 'a'. Then this new [JJ] matrix is shown by [JJ-a] matrix. The identification numbers of this [JJ-a] matrix are determined. These identification numbers 'SCPC-a' and 'MCPC-a' are the unique for this mechanism. In the same way, other bars are fixed alternatively.

In n-bar mechanisms; let

N_e = number of sets of equivalent edges

N_d = number of bars having distinct identification number

N_t = total number DMs

So, $N_t = (N_e + N_d)$

VI ILLUSTRATIVE EXAMPLE-1:

The first example concerns with 8–bars, 10- kinematic pairs, 1- DOF shown in Fig.1.

The DOF of bars of KC shown in Fig.1 is:

$l_a = l_b = l_c = l_d = 3$ and $l_e = l_f = l_g = l_h = 2$.

The [JJ] matrix of the KC [Fig.1] is $[JJ]_1$.

$$[JJ]_1 = \begin{pmatrix} 0&3&0&3&3&0&3&0&0&0\\ 3&0&3&0&0&0&3&3&0&0\\ 0&3&0&3&0&0&0&3&0&3\\ 3&0&3&0&3&0&0&0&0&3\\ 3&0&0&3&0&2&0&0&0&0\\ 0&0&0&0&2&0&2&0&0&0\\ 3&3&0&0&0&2&0&0&0&0\\ 0&3&3&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&2&0&2\\ 0&0&3&3&0&0&0&0&2&0 \end{pmatrix}$$

The set of Identification Numbers for the $[JJ]_1$ matrix are:

SCPC = 3.0011e+005 and MCPC =134784

Matrix of the Mechanisms

First of all, we fixed bar 'a', This bar 'a' is a ternary bar having kinematic pairs 1, 2 and 7. So ,we exchange the diagonal element L_{11}, L_{22} and L_{77} from 0 to 1 and [JJ-a] is the matrix of this first mechanism.

$$[JJ\text{-}a] = \begin{pmatrix} 1&3&0&3&3&0&3&0&0&0\\ 3&1&3&0&0&0&3&3&0&0\\ 0&3&0&3&0&0&0&3&0&3\\ 3&0&3&0&3&0&0&0&0&3\\ 3&0&0&3&0&2&0&0&0&0\\ 0&0&0&0&2&0&2&0&0&0\\ 3&3&0&0&0&2&1&0&0&0\\ 0&3&3&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&2&0&2\\ 0&0&3&3&0&0&0&0&2&0 \end{pmatrix}$$

The Identification Numbers of the first mechanism are :

SCPC -a = 3.4310e+005 , MCPC -a = 1.6407e+005

In the same way, the bars b, c, d, - - - etc are fixed alternatively and the Identification Numbers of the corresponding mechanisms are calculated. Identification Numbers of second, third, fourth, ------ mechanisms are as follows.

SCPC -b = 3.4310e+005, MCPC -b = 164070

SCPC -c = 3.4310e+005, MCPC -c = 1.6407e+005

SCPC -d= 3.4310e+005, MCPC -d = 1.6407e+005

SCPC -e = 4.5503e+005, MCPC -e = 2.3069e+005

SCPC -f = 4.5503e+005, MCPC – f = 2.3069e+005

SCPC -g = 4.5503e+005, MCPC – g = 2.3069e+005

SCPC -h = 4.5503e+005, MCPC -h = 2.3069e+005

Visualizing the identification numbers of the above eight mechanisms, we see that the Identification Numbers of bar – a, b, c and d are identical. So, they are identical bars and constitute one DM. In the same way, the identification numbers of bar - e, f, g and h are same, so, they also constitute one DM.

$N_e = 2$,$N_d = 0$, $N_t = 2+0=2$

Clearly, 2 DMs are achieved from KC shown in Fig.1

VI. ILLUSTRATIVE EXAMPLE -2

The 2nd example concerns with 9-bars, 11- kinematic pairs, and 2 -DOF KC shown in Fig.2.

Degree of the bars

The DOF of bars of KC shown in Fig.2 is:

$l_a = l_d = l_e = l_i = 3$ and $l_b = l_c = l_f = l_g = l_h = 2$.

[JJ] Matrix

The $[JJ]_2$ is matrix of the KC [Fig.2].

$$[JJ]_2 = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 \\ 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 3 \\ 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 0 & 3 \\ 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 \end{pmatrix}$$

Identification Numbers of Matric $[JJ]_2$ are:

SCPC = 4.2033e+006 MCPC = 1.8662e+006

The Identification Numbers of the other DMs are :

SCPC -a = 3.8866e+006, MCPC -a = 1.4446e+006

SCPC -b =5.2285e+006 , MCPC -b = 2.1002e+006

SCPC -c = 5.3292e+006 , MCPC -c = 2.1079e+006

SCPC -d= 4.1787e+006 , MCPC -d = 1.6383e+006

SCPC -e =3.8866e+006 , MCPC -e = 1.4446e+006

SCPC -f = 3.9733e+006 , MCPC – f = 1.4930e+006

SCPC -g =5.2285e+006 , MCPC – g = 2.1002e+006

SCPC -h =5.3292e+006 , MCPC -h = 2.1079e+006

SCPC -i = 4.1787e+006 , MCPC -i = 1.6383e+006

Visualizing the above identification numbers, we see that the identification numbers of bar ‘a’ and ‘e’ are same and they make only one DM. in the same way, the identification numbers of bar ‘b’ and ‘g’ are same and they also make another one DM., In the similar way bar ‘c’ and ‘h’, bar ‘d’ and ‘i’ make another third and fourth DM. bar ‘f’ has different identification numbers and makes the fifth DM.

$N_e = 4$,$N_d = 1$, $N_t = 4+1=5$

So, we are getting here 5 DM from given KC .

VII. ILLUSTRATIVE EXAMPLE-3

The third example concerns of 10-bar, 12-kinematic pairs, and 3-DOF, two KC shown in Fig.3 and Fig.4. We have to see that both these KC are isomorphic or not.

The DOF of the bars of KC shown in Fig.3 is

$l_a = l_e = l_f = l_h = 3$ and $l_b = l_c = l_d = l_g = l_i = l_j = 2$

Similarly, the DOF the bars of KC shown in Fig.4 is

$l_a = l_d = l_f = l_j = 3$ and $l_b = l_c = l_e = l_g = l_h = l_i = 2$

$[JJ]_3$ and $[JJ]_4$ are the matrices of the KC shown in Fig. 3 and Fig.4.

$$[JJ]_3 = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 2 & 0 & 0 & 0 \end{pmatrix}$$

$$[JJ]_4 = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 \end{pmatrix}$$

The identification numbers of the KC shown in Fig.3 and Fig.4 are as follows.

For Fig.3:

SCPC = 8.3734e+006 and MCPC =3.5938e+006

For Fig.4:

SCPC = 7.0147e+006 and MCPC =2.9393e+006

The identification numbers of the two KC are different. So, both the KC are non-isomorphic.

VIII. RESULTS

The suggested identification numbers SCPC and MCPC detect isomorphism in KC and even in KC successfully. The results of DM calculated from 1-DOF, 8-bar, and 1-DOF, 10-bar along with 2-DOF, 9-bar are with the author. A brief summary of these results are listed in Table-1.

IX. CONCLUSION

In this paper, suggested method is new and easy to apply for the identification of DM of a given KC. The two identification numbers SCPC and MCPC are determined polynomial of the [JJ] matrix. These identification numbers detect the isomorphism among the KC. Determination of DM using other methods already available in the literature are more time consuming and not easy to apply.

Table-1
Results obtained by applying the proposed method to several cases of structural analysis and synthesis treated in earlier literature.

Sl. No.	Structural synthesis and analysis problem	No. of simple jointed KC	DM by Ref[14]	DM by present method[19]	Remarks
1	1-F,8-link, simple jointed KC	16	71	71	For detail, see Table-2
2	1-F,10-link, simple jointed KC	230	1834	1842	For detail, see Table-2
3	2-F,9-link, simple jointed KC	40	254	264	For detail, see Table-3

Table-2
The number of DMs calculated from 1-DOF, KC of 8 and 10 links

Class	Group	No. of links	n	li=2	li=3	li=4	li=5	KC	DMs.	TDMs
III	a	8	10	4	4	-	-	9	35	71
	b	8	10	5	2	1	-	5	31	
	c	8	10	6	0	2	-	2	05	
IV	a	10	13	4	6	-	-	50	342	1842
	b	10	13	5	4	1	-	95	870	
	c	10	13	6	3	0	1	15	126	
	d	10	13	6	2	2	-	57	415	
	e	10	13	7	1	1	1	8	64	
	f	10	13	7	0	3	-	3	20	
	h	10	13	8	0	0	2	2	05	

Table-3
DMs of 9-link, 2-DOF, 40 KC.

KC no by Ref[18]	DM	KC no by Ref[18]	DMs	KC no by Ref[18]	DMs	KC no by Ref[18]	DMs
1	5	11	5	21	6	31	8
2	6	12	5	22	9	32	9
3	6	13	5	23	7	33	8
4	4	14	5	24	9	34	7
5	5	15	5	25	4	35	5
6	4	16	4	26	9	36	9
7	6	17	9	27	9	37	6
8	8	18	6	28	8	38	8
9	5	19	7	29	9	39	9
10	5	20	9	30	5	40	6

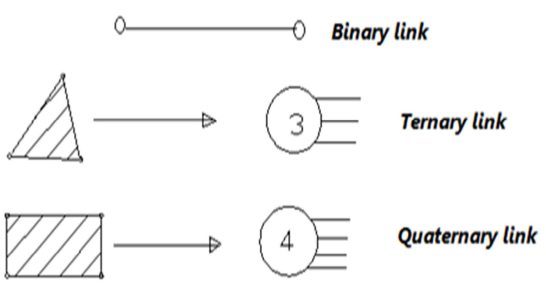

Figure (b): types of links

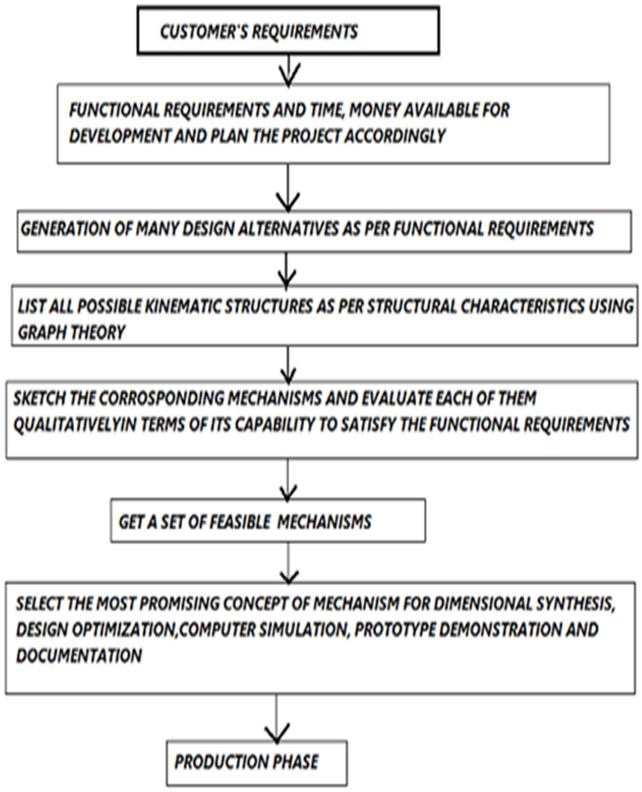

Figure (a): Basic Steps in Systematic Mechanism design

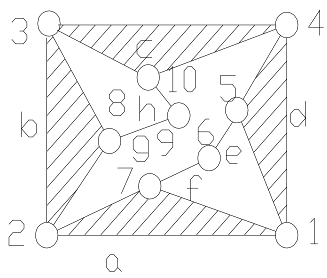

Fig.1:Eight-link,single degree of freedom KC

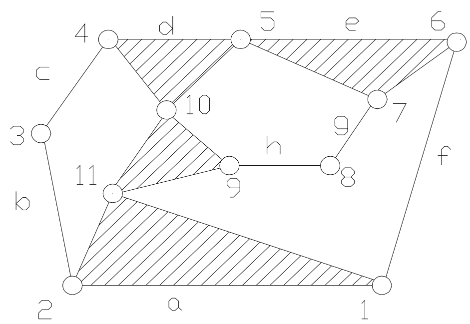

Fig.2:Nine-link,two-degree of freedom KC

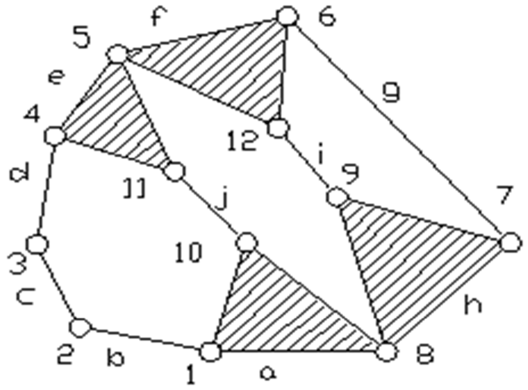

Fig.3: ten-bar chain, three freeodm

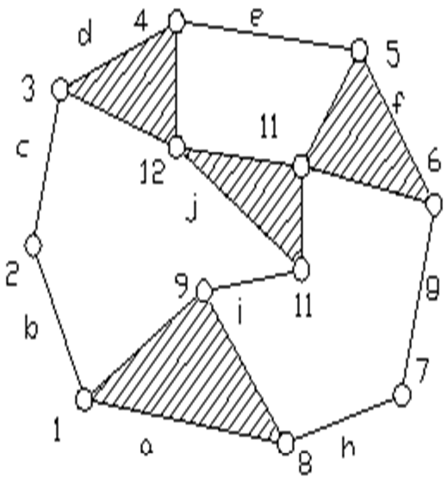

Fig.4: ten-bar choin, three freeodm

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