Loop Based Algorithm for Automatic Sketching of Planar Kinematic Chains

Gurusharan Singh Bedi
Department of Mechanical Engineering
Government Polytechnic
Khairagarh, India
g_sbedi@rediff.com

Shubhashis Sanyal
Department of Mechanical Engineering
National Institute of Technology
Raipur, India
Sanyal5366@gmail.com

***Abstract*— Methods for systematic enumeration of kinematic chains essentially involves sketching of kinematic chains. Search for a simple and reliable procedure for automatic sketching of mechanisms is still on. In the present work a simple and reliable method, that identifies the joints and loops as the basic constituents of a kinematic chain, is presented. The development and application of the algorithm is demonstrated with the help of several examples.**

Keywords—automatic sketching; basic loop; kinematic chain; known joint; unknown joint; joint–loop adjacency matrix

I. INTRODUCTION

An automated sketching procedure is always desirable to enable the designer to visualize the topology of kinematic chains and select a suitable kinematic chain for a specific application. Various researchers have proposed methods for the automatic sketching of kinematic chains. Freudenstein and Dobrjansky [1] were the first to attempt to have a graphical output of the kinematic chains. Woo [2] adopted a rule based approach for sketching of chains without crossing of links. Olson et al [3] proposed a method based on heuristics to avoid crossing of links. Cheing and Hoeltzel [4] also proposed a method to avoid crossing of links. Belfiore and Pennestri [5] proposed a procedure using graph mapping and embedding techniques. All these works made use of graph theory for the automatic sketching of kinematic chains. Mauskar and Krishnamurthy [6] used an entirely different approach based on inter-loop relationships for automatic sketching of chains. Xiao et al [7] presented an improved approach to automatic sketching of planar kinematic chain from its adjacent matrix. The basic loops of the graph, which correspond to the adjacent matrix of kinematic chain, are derived by means of breadth-first spanning tree. Then these loops are standardized and configured. Finally, the graph of kinematic chain is automatically sketched in terms of the configured loops. Ding et al [8] presented the synthetic degree-sequence based on canonical perimeter topological graph of kinematic chains. The characteristic adjacency matrix is derived from the canonical perimeter topological graph and a characteristic representation code is created, through the combination of the synthetic degree-sequence and the characteristic adjacency matrix, to represent the kinematic chains. On the basis of the characteristic representation code, the chains are sketched. Hsieh et al [9] using Visual Basic package developed a computer program for generating and sketching of kinematic chains. However the methods referred above are complex and involves lot of mathematical manipulations.

In the proposed work, a simple method, using the basic constituents of a kinematic chain – the joints and loops, is presented to sketch the parent kinematic chain. The kinematic chain is constructed loop by loop by sketching the joints present in that particular loop since each loop is unique in nature with respect to the joints contained. The Joint – Loop adjacency and Joint – Joint adjacency matrices are used as input to establish the connectivity of the kinematic chain. The algorithm is coded in ‘C’ language.

II. TERMINOLOGY

The terms used in the paper are defined below:

A. Loops: A loop in a kinematic chain can be defined as a closed polygon wherein one can travel from one joint and back to it traversing at least four joints.

The number of loops in a multi loop kinematic chain is given by (1):

$$L = ((N + 1) - F)/2 = J - N + 2 \quad (1)$$

Where, J = Number of joints
F = Degree of freedom of the chain
N = Number of links and
L = Number of independent loops plus one peripheral loop

B. Independent loop: The independent loop does not contain any other loops within. These loops for any chain can be easily identified by visual inspection and do not depend upon the manner in which the chain is drawn.

C. Sub loop: The sub loops are combinations of the independent loops.

D. Peripheral Loop: The sub loop for any chain that includes all the loops is known as the peripheral loop. It is in fact a combination of the independently existing loops and contains maximum number of joints in it.

E. Maximum Possible loops: The total number of maximum possible loops $\mathbf{N_L}$ in a planar kinematic chain can be obtained by (2):

$$N_L = {}^kC_1 + {}^kC_2 + \text{------------} + {}^kC_n \quad (2)$$

Where, k = L – 1 (i.e. number of independent loops) and ${}^kC_n$ is 'n' number of loops taken together.

F. Basic loops: Basic loops in a kinematic chain consist of the independent loops and the peripheral loop taken together.

G. Joint–Loop Adjacency Matrix (JLAM): Joint–Loop Adjacency Matrix is a J x L matrix, elements of which are 0 or 1 as defined by (3):

JLAMij (J x L) =1, if i^{th} *joint is a constituent of* j^{th} *loop,*
= 0, otherwise. (3)

Where, J = Number of joints in kinematic chain and
L = Number of basic loops in kinematic chain

H. Joint–Joint Adjacency Matrix (JJAM): Joint –Joint Adjacency Matrix is a square and symmetrical matrix of order J, elements of which are 0 or 1 as defined by (4):

JJAMij (J x J) = 1, if i^{th} *joint is connected to* j^{th} *joint,*
= 0, otherwise. (4)

For example, consider the eight-link single degree of freedom kinematic chain with 10 joints shown in Fig.1:

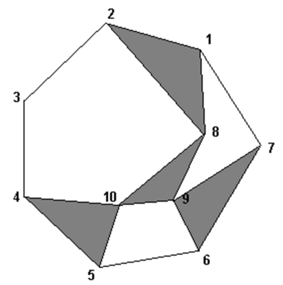

Fig. 1. Eight-link single degree of freedom kinematic chain

$L = J - N + 2$ = 10 – 8 + 2 = 4 (i.e. 3 independent loops plus 1 peripheral loop)

$k = L - 1$ = 4 – 1 = 3

$N_L = {}^3C_1 + {}^3C_2 + {}^3C_3$ = 3 (individual loops) + 3 (loops with two individual loops taken together) + 1(one loop with three individual loops taken together) = 7

Thus the chain can have 7 maximum possible loops, which can be identified in terms of joints as

L1 = 1-7-8-9
L2 = 2-3-4-8-10
L3 = 5-6-9-10
L4 = 1-2-3-4-5-6-7 (Peripheral Loop)
L5 = 1-2-3-4-10-9-7
L6 = 2-3-4-5-6-9-8
L7 = 1-8-10-5-6-7

Loops L1, L2 and L3 are independent loops whereas Loops L4, L5, L6 and L7 are sub loops i.e. combinations of loops L1, L2 and L3 and are not required for sketching process. Only Basic loops are taken for sketching of the chain.

The JLAM and JJAM are shown below by (5) and (6).

JOINTS	LOOPS			
	1	2	3	4
1	1	1	0	0
2	1	0	1	0
3	1	0	1	0
4	1	0	1	0
5	1	0	0	1
6	1	0	0	1
7	1	1	0	0
8	0	1	1	0
9	0	1	0	1
10	0	0	1	1

(5)

Joints	1	2	3	4	5	6	7	8	9	10
1	0	1	0	0	0	0	1	1	0	0
2	1	0	1	0	0	0	0	1	0	0
3	0	1	0	1	0	0	0	0	0	0
4	0	0	1	0	1	0	0	0	0	1
5	0	0	0	1	0	1	0	0	0	1
6	0	0	0	0	1	0	1	0	1	0
7	1	0	0	0	0	1	0	0	1	0
8	1	1	0	0	0	0	0	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	1	1	0	0	1	1	0

(6)

III. Procedure for Sketching of Chains

The process of sketching kinematic chains begins with preparation of Joint - Loop and Joint – Joint adjacency matrices. The step by step procedure for sketching of kinematic chains is detailed below:

Step 1: The JLAM and JJAM required for sketching is prepared.

Step 2: The 'Basic Loops' required for sketching are identified from JLAM. The column wise non-zero entries in the JLAM gives the loops required for sketching.

Step 3: The joints in all the loops are arranged in cyclic order of connectivity. Step by step the connectivity of each joint with the one immediately preceding is established using the JJAM. The joint which is not directly connected to the previous is shifted to the end and the process continues until the cyclic order of connectivity is established.

Step 4: The loops are then arranged in descending order of number of joints.

Step 5: The loop containing the maximum number of joints is termed as the 'Peripheral Loop' and is drawn first as a regular polygon. If two or more loops have equal number of joints, the loop that comes first in order in JLAM is sketched as peripheral loop.

Step 6: Next the known and unknown joints of each loop are identified. Known joints are those joints whose locations has been already identified and Unknown joints are those whose locations are to be identified.

Step7: On the basis of number of known and unknown joints the remaining loops are categorized and the method of obtaining their locations is detailed below:

(i) Loops with 1 Unknown joint.

In loops with one unknown joint the location of unknown joint is obtained as the center of polygon, formed by known joints of loop and center of peripheral loop, Fig. 2.

Peripheral loop: a-b-c-d-e-f-g-h

Inner loop: a-b-c-d-i with unknown joint i.

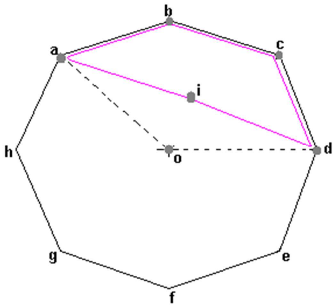

Fig. 2. Loop with one unknown joint

(ii) Loops with equal number of Known and Unknown joints.

The locations of unknown joints are obtained as the midpoint of line joining center of peripheral loop and corresponding known joint, Fig.3.

Peripheral loop: a-b-c-d-e-f-g-h

Inner loop: a-b-c-k-j-i, with unknown joints i, j, k.

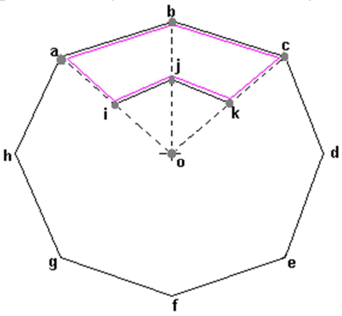

Fig. 3. Loops with equal number of Known and Unknown joints

(iii) Loops with number of Known joints greater than number of Unknown joints.

The locations of unknown joints are obtained as the centroid of the triangles formed using 2 known joints and center of peripheral loop. The number of triangles formed will be equal to number of unknown joints, Fig. 4.

Peripheral loop: a-b-c-d-e-f-g-h

Inner loop: a-b-c-i-j, with unknown joints i, j.

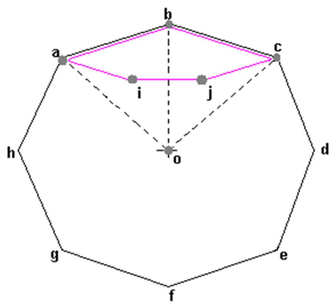

Fig. 4. Loops with number of Known joints greater than number of Unknown joints

(iv) Loops with number of Unknown joints greater than number of Known joints.

In this case locations of unknown joints are obtained using following steps:

(a) The two extreme unknown joints are obtained as the midpoint of line joining center of peripheral loop and corresponding known joint. Thus now number of unknown joints is reduced by 2 and number of known joints is increased by 2, Fig. 5(a).

(b) Even after resetting joints, if the number of known joints is less than the number of unknown joints, then step 1 is repeated till any of the first three cases is obtained and after that remaining unknown joints of loop are obtained using applicable case, Fig. 5(b).

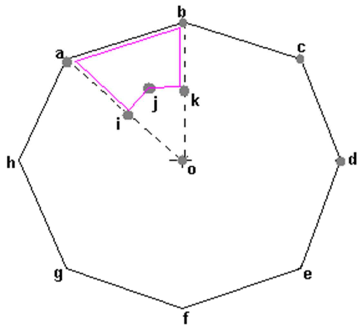

Fig. 5(a)

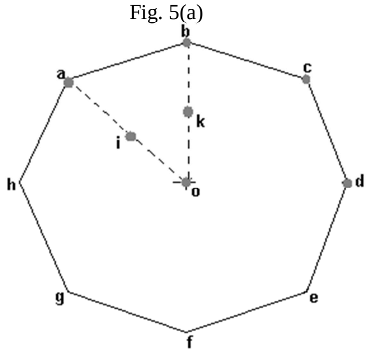

Fig. 5(b)

Step 8: The loops are drawn in the same sequence as mentioned above. After sketching a loop the number of known and unknown joints in remaining loops is recalculated and the procedure is repeated until all the loops are sketched.

Step 9: Once all the joints of kinematic chain has been obtained, the polygonal links are identified by colouring the loops.

IV. APPLICATIONS

A. The entire process of sketching, as detailed above, is explained with an example of eight-link single degree of freedom planar kinematic chain, Fig.1. For sake of brevity, the JLAM and JJAM are not reproduced here again.

Step 1: Identification of basic loops from JLAM.

The basic loops as identified from the JLAM are:

L1 – 1-2-3-4-5-6-7
L2 – 1-7-8-9
L3 – 2-3-4-10-8

L4 – 5-6-9-10

Step 2: Arrange the joints in all the loops in cyclic order of connectivity. For example the order of occurrence of the joints in the Loop 2 as shown above, Step 1, is 1-7-8-9. The order of connectivity is continuously reshuffled until the cyclic order of connectivity is established i.e.

Initially we have joint order as 1-7-8-9

Starting from joint 1, since in JJAM (1, 7) = 1, no shuffling will take place, and Joint order will remain as 1-7-8-9

In JLAM (7, 8) = 0, so shift joint 8 to the end, the Joint order will look as 1-7-9-8

In JLAM (7, 9) = 1, again next pair of joints will be tested JLAM (9, 8) = 1, the final Joint order will be 1-7-9-8

We have all the joints of the loop arranged in cyclic order of connectivity. This process is performed for all the loops.

Step 3: The loops are arranged in descending order of number of joints:

L1– 1-2-3-4-5-6-7
L2 – 2-3-4-8-10
L3– 1-7-9-8
L4– 5-6-9-10

Step 4: Loop L1, containing maximum number of joints, is drawn as peripheral loop, Fig. 6(a).

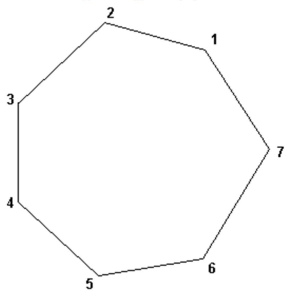

Fig. 6(a) Peripheral Loop, L1

Step 5: Now the numbers of known and unknown joints are identified. L2 have 3 known and 2 unknown joints and both L3 and L4 have 2 known and 2 unknown joints each. Loop L4 is drawn first, Fig. 6(b).

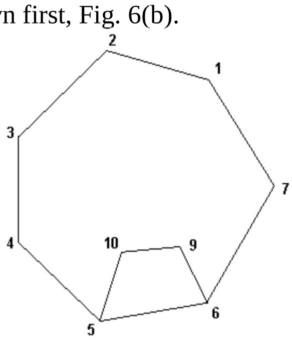

Fig. 6(b) Loop L4

Step 6: Now, only one joint i.e. 8 remains to be drawn and both loops L2 and L4 have now one unknown joint. Loop L3 is drawn next, Fig. 6(c).

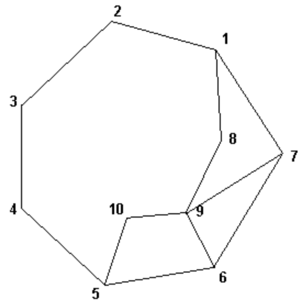

Fig. 6(c) Loop L3

1-7-8-9

Finally the remaining loop L2 (2-3-4-8-10) is drawn, Fig. 6(d).

1-7-9-8

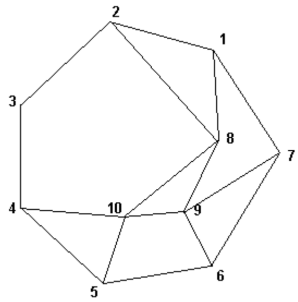

Fig. 6(d) Loop L2

Step 7: Now polygonal links are identified by coloring the loops in the order they are made, Fig. 6(e). 1-2-8, 4-5-10, 6-7-9 and 8-9-10 are the polygonal links (Ternary links in this case).

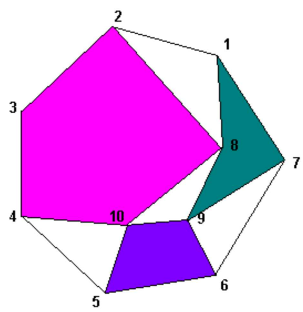

Fig. 6(e) Coloring the Loops

B. Sketching of Fractionated Chains

Certain chains of ten - link Single Degree of Freedom kinematic chains are Joint-fractionated chains. In joint-fractionated chains a joint divides the chain into two closed independent kinematic sub-chains. These chains have been termed as Double Periphery chains as they have two peripheral loops.

In these chains at least one loop called, Isolated Loop does not have any joint in common with any of the remaining loops including the peripheral loop. For sketching such chains one extra loop is added as second peripheral loop, which has one joint common with the first peripheral loop i.e. the cut-set joint, and one or more joints common with the isolated loop.

A ten – link single degree of freedom fractionated kinematic chain is shown in Fig.7.

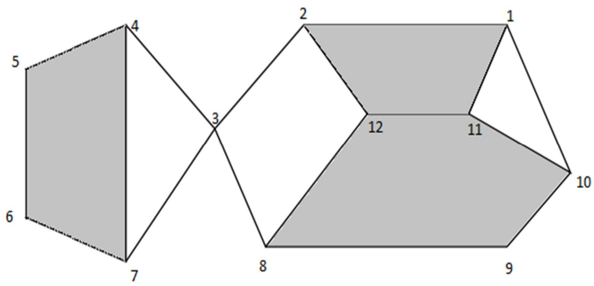

Fig. 7 Ten - link Single Degree of Freedom Fractionated kinematic chain

The JLAM and JJAM for the chain are given below:

JOINTS	LOOPS				
	1	2	3	4	5
1	1	0	1	0	0
2	1	0	1	0	0
3	1	0	0	0	1
4	0	0	0	1	1
5	0	0	0	1	1
6	0	0	0	1	1
7	0	0	0	1	1
8	1	1	0	0	0
9	1	1	0	0	0
10	1	1	0	0	0
11	0	1	1	0	0
12	0	1	1	0	0

JLAM

JOINTS	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	0	0	0	0	0	0	0	1	0	1
2	1	0	1	0	0	0	0	0	0	0	1	0
3	0	1	0	1	0	0	1	1	0	0	0	0
4	0	0	1	0	1	0	1	0	0	0	0	0
5	0	0	0	1	0	1	0	0	0	0	0	0
6	0	0	0	0	1	0	1	0	0	0	0	0
7	0	0	1	1	0	1	0	0	0	0	0	0
8	0	0	1	0	0	0	0	0	1	0	1	0
9	0	0	0	0	0	0	0	1	0	1	0	0
10	1	0	0	0	0	0	0	0	1	0	0	1
11	0	1	0	0	0	0	0	1	0	0	0	1
12	1	0	0	0	0	0	0	0	0	1	1	0

JJAM

The Basic loops for the chain shown above are identified as:

L1 – 1-2-3-8-9-10
L2 – 8-9-10-11-12
L3 – 1-2-11-12
L4 – 4-5-6-7
L5 – 3-4-5-6-7

Since this is a case of double periphery, first 3 loops will be part of first periphery and remaining will be of second. The rest of the procedure for sketching the chain is same as explained above in section IV A.

The proposed method is successfully applied to sketch single degree of freedom planar kinematic chains with six, eight and ten links, two degrees of freedom with nine links and three degrees of freedom with ten links. The rule based algorithm for the sketching of planar kinematic chains presented here provides a systematic and consistent way for sketching single and multiple degree of freedom chains. The algorithm is coded in 'C'. The interactive program requires minimal computation and is robust. The time required on Pentium PIV was less than a second. The results indicate that the method is valid and efficient in generating the sketches of kinematic chains irrespective of number of links or degrees of freedom. It is expected that the integration of this procedure in mechanism design will enable the visual inspection of wider selection of candidate chains

CONCLUSIONS

1. The method uses the joints and loops, the basic constituents of a kinematic chain, to sketch the parent kinematic chain.
2. The method has been successfully applied to planar kinematic chains with six – links, eight – links, nine – links and ten – links.
3. The method is simple and reliable.
4. The kinematic chains including the crossed link kinematic chains are sketched loop by loop with ease.

REFERENCES

[1] L. Dobrjanskyi and F. Freudenstein, "Some applications of graph theory to the structural analysis of mechanisms," ASME Journal of Engineering Industry, vol. 89, 153-158, 1967.

[2] L. S. Woo, "Type synthesis of plane linkages," ASME Journal of Engineering Industry, vol. 89, 159-172, 1967.

[3] D. G. Olson, T. R. Thompson, D. R. Riley and A. G. Erdman, "An algorithm for automatic sketching of planar kinematic chains," ASME Journal of Mechanisms, Transmission and Automation in Design, vol. 107, 106-111, 1985.

[4] W. Cheing and D. A. Hoeltzel, "Sketching of planar kinematic chains and epicyclic gear trains," ASME Journal of Mechanical Design, vol. 112, 6-15, 1990.

[5] N. P. Belfiore and E. Pennestri, "Automatic sketching of planar kinematic chains," Mechanism and Machine Theory, vol. 29, 177-193, 1990.

[6] S. Mauskar and S. Krishnamurty, "A loop configuration approach to automatic sketching of mechanisms," Mechanism and Machine Theory, vol. 31, 423-437, 1996.

[7] R. Xiao, Y. Liu and D. Wang, "An improved approach to automatic sketching of mechanism based on loop configuration and its implementation," Proceedings of DETC'02, ASME 2002 Design Engineering Technical Conferences and Computers and Information in Engineering Conference, 2002.

[8] H. Ding and Z. Huang, "A new theory for the topological structure analysis of kinematic chains and its applications," Mechanism and Mechanism Theory, vol. 42, 637-651, 2006.

[9] C. F. Hsieh, Y. W. Hwang, H. S. Yan, "Generation and sketching of generalized kinematic chains," Proceedings of the ASME 2008 Design Engineering Technical Conferences and Computers and Information in Engineering Conference, 2008.