Synthesis of Adjustable Offset Slider-Crank Mechanism for Simultaneous Generation of Function and Path using Variable-Length Links

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***Abstract*— This paper presents a new method to design an adjustable offset slider-crank mechanism to generate a function and a path simultaneously with the lengths of the input link and the link representing offset (henceforth called offset link) varying, without any limitation on the number of precision points. The mechanism comprises of the original offset slider crank mechanism along with a ternary link, a roller link and a guiding slot in the fixed link for each of the variable-length links. An n-degree polynomial is used to design the contours of the guiding slots, n being the number of precision points. A case study is provided to verify the feasibility of this new synthesis method.**

Keywords*— adjustable mechanism; offset slider-crank mechanism; simultaneous task; variable-length links

I. INTRODUCTION

While using kinematic devices or mechanisms, certain applications may not be represented by a single task. It is conceivable that a task may require an object to be moved along a trajectory on which the orientation of the object may be important at a few points while restriction on orientation could be relaxed at others. Furthermore, the task may require that a functional input/output relation exists at a few points along the trajectory. This scenario calls for hybrid task synthesis where the entire motion cycle becomes active, i.e., during a single crank rotation the same mechanism performs different subtasks. Peñuñuri, Peón-Escalante, Villanueva and Pech-Oy [1] developed an optimum synthesis method of mechanism for single and hybrid tasks using differential evolution (DE). Smaili and Diab [2] applied an ant-gradient (AG) algorithm to the optimum synthesis of hybrid task mechanisms. Virtually little work has been done in this area particularly where the subtasks are performed simultaneously, i.e., in the same range of motion of the crank.

Adjustable mechanisms allow any number of precision points to be utilized in any kind of task (function/path/motion generation). Numerous works are found on adjustable mechanisms generating flexible outputs with the same set of hardware with one or more parameters adjustable. Zhou and Ting [3] introduced an optimal synthesis model of adjustable slider-crank linkages for multiple path generation based on position structural error of slider guider. Soong and Wu [4] presented a method for designing variable coupler curve 4R mechanisms with one link replaced by an adjustable screw-nut link and driven by a servomotor. Soong and Chang [5] proposed a design method to solve function generation problems for 4R linkages using variable-length driving link. Zhou [6] developed a synthesis method for adjustable function generation linkages using the optimal pivot adjustment. Zhou and Cheung [7] put forward an optimal synthesis method of adjustable 4R linkages for multiphase motion generation. Many other published works are there in the field of adjustable mechanisms but none of them is on synthesis of adjustable linkages for simultaneous task generation.

The objective of the present work is to introduce a design process of adjustable offset slider-crank mechanism for function generation along with path generation with prescribed timing simultaneously. The lengths of input link and offset link are made adjustable. While the coupler point traces a desired path with prescribed timing (i.e. coordinated with the input link rotation), the slider displacement generates desired input-output functional relation. The desired function and path are generated simultaneously in the same range of input motion.

This new method of design helps in simultaneous generation of a path and a function by a single mechanism. A case study is done to demonstrate the effectiveness of the proposed approach of synthesis. Transmission angle [8] is used as the measure of the quality of motion transfer of the mechanism. An AutoLisp code is generated and is run on AutoCAD 2007 platform to give an effect of virtual animation in order to simulate the linkage that is synthesized here. The new mechanism and the new design process are presented in the following sections.

II. VARIABLE-LENGTH LINKAGE FOR SIMULTANEOUS GENERATION OF FUNCTION AND PATH

Fig. 1 represents the adjustable offset slider-crank mechanism for simultaneous generation of function and path (with prescribed timing). The lengths of the input link 2 and the offset link 4 are made variable. The adjustment for the length of the input link 2 is achieved by sliding a ternary link 6 over the input link 2. The ternary link 6 is again connected to the coupler 3 through the simple hinged joint and a roller link 7 by its protruded part at the back surface, as shown in the exploded view in Fig. 1. When the input link 2 is rotated, the roller link 7 moves along the fixed guiding slot 2. This constrained movement results in the sliding action of the ternary link 6 over the length of the input link 2. The guiding slot 2 functions as a fixed cam while the roller link 7 moves over it as a follower. While adjusted lengths of the input link 2 are achieved dynamically by driving it according to required input angle, desired length-variations of the offset link 4 can be obtained from the corresponding angular positions of the same or that of the slider guider 1, which is rigidly fixed to the binary link 8 over the offset link 4 and is thus always perpendicular to the latter. This adjustment works in the same way as that of the input link 2, i.e. by the constrained movement of the roller link 9 along the guiding slot 4 and the sliding action of the binary link 8 over the length of the offset link 4. So the angular positions of the offset link 4 are not inputs but are only obtained by using programmed servomotors to rotate it through the required angle to automatically adjust the length of the same. Coordinate system for the adjustable mechanism is shown in Fig. 2.

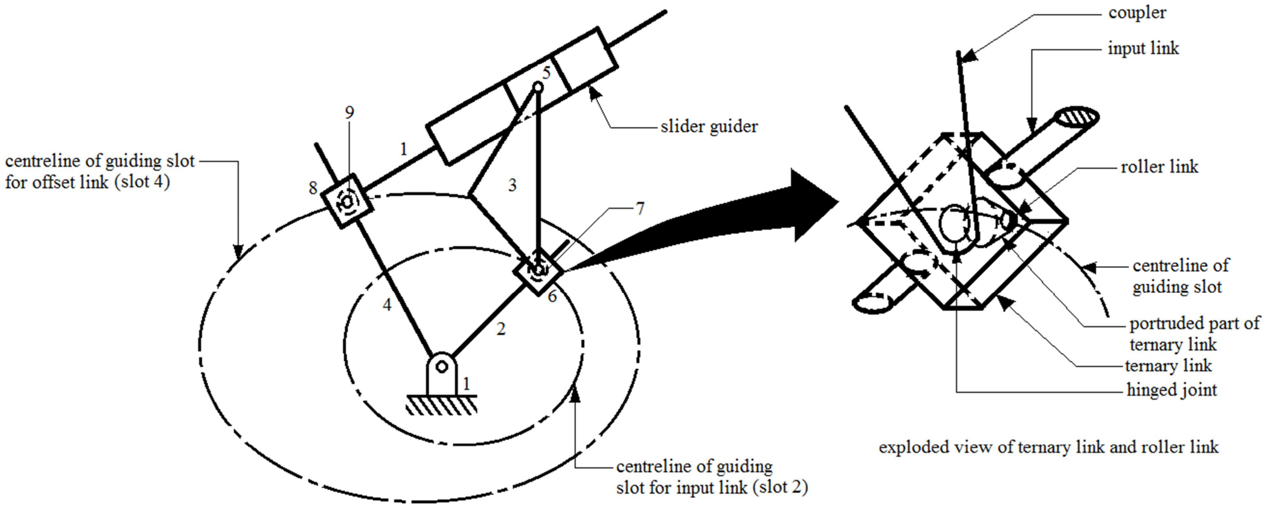

Fig. 1. Original mechanism with adjustability

Without the length adjustments, the desired functional relationship $s = f(\theta_2)$ and the desired path can only be generated approximately by the linkage. The actual function and path generated by the non-adjustable linkage matches the desired function and path only at a limited number of precision points. The difference between the actual function and the desired function is the structural error, which varies between the precision points and depends on where the precision points are placed in the range. To minimize the magnitude of the error the optimum spacing of accuracy points is that for which all the maxima and minima of the error curve as well as the error values at the extremities of the range of crank rotation are of the same magnitude. The exact spacing of accuracy points yielding the optimum situation depends on the task to be generated as well as the mechanism itself. To reduce the difference between the actual function/path generated and the desired function/path, the precision points may be chosen through Chebyshev spacing [9].

Problems of function generation and path generation (with prescribed timing) are solved for the same set of precision points for the two outputs. As both the output displacement S and the coupler point P are coordinated with the input angle θ_2, the precision points of the two outputs can be obtained from that of the input. The set of values of precision points of input angle for the two problems are same, i.e. they belong to the same domain. In both the cases, the precision points are obtained by Chebyshev spacing but between the same range, thus, making the values same for the two cases. Since this is an adjustable mechanism, there is no limitation on the number of precision positions to be used.

III. DESIGN PROCEDURE

The new procedure of design proposed in the present work is illustrated in Fig. 3. The design process is presented here in the form of a flowchart.

The original non-adjustable linkage is first synthesized using displacement equation and dyad or standard form equation [10] by 3-precision point synthesis and 2-precision point synthesis respectively. Then the adjustable link-lengths are synthesized to form the desired mechanism. This new method of design provides with the means for simultaneous task generation without any limitation on the number of precision positions.

IV. KINEMATIC ANALYSIS

Once the non-adjustable link dimensions are obtained by displacement equation and standard (or dyad) form equation and coordinate system established (as shown in Fig. 2), the following vector loop equation can be written:

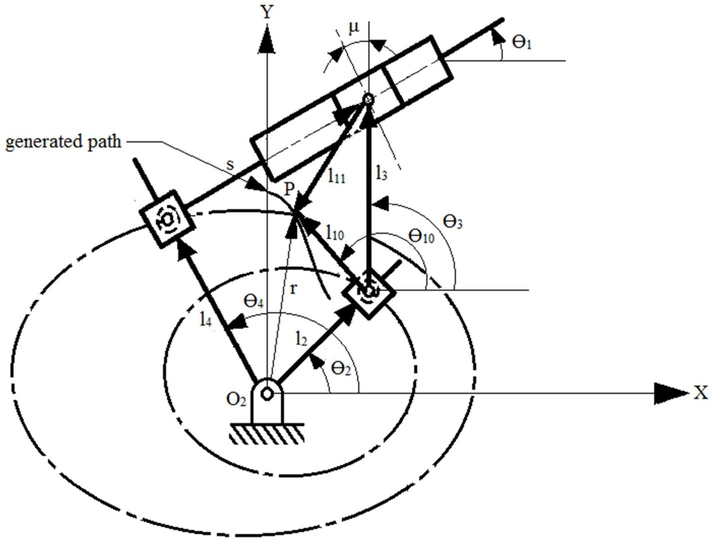

Fig. 2. Coordinate system of the mechanism

$$\vec{l_2}+\vec{l_{10}}-\vec{r}=0 \tag{1}$$

This vector equation corresponding to k^{th} precision point can be separated into following scalar component equations in the X and Y directions:

$$l_{2k}\cos\theta_{2k}+l_{10}\cos\theta_{10k}-P_{xk}=0, \quad k=1\sim n \tag{2}$$

$$l_{2k}\sin\theta_{2k}+l_{10}\sin\theta_{10k}-P_{yk}=0, \quad k=1\sim n \tag{3}$$

Rearranging, squaring and adding (2) and (3), the following equation is obtained:

$$\begin{aligned} &l_{2k}{}^2-2l_{2k}\left(P_{xk}\cos\theta_{2k}+P_{yk}\sin\theta_{2k}\right)\\ &+\left(P_{xk}{}^2+P_{yk}{}^2-l_{10}{}^2\right)=0 \end{aligned} \tag{4}$$

For known values of θ_{2k}, P_{xk}, P_{yk} and l_5, corresponding variations of length of the input link are obtained by:

$$l_{2k}=\frac{-E\pm\sqrt{E^2-4F}}{2} \tag{5}$$

where,

$$E=-2\left(P_{xk}\cos\theta_{2k}+P_{yk}\sin\theta_{2k}\right)$$

$$F=P_{xk}{}^2+P_{yk}{}^2-l_{10}{}^2$$

The angular positions of l_{10} are obtained by rearranging and dividing (2) and (3) as:

$$\theta_{10k}=\tan^{-1}\left(\frac{P_{yk}-l_{2k}\sin\theta_{2k}}{P_{xk}-l_{2k}\cos\theta_{2k}}\right) \tag{6}$$

The angular positions of l_3 are determined from:

$$\theta_{3k}=\theta_{10k}-\gamma \tag{7}$$

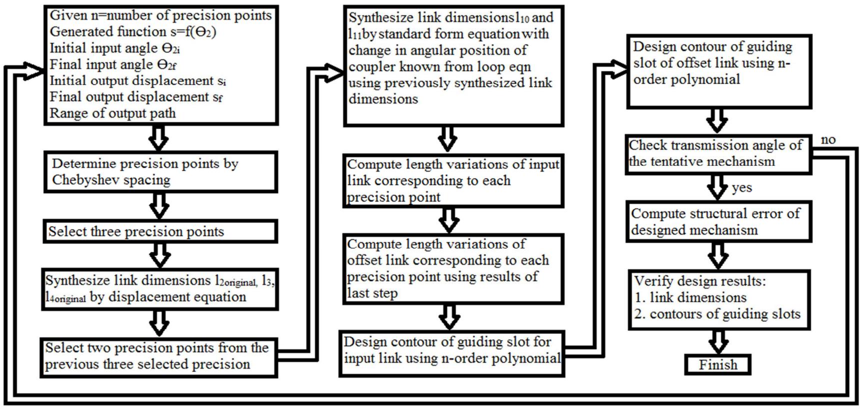

Fig. 3. Design process

where, γ is the coupler angle synthesized using standard form equation.

Also, the following vector loop equation can be written:

$$\vec{l}_2+\vec{l}_3-\vec{s}-\vec{l}_4=0 \tag{8}$$

This vector equation corresponding to k^{th} precision point can be separated into following scalar component equations in the X and Y directions:

$$\begin{aligned} & l_{2k}\cos\theta_{2k}+l_3\cos\theta_{3k}-s_k\cos\theta_{1k} \\ & -l_{4k}\cos\left(\frac{\pi}{2}+\theta_{1k}\right)=0, \qquad k=1\sim n \end{aligned} \tag{9}$$

$$\begin{aligned} & l_{2k}\sin\theta_{2k}+l_3\sin\theta_{3k}-s_k\sin\theta_{1k} \\ & -l_{4k}\sin\left(\frac{\pi}{2}+\theta_{1k}\right)=0, \qquad k=1\sim n \end{aligned} \tag{10}$$

Rearranging, dividing and again rearranging (7) and (8), the following equation can be written:

$$\begin{aligned} & \left(l_{2k}\cos\theta_{2k}+l_3\cos\theta_{3k}\right)\cos\theta_{1k} \\ & +\left(l_{2k}\sin\theta_{2k}+l_3\sin\theta_{3k}\right)\sin\theta_{1k}-s_k=0 \end{aligned} \tag{11}$$

For known values of s_k, θ_{2k}, θ_{3k}, l_{2k} and l_3, the corresponding angular positions of the slider guider are determined by:

$$\theta_{1k}=2\tan^{-1}\left(\frac{-B\pm\sqrt{B^2-C^2+A^2}}{C-A}\right) \tag{12}$$

where,

$$A=l_{2k}\cos\theta_{2k}+l_3\cos\theta_{3k}$$

$$B=l_{2k}\sin\theta_{2k}+l_3\sin\theta_{3k}$$

$$C=-s_k$$

The angular positions of the offset link are determined from:

$$\theta_{4k}=\theta_{1k}+\frac{\pi}{2} \tag{13}$$

Rearranging, squaring and adding (7) and (8) as:

$$\begin{aligned} l_{4k}{}^2 &= \left(l_{2k}\cos\theta_{2k}+l_3\cos\theta_{3k}-s_k\cos\theta_{1k}\right)^2 \\ & +\left(l_{2k}\sin\theta_{2k}+l_3\sin\theta_{3k}-s_k\sin\theta_{1k}\right)^2 \end{aligned} \tag{14}$$

For known values of l_{2k}, l_3, s_k, θ_{2k}, θ_{3k} and θ_{1k}, the corresponding variations of length of the offset link are determined by:

$$l_{4k}=\begin{bmatrix} l_{2k}{}^2+l_3{}^2-s_k{}^2+2l_{2k}l_3\cos\left(\theta_{2k}-\theta_{3k}\right) \\ -2l_{2k}s_k\cos\left(\theta_{2k}-\theta_{1k}\right)-2l_3s_k\cos\left(\theta_{3k}-\theta_{1k}\right) \end{bmatrix}^{\frac{1}{2}} \tag{15}$$

The free-running quality of the tentative mechanism can be measured by the transmission angle μ_k, which can be expressed as:

$$\mu_k=\sin^{-1}\frac{s_k{}^2+l_3{}^2-l_{2k}{}^2-l_{4k}{}^2+\cos\left(\theta_{4k}-\theta_{2k}\right)}{2s_kl_3} \tag{16}$$

V. Contour Design of Guiding Slots

The contours of the guiding slots are designed using n-degree polynomials of the input angle. The variable lengths of the input link 2 and the offset link 4 corresponding to all precision points can be expressed as:

$$l_{2k}\left(\theta_{2k}\right)=c_0+c_1\theta_{2k}+c_2\theta_{2k}{}^2+\ldots\ldots\ldots+c_{n-1}\theta_{2k}{}^{n-1} \tag{17}$$

$$l_{4k}(\theta_{4k}) = d_0 + d_1\theta_{4k} + d_2\theta_{4k}^{\ 2} + ... + d_{n-1}\theta_{4k}^{\ n-1} \quad (18)$$

Here θ_{2k} and θ_{4k} are the angular positions of input link and offset link having corresponding variable lengths l_{2k} and l_{4k} respectively and c_0, c_1,..., c_{n-1}, d_0, d_1,..., d_{n-1} are constants. The polynomials represent advanced cam curves where the follower displacements are expressed as polynomial functions of input rotation. Here guiding slots in the fixed link are fixed cams and the roller links are followers moving over the fixed cam-slots.

Once the variable lengths l_{2k} and l_{4k} are determined, the k^{th} point on the contours of the corresponding guiding slots having co-ordinates (p_{2xk}, p_{2yk}) and (p_{4xk}, p_{4yk}) respectively can be written as follows:

$$p_{2xk} = l_{2k}\cos\theta_{2k} \quad (19)$$

$$p_{2yk} = l_{2k} sin\theta_{2k} \quad (20)$$

and

$$p_{4xk} = l_{4k}\cos\theta_{4k} \quad (21)$$

$$p_{4yk} = l_{4k} sin\theta_{4k} \quad (22)$$

VI. CASE STUDY

A quadratic function and an elliptic path (with prescribed timing, i.e., coordinated with the crank rotation) with centre at (-0.4, 5.5) and major and minor axes of magnitudes 2.4 and 0.35 units respectively, is to be generated over the range of 45^0 to 105^0 crank rotation with simultaneous translation of the slider along the guider from 7 units to 6 units. The coordination of the input angle and the desired path is prescribed in a way that the parametric angle of the elliptic path is always 45^0 less than the input angle.

The number of precision points is taken as 12. The quadratic function to be generated by the mechanism is expressed as $\frac{s - s_i}{s_f - s_i} = \left(\frac{\theta_2 - \theta_{2i}}{\theta_{2f} - \theta_{2i}}\right)^2$, where subscripts i and f denote initial and final values respectively. The constraint on transmission angle is $25° \le \mu_k \le 155°$.

The first, sixth and end precision points are chosen to synthesize the link dimensions by displacement equation and left-hand dyad method. The original link lengths thus synthesized are:

$$\begin{bmatrix} l_{2original} & l_3 & l_{4original} & l_{10} & l_6 \end{bmatrix} = \begin{bmatrix} 1.5 & 9.1 & 7.9 & 3.6 & 6.0 \end{bmatrix}$$

The variable lengths l_{2k} and l_{4k} corresponding to each precision point are found from (5) and (15) respectively. The results are given in Table I.

The designed guiding slots for input link and offset link are obtained from (19-22). Structural error of the function is represented by Chebyshev polynomial for comparison. The structural error curves of the generated function without and with adjustments, the desired and generated paths, guiding slot contours of the adjustable links and transmission angle of the tentative mechanism are shown in Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8 and Fig. 9 respectively.

The maximum value of absolute structural error for the generated function obtained by 3-precision point synthesis is 0.0359. The maximum absolute structural error for the generated function with adjustments, thus using 12 precision points is 0.00000020733, which is about 0.0005% of 0.0359, that obtained by the conventional 3-precision point solution.

The lengths of input link and offset link vary smoothly within the range of motion as shown in Table I and thus can be achieved by designing corresponding guiding slots (as plotted in Fig. 7 and Fig. 8). The guiding slots are provided only for the required range of crank rotation and can be extended by a reference circle to count in the full rotation of input link.

TABLE I. VARIABLE LENGTHS OF THE INPUT LINK AND THE OFFSET LINK CORRESPONDING TO EACH PRECISION POSITION FOR THE CASE STUDY.

Precision Point (k)	**Link Lengths**	
	input link (l_{2k})	***offset link (l_{4k})***
1	2.67	9.32
2	2.61	9.29
3	2.51	9.25
4	2.42	9.18
5	2.35	9.10
6	2.32	9.02
7	2.32	8.95
8	2.36	8.88
9	2.42	8.82
10	2.49	8.77
11	2.56	8.73
12	2.60	8.71

For the path to be generated, the absolute error between the co-ordinates of points on generated path and desired path is very small, i.e. nearly zero (as plotted in Fig. 6). This indicates that the generated path nearly coincides with the desired path. Transmission angle of the designed mechanism remains in an acceptable range, from 32.47^0 to 47.11^0 (as plotted in Fig. 9).

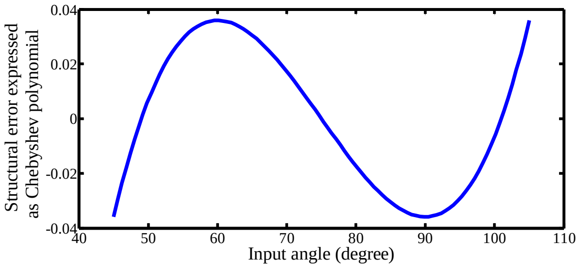

Fig. 4. Structural error curve of the generated function without adjustment

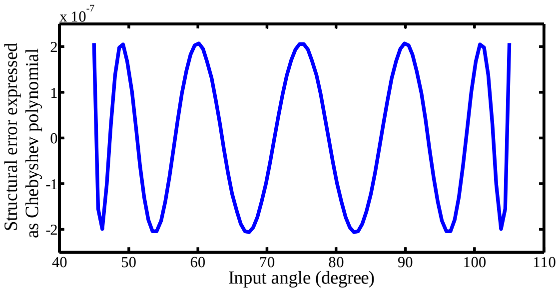

Fig. 5. Structural error curve of the generated function with adjustment

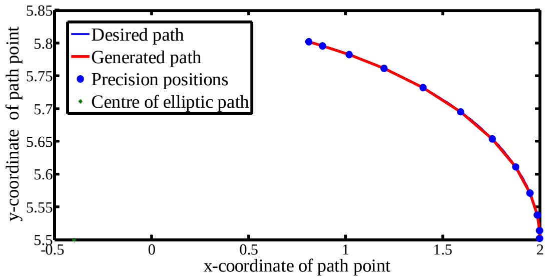

Fig. 6. Desired and approximately generated path

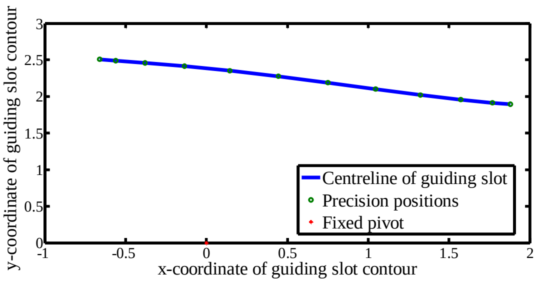

Fig. 7. Guiding slot contour of the input link

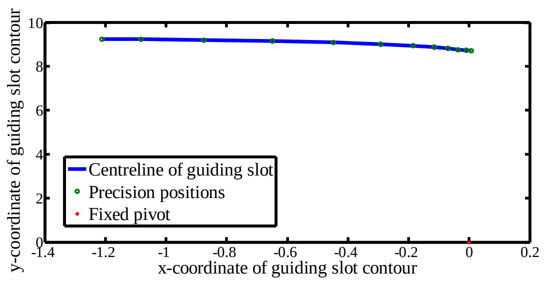

Fig. 8. Guiding slot contour of the offset link

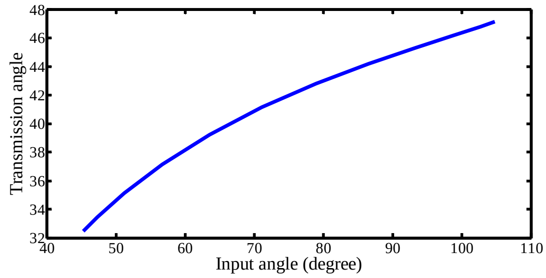

Fig. 9. Transmission angle of the designed mechanism

VII. SIMULATION

Using AutoLISP code generated for the purpose, the motion of the synthesized mechanism tracing the desired path and correlating the slider displacement with the input rotation according to the required functional relationship is simulated. Drawing an entity and at the same time erasing the previous entity on the graphics screen are done rapidly at a selected speed. This gives an effect of animation of the synthesized linkage on the graphics screen. AutoCAD Fig. 10, Fig. 11 and Fig. 12 show configurations of the mechanism in three positions during the simulation.

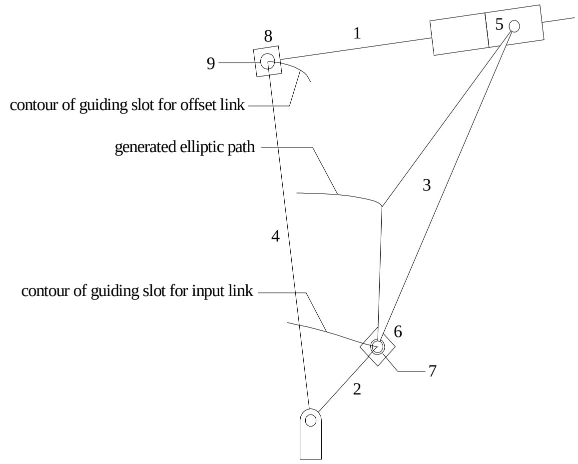

Fig. 10. Configuration of the synthesized mechanism at 1st precision position

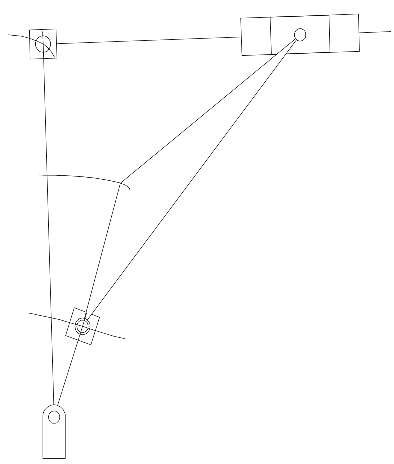

Fig. 11. Configuration of the synthesized mechanism at 6th precision position

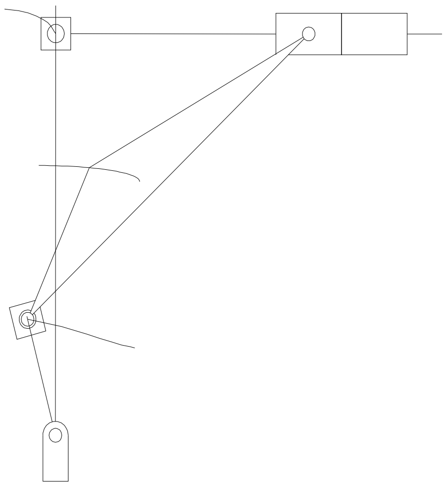

Fig. 12. Configuration of the synthesized mechanism at 12th precision position

VIII. CONCLUSION

This paper presents a new procedure of design for adjustable offset slider-crank mechanism for simultaneous generation of function and path (with prescribed timing) using variable-length links. The design process helps in simultaneous task generation using a single mechanism. The concept of hybrid task synthesis is taken to a higher level where the subtasks are performed simultaneously. The lengths of each of the input link and the offset link are adjusted using a ternary link, a roller link and guiding slot in the fixed link. An n-degree polynomial was employed to design the contours of the guiding slots for the two adjustable links. The dynamic length adjustment of input link and use of servomotor for length adjustment of offset link provide allowance for usage of any number of precision positions or accuracy points for decreasing maximum absolute structural error. Thus a function and a path (with prescribed timing) can be generated simultaneously by the single mechanism over the range of motion. The effectiveness of the proposed approach of synthesis is verified by a case study. The structural errors of the generated function, expressed as Chebyshev polynomial, are found to be significantly smaller than the errors produced by the conventional 3-precision point solution due to the use of greater number of precision positions. Also the generated path approximates the desired path satisfactorily as demonstrated in the simulation by AutoLISP code on AutoCAD platform.

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