Towards Synthesis of Tensegrity Structures of Desired Shapes

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***Abstract*—In this paper, we formulate an optimization problem to synthesize tensegrity structures of desired shapes. In particular, we consider class 1 tensegrity where no two compression elements have a common vertex, by designating all members in the desired shape a priori as either a bar in compression or a cable in tension. We solve static equilibrium equations at the vertices in the desired configuration subject to constraints on force densities, which are the variables in the synthesis problem. The reason for using the force density, defined as the force per unit length in the desired configuration, is twofold: (i) to directly impose constraints of positive and negative force densities in tension and compression elements, respectively, and (ii) to obtain free lengths of all the members using the force densities. We use this method to synthesize a previously known semi-toroidaltensegrity arch with 24 bars and 102 cables and a hitherto unknown tensegrity of biconcave shape similar to that of a red blood cell comprising 24 bars and 112 cables. We also present static analysis of a tensegrity structure by minimizing of potential energy with unilateral constraints on the lengths of the cables, which cannot take compressive loads. We also extend the method to synthesize a tensegrity table of desired height and area with three bars and nine cables under a predefined load. The prototypes of all three synthesized tensegrities are made and tested.**

Keywords—tensegrity;biconcave shape.

I. INTRODUCTION

Tensegrity structures are made of tension and compression elements [1]. The tension elements (cables) take tensile loads and compressive elements (bars) take compressive loads.Tensegrity structures use internal preload to assume a stiff and stable equilibrium shape under no external loads. Thus, they have applications in deployable devices used for example, in space structures [2]. Cytoskeleton models based on the concept of tensegrity are also used to model the mechanical response of biological cells [3]. Tensegrity structures have applications in the field of architecture, where different types of tensegrity domes and arches are built [4].As the large deformations in a tensegrity are controllable, they find application in robotic arms [5]. There are different classes of tensegrity structures; class-n tensegrity structures are structures where 'n' bars share a single node [1]. We consider class 1 tensegrity structureswith and without external loads.

There are different methods to find the equilibrium state attained by atensegrity structure for given connectivity and stiffness properties of the constituent bars and cables. These methods are generally known as *form-finding*methods [6]. They are classified into two categories: kinematic methods and static methods [6]. Some of the kinematic methods of form-finding haveanalytical solutions [6]. Such methods are used for form-finding of prismatic tensegrities where the cable lengths and the polygon face of the tensegrity prism are given, using which the lengths of the bars are calculated to get a stable tensegrity prism. Other kinematic methods use nonlinear programming [7], by posing a constrained minimization problem, where connectivity and nodal points are known a priori and the lengths of the bars are maximized until a stable structure is obtained. The third class of kinematic form-finding methodd usesdynamic relaxation [8] of different forms by varying the positions of the vertices of the system and solving the equilibrium equations at each node to find the unbalanced forces iteratively. The form is modified using the dynamic equations of motion. Although the kinematic methods have good convergence properties, they are only effective when the number of vertices in the structure is low.

Static methods of form-finding are of four types.In the force density method [9], for given connectivity of a tensegrity structure, the force density of each of the members is predefined. The nodal point coordinates are then calculated for the corresponding force densities that satisfy static equilibrium. In the analytical solution method [10], stable form of a rotationally symmetric tensegrity (e.g. a tensegrity prism) is found. Here, the nodal force-balance is used to arrive at the angles subtended by the bars for given lengths of cables. In the energy method [11], the form of the tensegrity is found for which the total potential energy is minimum. Here the potential energy of both cables under tension and bars under compression are considered. In the reduced co-ordinate method [12], the bars are considered to be rigid and the equilibrium equations pertaining only to the cables are formulated with the help of the principle of virtual work.

In all the aforementioned methods, the connectivity of the member elements and the stiffness property of the materials are given and we find the form that satisfies the equilibrium equations. It may be noted that form-finding addressed by these methods is a *forward problem* in the sense that the aim is to obtain a stable tensegrity form using given number of bars and cables and their connectivity and stiffness properties. The inverse problem is determining the geometry and force densities to obtain a prescribed shape. Form-finding of tensegrity structures with desired shape has been addressed by Masic [13], where the material stiffness properties and connectivity of the structure are known a priori. The difference between the coordinates of the vertices of the equilibrium form of the tensegrity and the coordinates of the prescribed vertices is minimized. The coordinates of the vertices and the force densities are the variables in this problem. This is one of few methods that address the inverse or the synthesis problem. In this paper, we synthesize a tensegrity structure of desired shape, where the nodal points known a priori, the connectivity of the elements in the tensegrity structure is arrived at by considering the Maxwell's rule for trusses, and we find the stiffness properties of bars and cables of the tensegrity structure, which give the desired form for the tensegrity structure.

We synthesize a free-standing tensegrity structure (i.e., with no external loads) of desired shape by solving the force density method as an optimization problem with constraints on the force densities. We synthesize the tensegrity structures with external loads, which attain a predefined geometric constraint (e.g. height) under the application of the load, in two steps. The first step involves solving the force densities in an optimization problem to get the free-standing tensegrity. The second step involves static analysis, where we minimize the potential energyof the system under the externally applied loads with a reduced coordinate system. We use unilateral constraints on the cables in the form of a sigmoid-type function to account for the slack condition of the cables under compressive loads. We check for the geometric constraint and update the tensegrity configuration and repeat the above steps until the geometric constraint is satisfied. We synthesize different desired shapes such as a semi-toroidal arch, biconcave tensegritysimilar to the cytoskeleton of a red blood cell, and a table of desired height under the application of a prescribed external load.

II. METHODS

For a free-standing tensegrity to be in static equilibrium, the net forces acting at each vertex should be zero. For this, the internal and external forces in the member elements are resolved in the three directions and the sum of these resolved forces at each vertexis equated to zero to get three equilibrium equations at each node. The equilibrium equations at vertex 1in Fig. 1 can be written as:

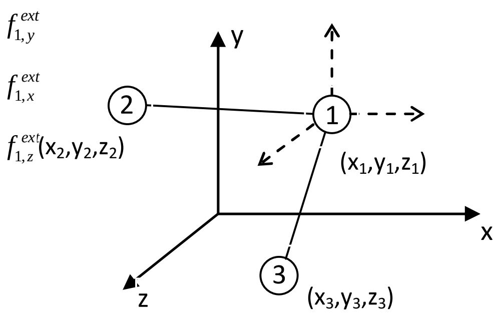

Fig. 1. A typical compliant mechanism-based cell manipulation setup

$$
\begin{aligned}
\frac{(x_1-x_2)\,f_{1-2}}{l_{1-2}}+\frac{(x_1-x_3)\,f_{1-3}}{l_{1-3}} &= f_{1,x}^{ext}\\
\frac{(y_1-y_2)\,f_{1-2}}{l_{1-2}}+\frac{(y_1-y_3)\,f_{1-3}}{l_{1-3}} &= f_{1,y}^{ext}\\
\frac{(z_1-z_2)\,f_{1-2}}{l_{1-2}}+\frac{(z_1-z_3)\,f_{1-3}}{l_{1-3}} &= f_{1,z}^{ext}
\end{aligned}
\tag{1}
$$

where f_{i-j} and l_{i-j} are the internal force and length of an element connecting vertices i and j. For ease of calculation, we denote $\frac{f_{i-j}}{l_{i-j}}$ with q_{i-j}, where q_{i-j} is called the *force density*, i.e., force per unit length of the deformed element. Thus, for the case where external loads are absent, we get:

$$
\begin{aligned}
(x_1-x_2)q_{1-2}+(x_1-x_3)q_{1-3} &= 0\\
(y_1-y_2)q_{1-2}+(y_1-y_3)q_{1-3} &= 0 \ (2)\\
(z_1-z_2)q_{1-2}+(z_1-z_3)q_{1-3} &= 0
\end{aligned}
\tag{2}
$$

When there are n vertices, there will be $3n$ equations to be solved in m unknowns, m being the number of elements connecting n vertices. The number of elements in a tensegrity structure is selected by using Maxwell's rule [14], which states that the number of elements should be at least as many as $3n-6$ for a structure to be just stiff, which means that it is statically determinate.

By Maxwell's rule it can be also seen that when $m>3n-6$, the structure will have $m-(3n-6)$ *states of self-stress (i.e*., pre-stress). Similarly when $m<3n-6$, the structure will have $(3n-6)-m$ *infinitesimal mode (i.e., rigid-body degree of freedom as in mechanisms).* A structure satisfying Maxwell's rule and having an infinitesimal mode will also have a corresponding state of self-stress or prestress. We choose m such that we have certain number of self-stressed elements in our structure. As the tensegrity we are synthesizing is of class 1, no two bars will share a common vertex. Hence the number of bars in the structure will be $n/2$. The rest of the elements will constitute the cables of the structure.

The unknowns in (2) are the force densities $q_1, q_2, ..., q_m$ when we are synthesizing a tensegrity structure. The resulting equations can be expressed in matrix form with the help of a connectivity matrix $\mathbf{C}$. Each row of matrix $\mathbf{C}$ represents the connectivity of the elements; each element in a particular row describes whether an element of the tensegrity structure is absent or an element begins or ends at that particular vertex. $\mathbf{C}$ is an $m\times n$ sparse matrix with m^{th} row having value 1 at the column corresponding to the vertex in which the member element emanates and -1 at the column corresponding to the node in which the element terminates with other elements of the row equal to zero. The sum of a row in matrix $\mathbf{C}$ should be zero. Thus we have:

$$\mathbf{C}^T diag(\mathbf{CX})\mathbf{q} = 0$$
$$\mathbf{C}^T diag(\mathbf{CY})\mathbf{q} = 0 \ \ (3) \qquad (3)$$
$$\mathbf{C}^T diag(\mathbf{CZ})\mathbf{q} = 0$$

where $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ are $n \times 1$ vectors containing the x, y and z coordinates of the nodes respectively. $diag(\mathbf{CX})$ is an $m \times m$ diagonal matrix with vector $\mathbf{CX}$ constituting it's diagonal entries. Equation (3) can be solved using singular value decomposition (SVD), but the resulting structure will be a tensegrity structure only for particular $\mathbf{C}$ matrices where force densities are positive for members designated as cables and negative for those designated as bars. In other case,s although we may get solutions, it would merely result in trusses rather than a tensegrity structure. The tensegrity structure we solve for will be a linear combination of the solutions we get. Thus, we need to give constraints on the values of $\mathbf{q}$ so that the cables and bars have positive and negative values for q to ensure the tensegrity condition. So, we pose the problem as an optimization problem with constraints on the unknown q s.

$$\underset{\mathbf{q}}{Min} \qquad (\mathbf{A} - \mathbf{f}^{ext})^2$$
$$where \qquad \mathbf{A} = \begin{bmatrix} \mathbf{C}^T diag(\mathbf{CX})\mathbf{q} \\ \mathbf{C}^T diag(\mathbf{CY})\mathbf{q} \\ \mathbf{C}^T diag(\mathbf{CZ})\mathbf{q} \end{bmatrix}, \mathbf{f}^{ext} = \mathbf{0}$$
$$\text{Subject to} \quad q_i < 0 \qquad \forall i \in N_{bar}$$
$$-q_i < 0 \qquad \forall i \in N_{cable}$$
$$Data \qquad \mathbf{C}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{f}^{ext}, N = N_{bars} \cup N_{cables}$$

(4)

where N is the set of elements on the structure and N_{bar} is the set of elements that are bars and N_{cables} is the set of elements that are cables. After finding the force densities of each of the elements of the tensegrity structure we can find the free length of the individual elements for given cross-sectional area and Young's modulus of the material by fixing any two among the three of free length, cross-sectional area, and Young's modulus and finding the third one. We solve the minimization problem using the fmincon subroutine in MATLAB [15].

The second type of problem solved involves external forces. Here, we aim for the desired shape of the tensegrity under specified external load. The synthesis procedure for this is explained next.

Upon getting the desired structure, static analysis is done by minimization of potential energy approach. In static analysis of the tensegrity structure, we make following assumptions: (i) the bars are rigid and do not undergo any axial deformations or bending (ii) the strain energy of the cables and the work done by the external load contribute to the potential energy (iii) the cables can only take tensile load and become slack under compressive loads. We formulate the static problem as:

$$\underset{g_i}{Min} \qquad PE = \sum_{k=1}^{n} \frac{1}{2S} K_k (L(g_i, l)_k - L_{0k})^2 - \sum_{m=1}^{r} F_m d(g_i, l)_m$$
$$where \qquad S = 1 + e^{-sp\left(\frac{L(g_i)_k}{L_{0k}} - 1\right)}$$
$$Data \qquad K, L_0, F, sp$$

(5)

where K_i and L_{0i} are the stiffnesses and free lengths of cables in the tensegrity structure, l_i is the length of the bar, g_i is the generalized coordinates of the tensegrity structure in terms of the angles made by the bars with datum and final lengths of some cables. $L(g_i, l)$ and $d(g_i, l)$ are the final lengths of the cables and the displacements of the vertices at which the loads act, respectively. Both of them are functions of the generalized coordinates and the lengths of the bars. It may be noted that S is a smoothing function used to account for the force-free slackness condition of the cables. The symbol sp is the steepness parameter of the smooth function.

The next step is to find the optimum height of a tensegrity structure which on application of a vertical load attains a desired shape. Equation (4) cannot be solved for all $\mathbf{f}^{ext} \neq 0$ (only a particular combination of the external load vector ($\mathbf{f}^{ext}$) can be solved) to get a tensegrity structure of desired shape. So we pose the problem by imposing constraints on the force densities.

$$\underset{a}{Min} \qquad f = \left| h(a) - h^* \right|$$
$$Subject\ to \qquad \underset{q}{Min} \qquad (\mathbf{A} - \mathbf{f}^{ext})^2$$
$$Subject\ to \quad q_i < 0 \qquad \forall i \in N_{bar}$$
$$-q_i < 0 \qquad \forall i \in N_{cable}$$
$$\underset{x_i}{Min} \qquad PE = \sum_{k=1}^{n} \frac{1}{2S} K_k (L(x_i, l)_k - L0_k)^2 - \sum_{m=1}^{r} F_m d(x_i, l)_m$$
$$where \quad S = 1 + e^{-sp\left(\frac{L(x_i)_k}{L0_k} - 1\right)}$$
$$Data \qquad \mathbf{C}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{f}^{ext}, F, sp, N = N_{bars} \cup N_{cables}$$

(6)

where K_i, L_{0i} and l_i are found from the output of (4) used in (6). h^* is the desired height the tensegrity structure should take under the application of load and a is the z coordinate of certain vertices of the initial free standing tensegrity. $h(a)$ is the height attained by the tensegrity structure which is a function of a. This is also an iterative process where the a changes until we get the required height.

III. RESULTS

We use (4) to synthesize a previously known semi-toroidaltensegrity arch and an unknown tensegrity of biconcave shape of the cytoskeleton of a red blood cell. The semi-toroidaltensegrity arch (see Fig. 2) [16] is synthesized by first fixing the vertices according to the height and width of the arch. We take 48 vertices to make the outline of the arch. For a class 1 tensegrity there is no common vertex for two bars, leading to 24 bars. The connectivity is made such that three of the bars and nine cables form a unit tensegrity prism. With eight such prisms we can make the semi-toroidaltensegrity arch having 24 bars and 102 cables. We choose the cross-sectional areas of the bars and cables and the material (Young's modulus) used to fabricate them. Then, their free lengths can be solved using the force densities. We solve the optimization problem (4) to get the force densities and then use them to calculate the free lengths of the individual elements. The given data and results are indicated in Tables 1, 2 and 3. The assumed Young's modulus is 9 GPa and 35 GPa; and the area of cross-section is 3.85e-5 m^2 and 7.85e-7 m^2 for bars and cables respectively.

Synthesis of the biconcave tensegrity (Fig. 3(a)) structure also involves the same steps as for the synthesis of the semi-toroidal arch. The nodal points are fixed on the surface generated by revolution of a Cassini's oval [17] corresponding to the size and shape of a red blood cell. We approximate the biconcave shape with 48 nodal points. We synthesize the biconcave tensegrity from three eight bar tensegrity prisms. By Maxwell's rule, the biconcave tensegrity to be stiff should have 144 elements. Considering the symmetry of the biconcave tensegrity we omit six cables and form the biconcave tensegrity with 24 bars and 112 cables. The connectivity of the elements of the biconcave tensegrity is assumed here, keeping in mind the ease of fabrication of such a structure and the symmetric distribution of the cables and bars. Tables 4 and 5 contain the data given to the problem. Table 6 contains the computed force densities and free lengths by solving the optimization problem of (4). Fig. 3(b) shows the prototype made of using wooden pencils and hemp thread. By fitting a taut triangular plane that replaces two tensioned cables and a bar, we can visualize the shape better than with the cables and bars. Such a model is shown in Fig. 3(c).

We also synthesized a tensegrity table that attains a desired height after application of load. The tensegrity table is made with three bars and nine cables and has six vertices. The vertices of the table are selected according to the area needed for the top of the table. The weight of a glass-top is the load. The heights (the z coordinates) of the first three vertices are selected to be as they rest on the ground (in x-y plane). The heights of the other three are randomly chosen for the initial analysis. The load on the table acts vertically down (i.e., in the $-z$ direction) at the three top vertices. The deformation of the table under load can be seen in the Figs. 4. (a) – (b). The table was synthesized such that the table has a height of 0.6 m for a load of 600 N. Tables 7 contains the coordinates of the vertices taken as an initial guess. Table 8 contains the connectivity data of the elements and the force densities and free lengths of the elements obtained after solving the optimization problem of (6).

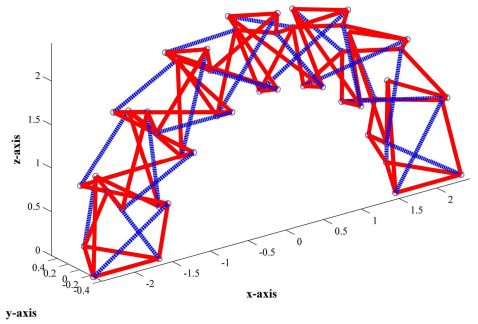

Fig. 2 A semi-toroidaltensegrity arch (blue dashed lines indicate the bars and the red lines indicate the cables).

TABLE 1. THE X, Y AND Z COORDINATES (IN M) OF THE 48 VERTICES USED TO CONSTRUCT THE TENSEGRITY ARCH.

SN	X	Y	Z	SN	X	Y	Z	SN	X	Y	Z
1	0.200	0.050	0.000	17	-0.093	-0.025	0.225	33	0.147	0.046	0.164
2	0.243	-0.025	0.000	18	-0.060	-0.025	0.145	34	0.076	-0.040	0.217
3	0.157	-0.025	0.000	19	-0.141	0.050	0.141	35	0.050	-0.005	0.142
4	0.185	0.050	0.077	20	-0.172	-0.025	0.172	36	0.073	0.046	0.208
5	0.225	-0.025	0.093	21	-0.111	-0.025	0.111	37	-0.013	-0.040	0.229
6	0.145	-0.025	0.060	22	-0.185	0.050	0.077	38	-0.008	-0.005	0.150
7	0.141	0.050	0.141	23	-0.225	-0.025	0.093	39	-0.012	0.046	0.220
8	0.172	-0.025	0.172	24	-0.145	-0.025	0.060	40	-0.100	-0.040	0.207
9	0.111	-0.025	0.111	25	-0.200	0.050	0.000	41	-0.065	-0.005	0.135
10	0.077	0.050	0.185	26	-0.243	-0.025	0.000	42	-0.096	0.046	0.199
11	0.093	-0.025	0.225	27	-0.157	-0.025	0.000	43	-0.171	-0.040	0.153

12	0.060	-0.025	0.145	28	0.207	-0.040	0.100	44	-0.112	-0.005	0.100
13	0.000	0.050	0.200	29	0.135	-0.005	0.065	45	-0.164	0.046	0.147
14	0.000	-0.025	0.243	30	0.199	0.046	0.096	46	-0.217	-0.040	0.076
15	0.000	-0.025	0.157	31	0.153	-0.040	0.171	47	-0.142	-0.005	0.050
16	-0.077	0.050	0.185	32	0.100	-0.005	0.112	48	-0.208	0.046	0.073

The designed tensegrity models are fabricated and tested. The semi-toroidaltensegrity arch and the biconcave tensegrity model of red blood cell are fabricated using wooden cylinders (pencil) and hemp thread (pencils act as bars and the hemp thread act as the cables). The tensegrity table is fabricated using aluminum rods for bars and braided stainless steel wire for cables. It is shown in Fig.5 (a-b). The table is stable and is currently used in our lab. As the weight of the glass top is nearly 40 kg, any small object (up-to a weight of 20 kg) placed on top of it does not disturb the stability of the structure.

TABLE 2. THE CONNECTIVITY OF 96 ELEMENTS THAT MAKE THE SEMI-TORROIDAL TENSEGRITY ARCH (LAST 24 ELEMENTS (I.E. 103-126) ARE BARS).

SN	Vertex 1	Vertex 2	SN	Vertex 1	Vertex 2	SN	Vertex1	Vertex2	SN	Vertex 1	Vertex 2
1	1	2	33	40	18	65	17	45	96	46	26
2	1	3	34	44	21	66	18	43	97	1	5
3	2	3	35	21	43	67	19	47	98	2	4
4	29	6	36	43	20	68	20	48	99	3	6
5	6	28	37	20	45	69	21	46	100	47	25
6	28	5	38	45	19	70	22	27	101	48	26
7	5	30	39	19	47	71	23	26	102	46	27
8	30	4	40	47	24	72	24	25	103	1	28
9	4	29	41	24	46	73	1	4	104	2	29
10	32	9	42	46	23	74	2	5	105	3	30
11	9	31	43	23	48	75	3	6	106	4	31
12	31	8	44	48	22	76	29	9	107	5	32
13	8	33	45	22	47	77	30	7	108	6	33
14	33	7	46	26	25	78	28	8	109	7	34
15	7	32	47	25	27	79	32	12	110	8	35
16	35	12	48	27	26	80	33	10	111	9	36
17	12	34	49	1	29	81	31	11	112	10	37
18	34	11	50	2	30	82	35	15	113	11	38
19	11	36	51	3	28	83	36	13	114	12	39
20	36	10	52	4	32	84	34	14	115	13	40
21	10	35	53	5	33	85	38	18	116	14	41
22	15	38	54	6	31	86	39	16	117	15	42
23	38	13	55	7	35	87	37	17	118	16	43
24	13	39	56	8	36	88	41	21	119	17	44
25	39	14	57	9	34	89	42	19	120	18	45
26	14	37	58	10	38	90	40	20	121	19	46
27	37	15	59	11	39	91	44	24	122	20	47
28	18	41	60	12	37	92	45	22	123	21	48
29	41	16	61	13	41	93	43	23	124	22	25
30	16	42	62	14	42	94	47	27	125	23	27
31	42	17	63	15	40	95	48	25	126	24	26
32	17	40	64	16	44						

TABLE 3. THE FORCE DENSITY AND LENGTH OF THE INDIVIDUAL ELEMENTS OF THE SEMI-TORROIDAL TENSEGRITY ARCH PROTOTYPE.

SN	Force density (N/m)	Length (m)	SN	Force density (N/m)	Length (m)	SN	Force density (N/m)	Length (m)	SN	Force density (N/m)	Length (m)
1	0.014	0.086	33	0.035	0.073	65	0.027	0.124	96	0.005	0.090
2	0.017	0.085	34	0.193	0.019	66	0.031	0.109	97	0.008	0.121
3	0.011	0.086	35	0.026	0.073	67	0.027	0.105	98	0.004	0.122
4	0.174	0.020	36	0.149	0.022	68	0.006	0.126	99	0.015	0.060
5	0.028	0.073	37	0.034	0.073	69	0.023	0.110	100	0.012	0.067
6	0.201	0.021	38	0.166	0.021	70	0.038	0.074	101	0.002	0.098
7	0.046	0.072	39	0.019	0.074	71	0.009	0.098	102	0.005	0.120
8	0.204	0.020	40	0.002	0.023	72	0.020	0.066	103	-0.036	0.135
9	0.020	0.074	41	0.017	0.074	73	0.014	0.077	104	-0.027	0.128
10	0.228	0.019	42	0.099	0.023	74	0.013	0.094	105	-0.036	0.126
11	0.028	0.073	43	0.038	0.073	75	0.015	0.060	106	-0.051	0.135
12	0.225	0.021	44	0.111	0.022	76	0.014	0.055	107	-0.048	0.128
13	0.050	0.072	45	0.011	0.074	77	0.009	0.073	108	-0.045	0.126
14	0.216	0.020	46	0.009	0.086	78	0.019	0.081	109	-0.056	0.135
15	0.020	0.074	47	0.011	0.086	79	0.031	0.054	110	-0.055	0.128
16	0.245	0.018	48	0.012	0.086	80	0.018	0.072	111	-0.059	0.127
17	0.032	0.073	49	0.017	0.106	81	0.023	0.080	112	-0.060	0.135
18	0.245	0.020	50	0.002	0.096	82	0.023	0.054	113	-0.059	0.128
19	0.057	0.072	51	0.019	0.111	83	0.023	0.072	114	-0.060	0.127
20	0.255	0.019	52	0.049	0.103	84	0.023	0.080	115	-0.056	0.135
21	0.022	0.074	53	0.012	0.126	85	0.021	0.054	116	-0.060	0.128
22	0.235	0.019	54	0.040	0.109	86	0.023	0.072	117	-0.053	0.127
23	0.017	0.074	55	0.051	0.102	87	0.028	0.080	118	-0.043	0.135
24	0.253	0.020	56	0.018	0.125	88	0.019	0.055	119	-0.054	0.128
25	0.059	0.072	57	0.042	0.108	89	0.017	0.072	120	-0.049	0.127
26	0.257	0.020	58	0.057	0.102	90	0.015	0.081	121	-0.032	0.135
27	0.031	0.073	59	0.019	0.125	91	0.012	0.055	122	-0.024	0.128
28	0.231	0.019	60	0.049	0.108	92	0.033	0.071	123	-0.035	0.126
29	0.015	0.074	61	0.059	0.102	93	0.009	0.081	124	-0.021	0.120
30	0.208	0.020	62	0.019	0.125	94	0.001	0.081	125	-0.029	0.126
31	0.044	0.073	63	0.039	0.109	95	0.004	0.118	126	-0.015	0.123
32	0.224	0.021	64	0.048	0.103						

Fig. 3. (a) A biconcave tensegrity structure of a red blood cell (blue lines indicate the bars and the red lines indicate the cables). (b) A red blood cell prototype (scaled up) made with pencil and hemp threads. (c) Solid model of a red blood cell.

Table 4. The X, Y and Z coordinates (in m) of the 48 vertices used to construct the tensegrity arch.

SN	X	Y	Z	SN	X	Y	Z	SN	X	Y	Z
1	0.057	-0.362	0	17	0.133	0.184	0.184	33	-0.053	0	0.008
2	0.133	-0.261	0	18	0.057	0.256	0.256	34	-0.053	0	-0.008
3	0.053	-0.008	0	19	-0.057	0.256	0.256	35	-0.133	0	-0.261
4	0.053	0.008	0	20	-0.133	0.184	0.184	36	-0.057	0	-0.362
5	0.133	0.261	0	21	-0.053	0.006	0.006	37	0.057	0.256	-0.256
6	0.057	0.362	0	22	-0.053	-0.006	-0.006	38	0.133	0.184	-0.184
7	-0.057	0.362	0	23	-0.133	-0.184	-0.184	39	0.053	0.006	-0.006
8	-0.133	0.261	0	24	-0.057	-0.256	-0.256	40	0.053	-0.006	0.006
9	-0.053	0.008	0	25	0.057	0.000	0	41	0.133	-0.184	0.184
10	-0.053	-0.008	0	26	0.133	0.000	0	42	0.057	-0.256	0.256
11	-0.133	-0.261	0	27	0.053	0.000	0	43	-0.057	-0.256	0.256
12	-0.057	-0.362	0	28	0.053	0.000	0	44	-0.133	-0.184	0.184
13	0.057	-0.256	-0.256	29	0.133	0.000	0	45	-0.053	-0.006	0.006
14	0.133	-0.184	-0.184	30	0.057	0.000	0	46	-0.053	0.006	-0.006
15	0.053	-0.006	-0.006	31	-0.057	0.000	0	47	-0.133	0.184	-0.184
16	0.053	0.006	0.006	32	-0.133	0.000	0	48	-0.057	0.256	-0.256

Table V. The connectivity of 136 elements to make the RBC biconcave model (last 24 elements (i.e. 113-136) are bars).

SN	vertex1	vertex2	SN	vertex1	vertex2	SN	vertex1	vertex2	SN	vertex1	vertex2
1	1	2	35	35	36	69	5	38	103	34	22
2	2	3	36	36	25	70	38	26	104	22	10
3	3	4	37	37	38	71	26	14	105	3	10
4	4	5	38	38	39	72	14	2	106	40	45
5	5	6	39	39	40	73	11	44	107	28	33
6	6	7	40	40	41	74	44	32	108	16	21
7	7	8	41	41	42	75	32	20	109	4	9
8	8	9	42	42	43	76	20	8	110	39	46
9	9	10	43	43	44	77	8	47	111	27	34
10	10	11	44	44	45	78	47	35	112	15	22
11	11	12	45	45	46	79	35	23	113	1	43
12	12	1	46	46	47	80	23	11	114	42	31
13	13	14	47	47	48	81	11	2	115	30	19
14	14	15	48	48	37	82	41	44	116	18	7
15	15	16	49	1	42	83	32	29	117	6	48
16	16	17	50	42	30	84	17	20	118	37	36
17	17	18	51	30	18	85	5	8	119	25	24
18	18	19	52	18	6	86	47	38	120	13	12
19	19	20	53	6	37	87	35	26	121	2	44
20	20	21	54	37	25	88	14	23	122	41	32
21	21	22	55	25	13	89	3	40	123	29	20
22	22	23	56	13	1	90	40	28	124	8	17
23	23	24	57	12	43	91	28	16	125	5	47
24	24	13	58	43	31	92	16	4	126	35	38
25	25	26	59	31	19	93	4	39	127	26	23

26	26	27	60	19	7	94	39	27	128	11	14
27	27	28	61	7	48	95	27	15	129	45	3
28	28	29	62	48	36	96	15	3	130	40	33
29	29	30	63	36	24	97	10	45	131	28	21
30	30	31	64	24	12	98	45	33	132	16	9
31	31	32	65	2	41	99	33	21	133	4	46
32	32	33	66	41	29	100	21	9	134	39	34
33	33	34	67	29	17	101	9	46	135	27	22
34	34	35	68	17	5	102	46	34	136	15	10

TABLE VII. THE FORCE DENSITY AND FREE-LENGTH OF THE INDIVIDUAL ELEMENTS OF THE RED BLOOD CELL BICONCAVE PROTOTYPE

SN	Force density (N/m)	Length (m)	SN	Force density (N/m)	Length (m)	SN	Force density (N/m)	Length (m)	SN	Force density (N/m)	Length (m)
1	0.008	0.125	35	0.008	0.125	69	0.004	0.198	103	0.223	0.005
2	0.004	0.263	36	0.009	0.113	70	0.004	0.198	104	0.223	0.005
3	0.033	0.016	37	0.008	0.125	71	0.004	0.198	105	0.113	0.090
4	0.004	0.263	38	0.004	0.263	72	0.004	0.198	106	0.113	0.090
5	0.008	0.125	39	0.033	0.016	73	0.004	0.198	107	0.113	0.090
6	0.009	0.113	40	0.004	0.263	74	0.004	0.198	108	0.113	0.090
7	0.008	0.125	41	0.008	0.125	75	0.004	0.198	109	0.113	0.090
8	0.004	0.263	42	0.009	0.113	76	0.004	0.198	110	0.113	0.090
9	0.033	0.016	43	0.008	0.125	77	0.004	0.198	111	0.113	0.090
10	0.004	0.263	44	0.004	0.263	78	0.004	0.198	112	0.113	0.090
11	0.008	0.125	45	0.033	0.016	79	0.004	0.198	113	-0.003	0.300
12	0.009	0.113	46	0.004	0.263	80	0.004	0.198	114	-0.003	0.300
13	0.008	0.125	47	0.008	0.125	81	0.002	0.265	115	-0.003	0.300
14	0.004	0.263	48	0.009	0.113	82	0.002	0.265	116	-0.003	0.300
15	0.033	0.016	49	0.002	0.276	83	0.002	0.265	117	-0.003	0.300
16	0.004	0.263	50	0.002	0.276	84	0.002	0.265	118	-0.003	0.300
17	0.008	0.125	51	0.002	0.276	85	0.002	0.265	119	-0.003	0.300
18	0.009	0.113	52	0.002	0.276	86	0.002	0.265	120	-0.003	0.300
19	0.008	0.125	53	0.002	0.276	87	0.002	0.265	121	-0.005	0.332
20	0.004	0.263	54	0.002	0.276	88	0.002	0.265	122	-0.005	0.332
21	0.033	0.016	55	0.002	0.276	89	0.222	0.005	123	-0.005	0.332
22	0.004	0.263	56	0.002	0.276	90	0.222	0.005	124	-0.005	0.332
23	0.008	0.125	57	0.002	0.276	91	0.223	0.005	125	-0.005	0.332
24	0.009	0.113	58	0.002	0.276	92	0.223	0.005	126	-0.005	0.332
25	0.008	0.125	59	0.002	0.276	93	0.222	0.005	127	-0.005	0.332
26	0.004	0.263	60	0.002	0.276	94	0.223	0.005	128	-0.005	0.332
27	0.033	0.016	61	0.002	0.276	95	0.222	0.005	129	-0.109	0.108
28	0.004	0.263	62	0.002	0.276	96	0.222	0.005	130	-0.109	0.108
29	0.008	0.125	63	0.002	0.276	97	0.222	0.005	131	-0.109	0.108
30	0.009	0.113	64	0.002	0.276	98	0.222	0.005	132	-0.109	0.108
31	0.008	0.125	65	0.004	0.198	99	0.222	0.005	133	-0.109	0.108
32	0.004	0.263	66	0.004	0.198	100	0.223	0.005	134	-0.109	0.108

33	0.033	0.016	67	0.004	0.198	101	0.223	0.005	135	-0.109	0.108
34	0.004	0.263	68	0.004	0.198	102	0.223	0.005	136	-0.109	0.108

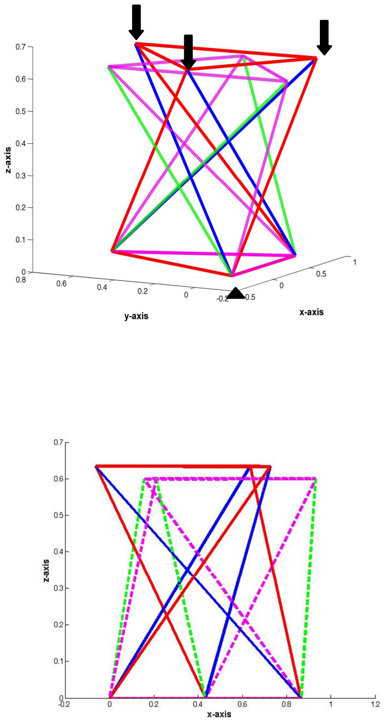

Fig. 4. Two views of the tensegrity table under the application of load. Free standing tensegrity table (blue lines indicate the bars and the red solid lines indicate the cables). The table under an applied external load (green dashed lines indicate the bars and the magenta dashed lines indicate the cables).

(a)

(b)

Fig. 5. Two views of the tensegrity table. The entire weight of the glass-top of the stable is supported by the tensegrity structure.

TABLE VII. THE X, Y AND Z COORDINATES OF THE 6 VERTICES USED TO CONSTRUCT THE TENSEGRITY TABLE. (Z COORDINATE OF THE VERTICES 4,5 AND 6 ARE ITERATED)

SN	X(m)	Y(m)	Z(m)
1	0	0	0
2	0.866	0	0
3	0.433	0.750	0
4	0.636	0.707	0.7
5	-0.064	0.198	0.7
6	0.727	-0.155	0.7

TABLE VIII. THE CONNECTIVITY OF 12 ELEMENTS (LAST 3(I.E. 10-12) ARE BARS), FORCE DENSITIES AND INITIAL LENGTHS OF INDIVIDUAL ELEMENTS TO MAKE THE TENSEGRITY TABLE.

SN	vertex1	vertex2	Force Density (N/m)	Length (m)
1	1	2	0.200	0.863
2	2	3	0.200	0.863
3	3	1	0.200	0.863
4	4	5	0.200	0.863
5	5	6	0.200	0.863
6	6	4	0.200	0.863
7	1	5	0.363	0.664
8	2	6	0.363	0.664
9	3	4	0.363	0.663
10	1	4	-0.348	1.144
11	2	5	-0.348	1.144
12	3	6	-0.348	1.144

IV. CONCLUSIONS

In this paper, we developed an optimization method to synthesize tensegrity structures of desired shapes. The coordinates of the vertices and the connectivity of the elements are the inputs to the problem, which we use to solve for the force densities in each of the member elements using the equilibrium equations at the vertices. Constraints are imposed on the force densities to accommodate tensions in bars and compressions in cables. The following conclusions can be drawn from the work presented here.

- The solution is unique for a given set of positions of the vertices and their connectivity with bars and cables. The free lengths of the elements change according to different materials assigned but the force densities do not change.
- Solving the constrained optimization problem posed for the purpose of synthesis of the tensegrity structure of desired shape is equivalent to constructing the null-space of the combined connectivity-coordinate matrix. When the rank of the null-space is more than unity, tensegrity structures can be constructed using linear

combinations of the basis vectors of the null-space. This will be explored further in future work

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