Coupler-curve synthesis via multi-objective optimisation using NSGA-II

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***Abstract*—This paper presents a study of a classical mechanism synthesis problem in the framework of multi-objective optimisation. In addition to the primary kinematic objective of reducing the structural error, staying away from the singular configurations is considered as a secondary objective. Two well-studied coupler-curve synthesis problems reported in existing literature are revisited for the purpose of application of the proposed formulations, and the results are obtained using the genetic algorithm-based optimiser, NSGA-II. Detailed analysis of the results show that the Pareto-optimal fronts obtained dominate the existing ones in terms of the secondary objective, while being comparable in the primary one.**

Keywords – Coupler-curve Synthesis; Multi-objective Optimisation, NSGA-II, Genetic Algorithms

I. Introduction

Synthesis of a four-bar mechanism for a given coupler curve is one of the classical problems in the domain of mechanism synthesis and design. It is known that the problem can be solved *exactly* for at the most nine arbitrary points specified in a plane, and reports exist in literature documenting how all the possible solutions can be found for this problem [1]. It may be noted that while such solution procedures are available, they are typically difficult to implement, as one inherent step in such formulations is the solution of a set of polynomial equations – a task that remains computationally challenging even in the modern times. Furthermore, often in a design problem, one tries to specify a number of points on the desired coupler curve from the perspective of defining the *overall* shape of the curve. In such a scenario, it is generally more useful to find a curve which passes *approximately* though a large (i.e., larger than nine) number of points, than passing exactly via nine specified points. These observations have led to the popularity of a number of optimisation-based approximate synthesis methods in the recent years.

Several probability-based optimisation methods have been applied to classical problems in mechanism synthesis, such as the *coupler-curve* synthesis of a planar four-bar mechanism [2], [3], [4], [5]. Optimisation techniques based on Genetic Algorithms (GA), Differential Evolution (DE), Particle Swarm Optimisation (PSO) etc. have several advantages over the classical gradient-based techniques in solving such problems. Firstly, depending upon the kinematic formulations, computing the gradients of the objective functions is typically a difficult task, if not practically impossible. Further, in most cases, *full-cycle mobility* of the crank (i.e., the ability of the actuated link to run through a complete cycle without getting locked) is a practical necessity, and the most common form of kinematic constraint governing this behaviour, namely the Grashof's condition, is procedural in nature, and hence cannot be cast in terms of *differentiable* functions. The same can be said about other commonly imposed constraints, e.g., packaging considerations [6]. Thus, the very nature of the standard (or, popular) formulations tend to favour methods of soft-computing, which can handle non-smooth objectives and constraints. Another major advantage of these schemes is that they are designed to explore the search space better and to locate the *global* minima.

As it turns out, practically *all* engineering problems (the one under consideration included) are essentially multi-objective in nature. For instance, while the primary objective in the problem at hand is to find a four-bar mechanism whose coupler curve approximates a given curve *as closely as possible* (in kinematics parlance, *minimising the structural error*), several secondary objectives are motivated by practical considerations, such as applicability and manufacturability. For instance, the mechanism should be as compact as possible; at all configurations it should be away from singularity; no links should be too long or too short, and so on.

In all such problems one is interested in obtaining the *Pareto-optimal front* for the two-objective or multi-objective problem representing the trade-offs between the two or more objectives over the relevant design domain. Thus, an optimiser with the capability of solving multi-objective problems would be ideally suited to study such problems. A GA-based optimiser, namely, Non-dominated Sorting Genetic Algorithm `NSGA-II`[1] [7] has been applied successfully to several multi-objective optimisation problems.

It is interesting that in spite of several applications of soft-computing tools to mechanism synthesis problems, much of the potentials of some of these methods in solving multi-objective problems in this domain remain to be exploited. In a recent study the problem of kinematic design of suspension mechanisms under conflicting objectives has been attempted and Pareto-optimal front obtained using `NSGA-II` [8]. Manufacturing and assembly tolerance and structural error have been used as conflicting objectives to obtain Pareto-optimal fronts in the coupler curve synthesis problem [9]. Another study reports arriving at the Pareto-optimal front for coupler synthesis considering the structural error and *transmission angle error* as conflicting objectives using a variant of `NSGA-II` along with a local optimiser to improve the Pareto-optimal front [10]. A related work [11], shows that by employing special constraint-handling methods and using optimal control parameters of `NSGA-II`, the results obtained for coupler-curve synthesis problem are comparable to the ones reported in paper [5] and superior to the ones reported in paper [2] for single objective formulation. The present work utilises the above developments to study the coupler-curve synthesis problem as a multi-objective optimisation problem. A *niching* strategy is adopted, so as to capture the Pareto-optimal front to the synthesis problem, focussing on the primary objective to reduce the structural errors and the secondary objective which strives to keep the mechanism as far away from *singularity* (i.e., locking) as possible. The results obtained are fairly encouraging, as the solutions compare favourably to those reported in existing literature in terms of the primary objective, while scoring better considering the secondary objective. Two case studies reported in [10] are used to demonstrate the proposed formulations, and detailed comparisons of the results obtained are presented. The rest of the paper in organised as follows: Section 2 presents the formulation of the kinematics objectives and constraints. Section 3 describes the multi-objective formulation. The results obtained are discussed in detail in Section 4, and finally the conclusions presented in Section 5.

[1]Developed at the Kanpur Genetic Algorithms Laboratory, Indian Institute of Technology Kanpur, India. Available online for free download at: http://www.iitk.ac.in/kangal/codes.shtml.

II. Formulation of the Coupler-Curve Synthesis Problem

The formulation for the coupler-curve synthesis problem is fairly standard (see, e.g., [5]) as the problem has been studied extensively. Fig. 1 shows the schematic of the mechanism under consideration. The *coupler point* $\boldsymbol{p}_c$ is required

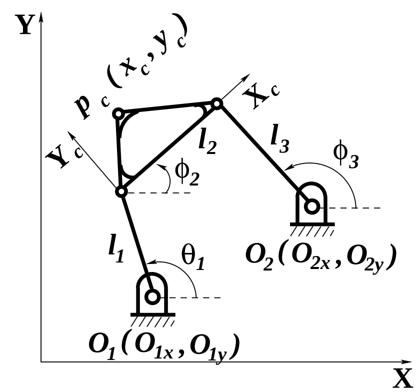


Fig. 1. A planar four-bar mechanism with rotary actuators

to describe a desired curve as the crank, i.e., link 1, runs through a specified interval. From Fig. 1, the coordinates of $\boldsymbol{p}_c$ in the XY frame are found as:

$$x = O_{1x} + l_1 \cos\theta_1 + x_c \cos\phi_2 - y_c \sin\phi_2, \qquad (1)$$
$$y = O_{1y} + l_1 \sin\theta_1 + x_c \sin\phi_2 + y_c \cos\phi_2. \qquad (2)$$

Typically, a number of *target points* are chosen to represent the desired coupler curve, and the original problem is considered to be solved adequately when the coupler curve generated by the synthesised mechanism passes through these points (denoted by (x_{di}, y_{di}), $i = 1, \ldots, n$) at certain specified values of the crank angle, $\theta_{1i}, i = 1, \ldots, n$.

The primary objective is to reduce the *structural error* computed as the sum of the squares of the Euclidean distances of the actual coupler points generated by the mechanism (denoted by (x_{gi}, y_{gi})) from the respective target points at the n specified crank locations:

$$E = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (x_{gi} - x_{di})^2 + (y_{gi} - y_{di})^2. \qquad (3)$$

Though many constraints may be applied on the synthesis problem, the only one considered here is *full-cycle mobility*, i.e., link 1 should be a proper crank, that can rotate through $360°$ without the mechanism getting locked ever [2]. This is possible when the link lengths satisfy the Grashof's conditions (see, e.g., [12]). However, as shown in [11], the same condition can be expressed in terms of the following

[2]This is primarily because in the subsequent sections, the results of the proposed method are compared with several others reported in the existing literature, and hence the problem formulation tries to replicate the existing ones to the extent possible so as to make the comparison meaningful.

set of inequality constraints:

$$g_1 \triangleq \left| \frac{l_0^2 + l_1^2 - (l_2 + l_3)^2}{2 l_0 l_1} \right| - 1 > 0, \tag{4}$$

$$g_2 \triangleq \left| \frac{l_0^2 + l_1^2 - (l_2 - l_3)^2}{2 l_0 l_1} \right| - 1 > 0, \tag{5}$$

$$g_3 \triangleq l_1 + l_2 + l_3 - l_0 > 0, \tag{6}$$

$$g_4 \triangleq l_0 + l_2 + l_3 - l_1 > 0, \tag{7}$$

$$g_5 \triangleq l_0 + l_1 + l_3 - l_2 > 0, \tag{8}$$

$$g_6 \triangleq l_0 + l_1 + l_2 - l_3 > 0. \tag{9}$$

In the above equations, l_0 refers to the length of the fixed base, i.e., the distance between the points $\boldsymbol{O}_1$ and $\boldsymbol{O}_2$ in Fig. 1. Using this form of the full-cycle mobility instead of the standard form of the Grashof's condition has many advantages. Specifically, the values of g_1 and g_2 can be used as measures of "distance" from the singularities. This is made use of specifically in the later part of the paper when the functions $g_1$and g_2 are used to construct a secondary objective, in addition to their use in the constraints.

A. Constraint-handling Scheme

The constrained optimisation problem as formulated above is converted into a unconstrained problem using a penalisation strategy. Each constraint is imposed through a corresponding penalty term, which is added to the objective function; the sum is then used to arrive at the total *fitness value* as required by `NSGA-II`, (i.e., the optimiser used in this work) to rank the individuals. It has been shown in [11] that the use of a *non-linear scaling* of the penalty terms help prevent the distortion of the original objective function, which may have introduced spurious local optima otherwise [13], [14]. The details of the scheme have been included in the Appendix for the sake of completeness.

B. Optimisation using NSGA-II

The optimisation problems formulated in this work are solved with `NSGA-II`. The convergence, as well as the rate of convergence of this method are affected by its internal control parameters, namely: probability of crossover p_c, probability of mutation p_m, distribution index for crossover η_c and the distribution index for mutation η_m. In addition, the other process parameters are: the population size N_{pop}, the number of generations N_{gen}, and the seed value for random number generation. In a related work [11], the authors have presented a systematic study on the sensitivity of the optimal results obtained by NSGA-II to its internal parameters, for the specific case of the coupler curve synthesis problem (treated as a single-objective problem). The parameter values found to provide good convergence therein is used throughout in the present work.

TABLE I. BOUNDS FOR GEOMETRIC DESIGN VARIABLES

Variable	l_1	l_2	l_3	x_c	y_c	O_{1x}	O_{1y}	O_{2x}	O_{2y}
Lower bound	0	0	0	-60	-60	-60	-60	-60	-60
Upper bound	60	60	60	60	60	60	60	60	60

III. MULTI-OBJECTIVE FORMULATION

The coupler curve synthesis problem is posed as a multi-objective problem with the primary objective as the minimisation of the structural error, E, as defined in Eq. (3), and secondary objective as maximising the "distance" from *singularity* (denoted by SD). The multi-objective problem is posed formally as:

$$\begin{aligned} &\text{Minimise} \quad E, \\ &\text{Maximise} \quad SD \triangleq \min(g_1, g_2), \\ &\text{Subject to} \quad g_i > 0, \quad i = 1, \ldots, 6. \end{aligned}$$

In a related work [10], the secondary objective is taken as the improvement of transmissibility, and it is quantified in terms of the departure (denoted by TAE, which is an acronym for "transmission angle error") of the minimum and the maximum transmission angle from the optimal value of $90°$:

$$TAE = (\gamma_{\max} - 90°)^2 + (\gamma_{\min} - 90°)^2, \tag{10}$$

$$\text{where,} \quad \cos\gamma_{\max} = \frac{l_2^2 + l_2^3 - (l_0 + l_1)^2}{2 l_2 l_3}, \tag{11}$$

$$\cos\gamma_{\min} = \frac{l_2^2 + l_2^3 - (l_0 - l_1)^2}{2 l_2 l_3}. \tag{12}$$

Although in this work the secondary objective is formulated so as to keep the mechanism away from the singularities, it is expected that the same would result in good transmissibility as well. In order to assess this in a quantitative manner, the measure TAE is also used to evaluate the quality of the mechanisms obtained in this paper.

IV. RESULTS AND DISCUSSIONS

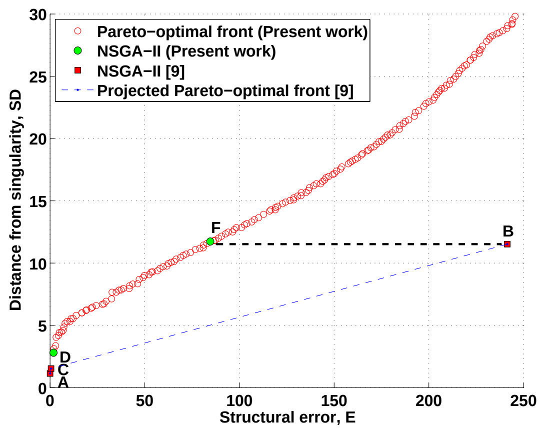
In this section, the formulations described above are illustrated via applications to two different problems from existing literature.

A. Problem 1

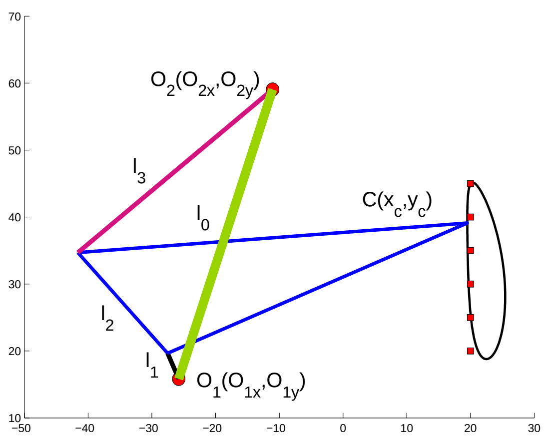
In this case the coupler point needs to pass through six prescribed locations on a straight line (adopted from Case 1 of [10]): $(20, 20)$, $(20, 25)$, $(20, 30)$, $(20, 35)$, $(20, 40)$, $(20, 45)$, respectively. The corresponding crank angles (denoted by $\theta_1^{(i)}$) are not specified, and thus they get included in the variable vector. The other design variables, i.e., the geometric parameters defining the mechanism, and their respective bounds are listed in Table I. The *Pareto-optimal front* obtained is shown in Fig. 2(a). Two solutions on the

TABLE II. PARETO-OPTIMAL POINTS FOR PROBLEM 1 (SEE FIG. 2(A))

	l_1	l_2	l_3	x_c	y_c	O_{1x}	O_{1y}	O_{2x}	O_{2y}	θ_1^1	θ_1^2	θ_1^3	θ_1^4	θ_1^5	θ_1^6
Point D	4.20	20.60	39.08	-17.96	-47.60	-25.78	15.83	-11.05	59.04	3.65	3.14	2.72	2.33	1.91	1.25
Point A [10]	7.13	26.78	22.82	39.24	28.07	-18.90	58.47	-32.01	32.38	5.09	6.63	7.15	7.64	8.17	9.22
Point F	2.29	35.95	50.00	-47.85	-60.0	-48.16	-2.31	-60.00	58.12	4.33	4.29	3.28	0.15	1.17	1.29
Point B [10]	2.13	37.76	37.76	9.93	24.41	-0.49	48.37	-47.48	23.44	5.09	5.12	5.16	8.20	8.25	8.28
Point C [10]	6.81	25.81	28.22	37.03	31.12	-20.07	57.45	-39.84	30.63	5.72	6.64	7.14	7.60	8.16	8.94



(a) Pareto-optimal front



(b) Optimal mechanism (corresponding to Point D)

Fig. 2. Pareto-optimal front and synthesised mechanism for Problem 1

Pareto-optimal front, namely, point D, which corresponds to the smallest structural error, and an intermediate point F, are tabulated in Table II. Three solution points obtained in the Pareto-optimal front for the same problem in [10] are marked as points A, B and C in Fig. 2(a), and also tabulated in Table II. The objective values corresponding to all these points are tabulated in Table III. As can be seen from these, the Pareto-optimal front obtained in the present method is *dominant* over the *projected* (i.e., linearly interpolated) Pareto-optimal front reported in [10]. The point F marked on the Pareto-optimal front dominates point B reported in [10], since it affords the *same* distance from singularity (SD), but with significantly less structural error than point B. The TAE index, used as the secondary objective in [10], is also compared at the corresponding points in Table III.

TABLE III. COMPARISON OF RESULTS FOR PROBLEM 1 (SEE FIG. 2(A))

	E	SD	TAE
Point D	1.83	2.8	444.02
Point A [10]	2.9e-7	1.13	1451.46
Point F	84.61	11.73	40.50
Point B [10]	241.41	11.52	41.68
Point C [10]	0.54	1.53	1023.29

The Pareto front obtained in this work is dominant in terms of the secondary objective (i.e., distance from singularity), and comparable in terms of structural error, the primary objective. The mechanism corresponding to solution point D in the front is shown in Fig. 2(b) along with the target points and the path traced.

B. Problem 2

This problem statement is identical with the Case 2 reported in [10]. Apart from the variables described in Table I, there is an additional variable in this case: the initial position of the crank, $\theta_1^{(1)} \in [0, 2\pi]$. The 18 points are to be reached at $20°$ increments of the crank angle, i.e., $\theta_1^{(i)} = \theta_1^{(i-1)} + \pi/9$, $i = 2, \ldots, 18$. The target points are given in Table IV. The Pareto-optimal front obtained for this problem is shown in Fig. 3. The objective values corresponding to two extreme solutions on the front, i.e., points A and B along with solutions points obtained in the Pareto-optimal front for the same case study in [10], namely D and E, are tabulated in Table V. As can be seen from Fig. 3 and Table V, the Pareto-optimal front obtained in the present work is *dominant* over the projected front reported in [10]. A comparison of both the SD and TAE indices show improvement over [10], whereas the structural error indices are comparable.

V. CONCLUSIONS

A multi-objective optimisation approach to the problem of coupler-curve synthesis of a four-bar mechanism is presented in this paper. The primary objective is to minimise the structural error, and the secondary objective to improve the kinematic performance of the mechanism, by keeping it away from singularities to the maximum

TABLE IV. Coupler points to be traced for prescribed angles in Problem 2

$\theta_1^{(i)}$	$\theta_1^{(1)}$	$\theta_1^{(2)}$	$\theta_1^{(3)}$	$\theta_1^{(4)}$	$\theta_1^{(5)}$	$\theta_1^{(6)}$	$\theta_1^{(7)}$	$\theta_1^{(8)}$	$\theta_1^{(9)}$	$\theta_1^{(10)}$	$\theta_1^{(11)}$	$\theta_1^{(12)}$	$\theta_1^{(13)}$	$\theta_1^{(14)}$	$\theta_1^{(15)}$	$\theta_1^{(16)}$	$\theta_1^{(17)}$	$\theta_1^{(18)}$
x_{di}	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.0	0.0	0.03	0.10	0.15	0.2	0.3	0.4	0.5	0.6	0.6
y_{di}	1.1	1.1	1.1	1.0	0.9	0.75	0.60	0.5	0.4	0.30	0.25	0.20	0.3	0.4	0.5	0.7	0.9	1.0

TABLE V. Pareto-optimal points and comparison of results for Problem 2 (see Fig. 3)

	l_1	l_2	l_3	x_c	y_c	O_{1x}	O_{1y}	O_{2x}	O_{2y}	$\theta_1^{(1)}$	E	SD	TAE
Point A	0.34	5.00	10.00	1.73	-4.16	4.64	1.67	-6.20	-2.82	0.36	0.069	9.95	151.43
Point D [10]	0.33	6.43	0.48	1.83	4.36	2.59	-3.43	6.72	1.33	1.29	9.8e-03	0.09	5454.51
Point B	0.30	10.00	10.00	2.22	-4.05	4.68	1.90	-7.32	-5.55	0.35	0.388	22.57	11.82
Point E [10]	0.30	5.24	5.29	0.74	-1.38	-0.61	-0.66	1.75	6.34	7.06	0.380	11.52	42.15

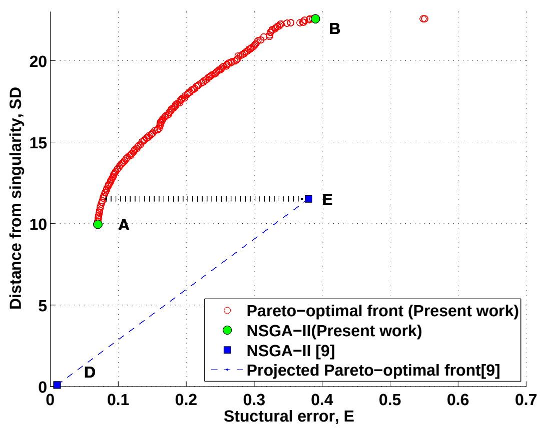


Fig. 3. Pareto-optimal points and comparison of results for Problem 2

extent possible. A GA-based multi-objective optimisation tool, namely, `NSGA-II` has been used in this work. Through the comparison with several existing solutions in reported literature, it is demonstrated that the results obtained in this work are better in terms of the secondary objective, while being comparable in terms of the primary objective of reducing the structural error. These problems are under further study to improve upon the results obtained.

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Appendix
Details of the constraint handling scheme

Consider a typical constrained optimisation problem:

$$\begin{aligned}
\text{Minimise} \quad & f(\boldsymbol{x}), \\
\text{subject to} \quad & g_i(\boldsymbol{x}) \geq 0, \quad i = 1, 2, ..., n; \\
& h_j(\boldsymbol{x}) = 0, \quad j = 1, 2, ..., m; \\
\text{with} \quad & x_k \in [a_k, b_k], \quad k = 1, 2, ..., p,
\end{aligned}$$

where $f(\boldsymbol{x})$ is the objective function; $\boldsymbol{x} = (x_1, x_2, x_3, \cdots, x_p)^T$ is the vector of design variables; $g_i(\boldsymbol{x})$

and $h_j(\boldsymbol{x})$ are the inequality and equality constraints respectively; and a_k and b_k are the upper and lower bounds on x_k, respectively. The proposed approach uses a penalisation strategy in order to convert the constrained optimisation problem to an unconstrained one. Each constraint is imposed through a corresponding penalty term which is added to the objective function; the sum is then used to arrive at the total *fitness value* required by the optimiser. Each penalty term modifies the objective function in the region where it is violated. If the contribution from this term is *too small* in comparison to the original objective, then it may fail to enforce the constraint strictly. On the other hand, if the penalty term is too high in value, the distortion of the original objective function may be so high as to introduce spurious local optima [13]. The inherent drawback in this approach is the difficulty in finding appropriate penalty functions and the corresponding numerical weights, which offer the best compromise for a given problem [14]. In partial solution to the above problem, a transformation of the penalty function is used which confines its values to the interval [0,1], irrespective of the form of the function. A single weight, α, is then used on the sum of the penalty terms. The heuristics in the process is therefore reduced to the choice of a single parameter α, so as to match the order of the nominal penalty terms to that of the unconstrained objective. The details of the scheme are described in the following.

The said transformation of the constraint functions has the following form:

$$\psi_\lambda(t) = \frac{t}{\lambda + t},\ \lambda > 0. \tag{13}$$

It has been used previously in a very different application, namely, navigation of mobile robots [15]. Obviously, $\psi_\lambda(t) \in [0,1]\ \forall t \in [0,\infty]$, i.e., for any non-negative real value of t, the function $\psi_\lambda(t)$ is confined to the interval $[0,1]$. Incorporating these details, the steps for the evaluation of the unconstrained objective for any given individual having the design variables $\boldsymbol{x}$ are as follows:

1) Compute the objective function value, $f(\boldsymbol{x})$.

2) Compute the penalty term quantifying the extent of violation of the equality constraints by the sum of the squares of the *scaled* residuals:

$$h_{eq}(\boldsymbol{x}) = \sum_{j=1}^{m} \psi_\lambda(h_j^2(\boldsymbol{x})). \tag{14}$$

3) Compute the penalty term for the inequality constraints in a similar manner, iff they are violated:

$$g_{eq}(\boldsymbol{x}) = \sum_{i=1}^{n} P(g_i(\boldsymbol{x})), \tag{15}$$

where the function $P(x)$ is defined as:

$$P(t) = \begin{cases} 0 & \text{if } t \geq 0, \\ \psi_\lambda(t^2) & \text{if } t < 0. \end{cases}$$

The function $P(t)$ acts as a switch, adding the penalty term corresponding to a constraint iff it is violated.

4) Compute the unconstrained objective $F(\boldsymbol{x})$ as a sum of the penalty terms and the original objective:

$$F(\boldsymbol{x}) = f(\boldsymbol{x}) + \alpha(g_{eq}(\boldsymbol{x}) + h_{eq}(\boldsymbol{x})). \tag{16}$$

In (16), the positive scalar α is used to weigh the penalty terms appropriately against the original objective.

It is expected that the via proper tuning of the parameters, λ and α, the effect of constraints can be imparted on the overall objective in a *better* manner.