Detection of Fractionated DOF and its Varieties using Modular Kinematics

G Kalyan Ramana
Centre for Product Design and Manufacturing
Indian Institute of Science,
Bangalore, India
gkramana@cpdm.iisc.ernet.in, gkalyanramana@gmail.com

Dr. Dibakar Sen
Centre for Product Design and Manufacturing
Indian Institute of Science,
Bangalore, India
dibakar@cpdm.iisc.ernet.in

***Abstract*—Modular kinematics is a recursive scheme which is typically used for kinematic analysis of mechanisms. This paper extends the scope of modular kinematics for efficient structural analysis, namely for detecting if a given planar kinematic chains (KCs) has fractionated degrees of freedom (d.o.f.) or not. Detailed literature survey reveals the availability of the methods based on loops, graph theory, link-link distances etc. The method introduced is reliable in terms of robustness and completeness. Modular kinematics is based on the fact that any KC can be constructed as a sequence of the classes of modules for example dyad, input, floating link transformations and constraint. This construction procedure is termed as module sequence. This module sequence is used in detection of fractionation in KCs. From this module sequence single d.o.f., subchains are identified. The common link between adjacent single d.o.f. subchains gives way to potential separation link, depending on the connectivity data between the links of the adjacent subchains. The method implemented not only detects whether fractionation in a KC exists or not but also gives details about separation link as well as types in fractionation. A new terminology to define order of fractionation is also introduced depending on the number of type of fractionations in the KC. A number of illustrative examples have been shown to explain the method in an efficient manner.**

Keywords – Fractionated degree of freedom; structural analysis; modular kinematics; fractionation varieties

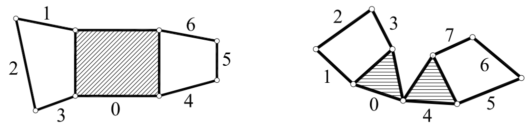
I. Introduction

The d.o.f. of a KC is defined as the number of independent parameters required to completely define the state of link members. A KC may have total, partial or fractionated d.o.f. For a f-d.o.f. KC with total freedom, any combination of f-joints can be selected for giving input to the mechanism; whereas in partial and fractionated freedom it is not so. Determination of type of freedom for a KC is a challenge in structural analysis. The test for total freedom and partial freedom can be converted to a problem of identification of the feasible input pairs. A fractionated d.o.f. KC is characterized by the presence of a separation link which can be cut into two segments to split the KC into two independent KCs such that if F is the d.o.f. of the original KC, and F_1 and F_2 are the d.o.f. of the component KCs, then, $F = F_1 + F_2$. However the KCs with simple joints have been dealt within the studies. Kinematic chains with fractionated d.o.f. have many applications in fields like robotics where decoupling of grasping and manipulation of robotic hand is very much necessary. But in some applications viz., of toggle mechanisms for switches [1] fractionated d.o.f. is unwanted. Hence, recognition of fractionated type of freedom is important.

Fractionation i.e. division of the KC into two different sub-chains can be done either by removing a joint or cutting a link. These were referred to as body fractionation, joint fractionation and fractionation into open chains in [2]. There is mention of another version of the third type in [3] as hybrid kinematic subchains. If a link is cut or fractionated to divide the KC into two subchains, it is called body fractionation and if a joint is removed, it is called joint fractionation to divide the KC into two subchains. In body fractionation the sum of d.o.f. of the two subchains obtained is equal to d.o.f. of the KC; but in joint fractionation it is not so. There exists a relationship between type of sub-chains obtained and connectivity of separation links. In body fractionation, to obtain two closed subchains connectivity of separation link must be $\geq$ four otherwise it will yield hybrid KC. For possibility of joint fractionation, there must exist at least two connected separation links. To obtain two closed subchains in joint fractionation the separation links must have connectivity $\geq$ three; otherwise it will yield hybrid KCs. All the varieties in fractionation have been shown in Fig. 1. In Fig. 1(a), body fractionation into two closed subchains is possible since the link 0 has connectivity as four. Two closed subchains by joint fractionation are obtained by removing 0-4 as shown in Fig. 1(b).

A. Literature Review

Test for fractionated d.o.f. has been studied extensively in the last five decades. The dependency of link member's motion was first observed by Crossley [4] where it was mentioned that for a two d.o.f. mechanism member's motion are dependent on both the inputs and gave examples justifying it. Since then structural analysis was rigorously studied. Then classification of d.o.f. into total and partial



(a) Body fractionation by link 0.

(b) Joint fractionation at joint 0-4.

Fig. 1: Varieties in fractionation.

d.o.f. was done by Hain [5]. Manolescu [6] introduced the concept of fractionated d.o.f. in multi-freedom mechanisms. Further, Davies [7] extended the Manolescu's classification and gave tests for detection of type of freedom based upon loops and graph theory. It was opined that fractionated d.o.f. depends on the choice of fixed link or inversion. The view on separation link that it must have atleast 4 pairs was criticized by Mruthyunjaya [8] who proposed a modified definition for the fractionated d.o.f. in KCs. The author also gave tests for detection of fractionated d.o.f. using the concept of cut vertex in graph theory and used the concept of computation of reachability matrix associated with each link. Each vertex in the graph (corresponding to a link in KC) was removed from the adjacency matrix and reachability was computed using any of the vertex (rows or columns). If all the matrix elements did not become zero then it was concluded that it is a separation link otherwise not. This process is computationally involved because all of the links have to be tried until a separation link is obtained. Agrawal and Rao [9] used path loop connectivity matrix for detection of fractionated d.o.f of multi-freedom KC. The authors also developed analytical methods for the same. Their method was based on detection of independent loops in a KC and proving that some of the loops do not have any connection between them. On similar lines, Liu and Yu [10] also used loop based procedures for the test. Hunt [2] classified the types of fractionation into body-fractionation, joint-fractionation and fractionation with open chains. Agrawal [11] introduced the concept of link-link distance matrix for freedom analysis. The method is similar to that of [8]. Hamming number technique was used by Rao [12] to detect fractionation in a KC. In this paper the test was based on relationship between connectivity of the KC, number of links and d.o.f. of the chain. The same author introduced the concept of loop value and link value in a loop based method for detection of fractionation in [13]. Here in this paper, fractionation in a KC was detected by analysing the difference obtained in loop values when observed and calculated.

B. Limitations of Previous Methods

Overall, the following observations were made regarding test for fractionated d.o.f.:

- Loop based procedures do not identify the separation link.
- The methods mentioned in [8] and [11] are computationally expensive.
- None of the methods mentioned above classify the type of fractionation in a KC.

In this paper, modular kinematics is used which is found to be effective and not computationally involved. A brief introduction to the modular kinematics and symbolic representation of the solution scheme is provided in the appendix section of this paper. A new approach of cutting a joint and link along with introducing a new terminology on order of fractionation is being explained in section 2. Section 3 explains how the modular kinematics is characterized to obtain the separation link. The idea behind the method proposed in this paper is explained in section 4. Section 5 gives the algorithm for detection, varieties and order of fractionation. In the last section, some illustrative examples with results and discussion along with future work are presented.

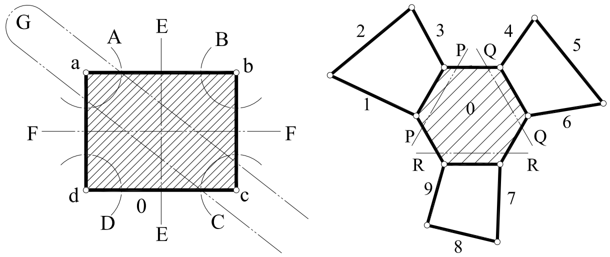
II. Variety in Fractionation

A KC can be uniquely represented by a graph whose vertices and edges correspond to links and joints of the chain [5]. A cut vertex or cut edge of a graph G is identified as an element upon whose removal disconnects G into two parts. To identify body fractionation presence, cutting of separation link is a must to obtain two independent KCs. There is also a provision of joint fractionation, in the KC which must be similar to cutting a joint to obtain two independent KCs which is not happening in [2] as they considered removal of joints completely. It is observed here that there is inconsistency in terms of cutting when body and joint fractionation of a KC into two independent KCs are considered. To bring some uniformity in terms of cutting following concepts are proposed.

A. Simple and Multiple Fractionation

The splitting of separation link in a fractionated d.o.f. KC can be viewed as sharing of the joints on the link among two components. This reduces the problem to that of finding those partitions of the joints that would give rise to the increase in number of components. For example, in Fig. 2(a), 0 is a quaternary link containing the joints a, b, c and d which can be partitioned into groups of (1,3) and (2,2) joints in seven ways viz. (a,(b,c,d)), (b,(a,c,d)), (c,(a,b,d)), (d,(a,b,c)), ((a,b),(c,d)), ((a,c),(b,d)) and ((a,d),(b,c)). Out of these, only ((a,b),(c,d)) leads to an increase in number of components; whereas in Fig. 2(b) there are three partitions which split the KC.

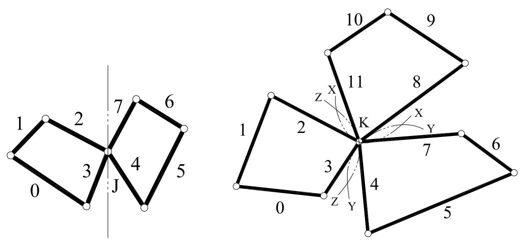
On similar lines, the notion can be extended to cut joints where instead of removing the joint, the links incident at the joint would be shared in the two components. Cutting a joint means sharing of links connected to it. Basically, it implies the cutting of the pin that holds the revolute joint (in this case) between the links thereby sharing the links of the KC. In Fig. 3(a) joint J has d.o.f. as three. On careful



(a) Simple body fractionation at E-E by link 0 of the KC shown in Fig. 1(a).

(b) Multiple body fractionations at P-P, Q-Q, and R-R by single separation link 0

Fig. 2: Cutting of separation links



(a) Simple joint fractionation to yield two independent KCs at joint J.

(b) Multiple joint fractionations by sections X-X, Y-Y and Z-Z at joint K.

Fig. 3: Cutting of separation joints.

observation, it can be seen that the links 2, 3, 4 and 7 are held by one pin only. For joint fractionations this pin can be cut in several ways i.e. joint pin between any two links is to be cut keeping the connectivity of others intact. Similar possibilities as mentioned above can be explored. Out of all the possibilities only one of them will yield two independent KCs i.e. two four bars {0,1,2,3} and {4,5,6,7}. Similarly, as in the case of multiple ways of cutting a separation link, there are multiple ways of cutting a joint to obtain variety of independent KCs. In Fig. 3(b) multiple independent KCs are obtained if the pin at the joint K is cut by sections X-X, Y-Y and Z-Z. Here, we can observe that fractionating a link or joint yields simple fractionations (see Fig. 2(a) and 3(a)) if there is only one way of obtaining independent KCs and multiple fractionations (see Fig. 2(b) and 3(b)) if many ways are found. In this work, only simple body and joint fractionation have been considered for their detection. The detection of other types of fractionating freedom using modular kinematics is under progress.

B. Order of Fractionation

As mentioned in section 1, joint fractionation is possible when two separation links are adjacent. To represent the details of number of body and joint fractionations in the KC, order of fractionation of the KC is being introduced. The order of fractionation is indicated with an ordered pair

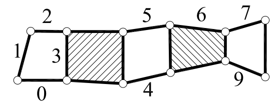
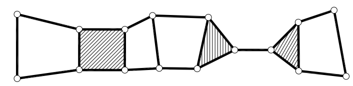
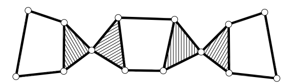


Fig. 4: Fractionated KC of order 2.0.



(a) KC with fractionating blocks of order 1.0 and 3.2.



(b) KC with fractionating blocks of order 2.1 and 2.1.

Fig. 5: KCs with same fractionating order 4.2.

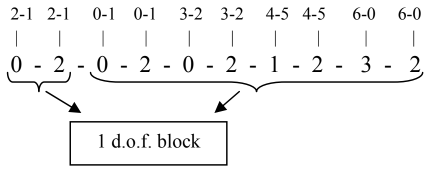
of integers (X,Y) where the first number corresponds to the number of separation links and second number corresponds to that of separation joints. For instance, order of fractionation for the KC shown in Fig. 1(b) and in Fig. 4 is (2,1) and (2,0) respectively. However, in case of fractionated KCs with only simple joints and simple fractionation inherent relationship between X and Y is observed. If there is joint fractionation it would be flanked by two separation links. A set of successive separation links and joints is referred to here as a fractionating block. Let A_i be the number of separation links in the i^{th} fractionating block, B_i number of separation joints in the same block and also r be the number of fractionating blocks. Then it is easy to see that

$$\begin{aligned} & B_i = A_i - 1, \text{ also } \sum_i^r A_i = X \text{ and } \sum_i^r B_i = Y \\ & \implies \sum_i^r (A_i - 1) = Y \implies \sum_i^r A_i - \sum_i^r 1 = Y \\ & \implies X - Y = r \end{aligned} \tag{1}$$

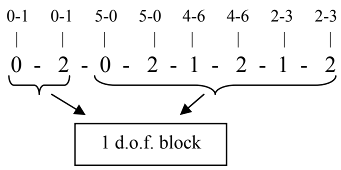
This implies that all the KCs with a given order of fractionation have the same number of fractionation blocks irrespective of the distribution of the fractionating links and fractionating joints in the blocks. For example, consider the KCs in the Fig. 5 whose order is 4.2. Work is under progress regarding producing of all the possibilities of fractionating blocks in the KC.

III. Characterization of Module Sequence for Fractionation

A fractionated kinematic chain is characterized by presence of separation link and partitioning of d.o.f. of KC. Consider, for example, the KC shown in Fig. 11 which has seven links and 2 d.o.f. If one of the pairs say 1-2 is



(a) Module Sequence if 2-1 is chosen as input pair.



(b) Module Sequence if 0-1 is chosen as input pair.

Fig. 6: Comparison of module sequence for KC in Fig. 11 with respect to link 1 as fixed.

frozen, it divides the KC into so called passive and active subchains. Passive subchain is that part of a KC (set of links) that gets frozen with the frozen pair and these set of links act as a single link for the rest of KC. In the given example, if pair 1-2 is frozen, links 1 and 2 form the passive subchain. The remaining set of links which can still have motion form the active subchain. For the sake of simplicity, let us assume that the passive subchain be named after the link that has the lowest index and a '$*$' as superscript; thus the passive subchain $\{1, 2\}$ is represented as 1^*. The active subchain is the set of links $\{1^*, 0, 3, 4, 5, 6\}$. On similar lines, if pair $\{0, 3\}$ is frozen in Fig. 1(a), $0^* = \{0, 3, 1, 2\}$ is the passive subchain and $\{0^*, 4, 5, 6\}$ forms the active subchain. The subchains formed in these cases are single d.o.f. subchains. This concept can be extended to n-d.o.f. KCs. On observation of the module sequence (see appendix for brief overview) in Fig. 11(g), the first single d.o.f. block is the passive subchain and the second block together with first block forms the active subchain. Therefore, a module sequence can be used nicely to reveal active and passive subchains as single d.o.f. blocks.

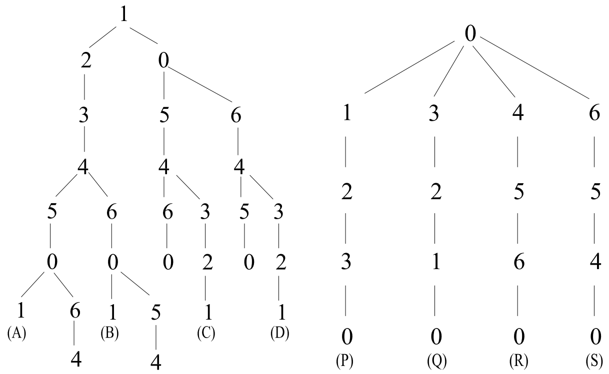
Same KC may have different module sequence if different fixed links and pairs as input modules are used (see appendix on how to obtain module sequence for a KC). For the same fixed link and different pairs as input modules different modules sequences are obtained. For example, in the case of KC shown in Fig. 11 if link 1 is fixed, module sequences are obtained as shown in Fig. 6. Module sequence for this KC has difference in the type of modules used. The module sequence obtained in Fig. 6(a) has links in single d.o.f. subchains as $\{1, 2\}$ and $\{0, 1, 2, 3, 4, 5, 6\}$. On comparing the single d.o.f. subchains, there are more than one links $(1$ and $2)$ common. Similarly, the module sequence in Fig. 6(b), has links in single d.o.f. subchains as $\{0, 1\}$ and $\{0, 2, 3, 4, 5, 6\}$ which yield only link 0 as common. Consider the module sequence in Fig. 11(g). Here single d.o.f. subchains are $\{1, 2\}$ and $\{0, 1, 3, 4, 5, 6\}$ has link 1 as common. This implies that for different inversions, different number of common links in module sequence are obtained. But for the mechanism in Fig. 1(a), no matter which inversion is considered for constructing the mechanism, link 0 is always the common link between the two blocks. It can be concluded that, if a given link is obtained as a common link in all the module sequences, then that link is the fractionating link and therefore the mechanism has a fractionated d.o.f. To determine fractionation, all the possibilities of module sequences are to be obtained until a different or more than one common link appears between the single d.o.f. blocks. If in any case more than one common link appears between the single d.o.f. blocks, then the KC does not have fractionated d.o.f. So, when only one common link is detected, instead of exploring all the possibilities in all module sequences for a given KC, the common link in a single module sequence can be analysed to detect the fractionation as described below.

IV. Analysis of the Common Link

It is interesting to note that when multiple links are common between the two sets of links in the adjacent blocks in the module sequence, the adjacency of the links themselves is not relevant to recognize the absence of fractionated d.o.f. In fact, it is the presence of joints between one link in one block and the other in the adjacent block that calls for additional test to detect fractionation. This is done using an adjacency tree. The adjacency tree is created with a given link as root and links adjacent to it as branches. The branches of next level are determined from adjacency of corresponding links. Care must be taken that the parent link does not appear again in the list of child links. At any level, if the link in any of the branch is same as the parent link, the construction of the tree is terminated. This will be clear by the following examples. Consider the mechanism shown in Fig. 11. The adjacency tree with common link 1, obtained between the two single d.o.f. blocks see Fig. 11(g), as root node is as shown in Fig. 7(a). If loops A and B are observed, the links 2 and 3 are also found in loops C and D which implies that loops A, B, C and D are interlinked or connected in some way or the other. Unlike in Fig. 7(a), the adjacency tree for mechanism in Fig. 1(a) is obtained as shown in Fig. 7(b). This tree is drawn with the common link 0 which is obtained between the two sets of single d.o.f. blocks in Fig. 12. Here we can observe that the links $1, 2$ and 3 (in loops P and Q) do not appear in loops R and S which implies that loops (P, Q) and (R, S) are not at all connected meaning link 0 is a separation link.

V. Algorithm for Analysis of D.O.F.

The construction of a given KC from a module sequence is explained in appendix. The module sequence can be used for testing whether a given KC has fractionated d.o.f. or not. It is assumed that all the rows and columns of matrices



(a) Adjacency tree for the mechanism in Fig. 11 with link 1 as root.

(b) Adjacency tree for the mechanism in Fig. 1(a) with link 0 as root.

Fig. 7: Adjacency tree analysis of the common link.

number start with zero rather than one. The method works in the following way:

1. Obtain the module sequence of the given KC in the symbolic way shown in Fig. 10 (see Appendix). .
2. Starting from end of the sequence, identify the single d.o.f. blocks using guidelines provided in Appendix.
3. Identify the links in each of the single d.o.f. blocks and store their ids in different sets say S_1, S_2, S_3, ..., S_n.
4. Compare each set with every other set like S_1 vs S_2, S_1 vs S_3, etc. and store their result in a matrix $S = (S_{ij})_{f\times f}$ according to the condition (where f is d.o.f. of the KC).
$$S_{ij} = \begin{cases} x & \forall\ x \in (S_i \cap S_j)\ if\ \|S_i \cap S_j\| = 1 \text{ and } i < j \\ -1 & \text{otherwise} \end{cases} \quad (2)$$
If all the elements in the matrix S are negative then the given KC is not fractionated.
5. Initialize matrix C to adjacency matrix A. All the entries of the common link index's row and column in C matrix are to be made equal to zero.
6. Consider the non negative entry from S . It indicates the common link index which is involved in both the single d.o.f. blocks (say i^{th} row and j^{th} column). Take each element from set S_i and multiply the element's row and column by two in matrix C. Similarly, each element of S_j is multiplied by $three$ in matrix C . If there exists any entry as a multiple of six in C then it can be concluded that the KC is not a fractionated KC; otherwise, it is a fractionated KC. Moreover, if the KC has been proved to be fractionated, then the common link is itself a separation link and insert this link's index in a set F. Hence, F contains separation link indices.
7. Repeat steps 5 and 6 until all the non-negative elements in S are covered. If there exists a separation link, while computing C from each combination it can be concluded that the given KC as fractionated otherwise not.
8. Obtain the order of fractionation in the ordered pair form (X, Y) where
$$X = \|F\| \text{ and } Y = \textstyle\sum_{i,j} A_{ij}, \text{ where } i{=}f_m,\ j{=}f_n, \\ \text{where } m, n{=}1,2,3,...\|F\| \text{ and } m < n\ \forall\ f_k \in F \quad (3)$$
Y represents the total number of joint fractionations which is possible in the KC. Here, X represents the number of body fractionations.

VI. Illustrative Examples, Results and Discussion

It is assumed that C matrix (and adjacency matrix) row and column numbers start with zero for the sake of simplicity.

Example 1. Module Sequence obtained (if link 2 is fixed) for the mechanism shown in Fig. 11(g). Set of links from each of the single d.o.f. blocks are S_1={$1, 2$}, S_2={$0, 1, 3, 4, 5, 6$} and common link = {1}.

$$S{=}(S_{ij})_{2\times 2} = \begin{array}{ccc} & S_1 & S_2 \\ S_1 & -1 & 1 \\ S_2 & -1 & -1 \end{array}$$

$$C \text{ for } S_2 \text{ vs } S_1 = \begin{bmatrix} 0&0&0&0&0&9&9 \\ 0&0&0&0&0&0&0 \\ 0&0&0&6&0&0&0 \\ 0&0&6&0&9&0&0 \\ 0&0&0&9&0&9&9 \\ 9&0&0&0&9&0&0 \\ 9&0&0&0&9&0&0 \end{bmatrix}$$

Row and column 1 entries are made equal to zero. Then elements in S_1 are multiplied by two and elements in S_2 by three. Element 2×3 is a multiple of six. Hence the given link is not a separation link; which implies that it has no fractionated d.o.f. Hence, there is no point in discussing about body or joint fractionation. The order of fractionation is (0, 0).

Example 2. Module Sequence obtained (if link 3 is fixed) for the mechanism in Fig. 1(a) is shown in Fig. 12. Set of links from each of the single d.o.f. blocks are S_1={$0, 1, 2, 3$}, S_2={$0, 4, 5, 6$} and common link = {0}

$$S{=}(S_{ij})_{2\times 2} = \begin{array}{ccc} & S_1 & S_2 \\ S_1 & -1 & 0 \\ S_2 & -1 & -1 \end{array}$$

$$C \text{ for } S_1 \text{ vs } S_2 = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&0&9&0&0&0 \\ 0&0&9&0&9&0&0 \\ 0&0&0&9&0&0&0 \\ 0&0&0&0&0&4&0 \\ 0&0&0&0&4&0&4 \\ 0&0&0&0&0&4&0 \end{bmatrix}$$

Here, we can observe that there is no element which is a multiple of six. Hence, we can conclude that the given KC is a fractionated one with link having index 0 as separation link. Body fractionation into two closed KC by this link is

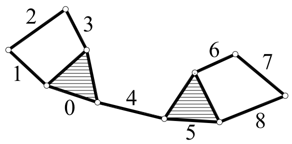


Fig. 8: Fractionated KC with binary link as separation link.

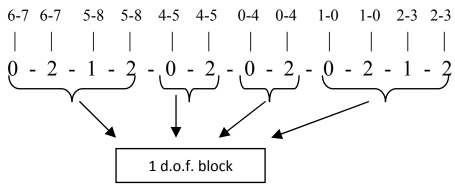


Fig. 9: Module sequence of for KC represented in Fig. 8.

possible since it has connectivity more than three. The order of fractionation is $(1, 0)$.

Example 3. Module Sequence obtained (if link 7 is fixed) for the mechanism shown in Fig. 8 is represented in Fig. 9. Set of links obtained in single d.o.f. blocks are S_1={5, 6, 7, 8}, S_2={4, 5}, S_3={0, 4} and S_4={0, 1, 2, 3}. On comparing the elements of each set, we get the following S matrix.

$$S = (S_{ij})_{4\times4} = \begin{array}{c|cccc} & S_1 & S_2 & S_3 & S_4 \\ S_1 & -1 & 5 & -1 & -1 \\ S_2 & -1 & -1 & 4 & -1 \\ S_3 & -1 & -1 & -1 & 0 \\ S_4 & -1 & -1 & -1 & -1 \end{array}$$

C for S_1 vs S_2 =
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \end{bmatrix}$$

Here, there is no entry which is a multiple of six, that implies that link 5 is a separation link.

C for S_2 vs S_3 =
$$\begin{bmatrix} 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 1 & 0 \end{bmatrix}$$

Absence of multiples of six in C implies that link 4 is a separation link.

C for S_3 vs S_4 =
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Here also, it can be observed that absence of multiples of six implies link 0 is a separation link. Hence, the given KC has fractionated d.o.f. with three separation links namely $0, 4$ and 5 and moreover they are connected to each other when one observes the adjacency matrix. On observation, we can obtain corresponding body and joint fractionations using the rules from section 1.

1. Body fractionation
   a. Link 0 and 5 have connectivity $= 3$, which implies that when link 0 or 5 have been fractionated or cut, there can be one closed and one hybrid subchains.
   b. Link 4 has connectivity two, which implies that when link 4 is cut or fractionated, two hybrid sub chains are obtained. Hence, total number of body fractionations is three.
2. Joint fractionation
   Here joints available for joint fractionations are 0-4 and 4-5. Either of them when fractionated gives two hybrid sub chains. Hence, total number of joint fractionations is two. The order of the fractionation is $(3, 2)$.

The algorithm is implemented in C++ language using in-house modular kinematics library. The inputs to the program are link-link adjacency data, joint data and fixed link index. The results obtained for one inversion are valid for all other inversions. This means that even though module sequence written with one fixed link differs (some cases may not) with that obtained when another link is fixed, separation link is obtained any way for fractionated d.o.f. and gives reliable results for non-fractionated d.o.f. KC also. The main point is testing for one inversion gives exact results, hence there is no need of verifying other inversions. This result disagrees with the claim made by Davies [7] that fractionated d.o.f. depends on inversion or the link which is fixed. The separation links obtained in example 3 exactly complies with that of shown in [8]. This implies that the method perfectly adheres to the definition of fractionated d.o.f. and agrees with the claim in [8] that a separation link need not have atleast four pairs; it can be a binary link also. The method developed works for any d.o.f. KC, provided there is proper link-link adjacency information given. It readily gives out the separation link data without considering any loops. It is observed that the step of identification of independent or basic loops is eliminated entirely. The trial and error has been minimized to a very few links (one in some cases) for proving it to be a separation link or not.

VII. Conclusion

In this paper, a new method for detection of fractionated d.o.f. has been automated and all the problems mentioned in section 1 are solved. The method is simple, reliable and gives quicker results when compared to loop based or link based procedures. Also, a new pattern of fractionations has been observed as simple and multiple fractionations. A new notion called order of fractionation has been introduced. This order of fractionation gives details about distribution of fractionating links in a simple jointed fractionated KC. The work on how it is distributed is under progress.

References

[1] Manan Deb, Systematic Synthesis and Analysis of Multi-DOF Toggle Mechanisms for Electrical Switches,M.Sc.(Engg.) Thesis. Indian Institute of Science, India, 2012.

[2] C. R. Tischler, A. E. Samuel, and K. H. Hunt, "Kinematic chains for robot hands-II. Kinematic constraints, classification, connectivity, and actuation", Mechanism and Machine Theory, vol. 30(8), pp. 1217-1239, 1995.

[3] D. Martins, R. Simoni, and A. P. Carboni, "Fractionation in planar kinematic chains: Reconciling enumeration contradictions", Mechanism and Machine Theory, vol. 45(11), pp. 1628-1641, 2010.

[4] F. E. Crossley, "A Contribution to Gruebler's Theory in the Number Synthesis of Plane Mechanisms", Journal of Engineering for Industry, vol. 86(1), pp. 1-5, 1964.

[5] T. S. Mruthyunjaya, "Kinematic structure of mechanisms revisited", Mechanism and Machine Theory, vol. 38(4), pp. 279-320, 2003.

[6] N.I. Manolescu, "A unitary method for construction of the kinematic plane chains and plane mechainsms with different degrees of mobility", Revue Roumaine des Sciences Techniques, Serie de Mecanique Applique, vol. 9, pp. 1263-1313, 1964.

[7] T. Davies , "An extension of Manolescu's classification of planar KCs and mechanisms of mobility M$\geq$ 1, using graph theory", Journal of Mechanisms, vol. 3(2), pp. 87-100, 1968.

[8] T. S. Mruthyunjaya and M. R. Raghavan, "Computer-aided analysis of the structure of kinematic chains", Mechanism and Machine Theory, vol. 19(3), pp. 357-368, 1984.

[9] V. P. Agrawal and J. S. Rao, "Fractionated freedom kinematic chains and mechanisms", Mechanism and Machine Theory, vol. 22(2), pp. 125-130, 1987.

[10] T. Liu and C. H. Yu, "Identification and classification of multi-degree-of-freedom and multi-loop mechanisms", Journal of mechanical design, vol. 117(1), pp. 104-111, 1995.

[11] J. N. Yadav, C. R. Pratap, and V. P. Agrawal, "Freedom analysis of planar kinematic chains using the concept of distance" Journal of mechanical design, vol. 118(3), pp. 367-371, 1996.

[12] A. C. Rao, "Hamming number technique-1. Further applications", Mechanism and Machine Theory, vol. 32(4), pp. 477-488, 1997.

[13] A. C. Rao and Prasad Raju Pathapati, "Loop Based Detection of Isomorphism Among Chains, Inversions and Type of Freedom in Multi Degree of Freedom Chain", Journal of Mechanical Design, vol. 122(1), pp. 31-42, 2000.

[14] V. S. C. Sekhar Chowdary, Modular Kinematic Analysis of Planar Linkages, M.Sc.(Engg.) Thesis. Indian Institute of Science, India, 2006.

[15] D. Sen, Shape-Synthesis of Workspaces of Planar Manipulators with Arbitrary Topology, Doctoral Thesis. Indian Institute of Science, India, 1996.

Appendix

Modular kinematics is a recursive scheme is typically used for kinematic analysis of mechanisms. This paper utilizes the symbolic description of modular kinematics for detecting if a given planar KCs has fractionated d.o.f or not. The concept of kinematic analysis using modules is not new. It is based on the set of constraints that define the topology of the mechanism and can be resolved into smaller sets of constraints, each of which can be solved in closed form or iteratively ([14]). This implies that solving the kinematics of these independent constraints (called modules) in a particular order leads to solving of the kinematics of the mechanism.

A. *Types of modules*

Any KC can be represented in terms of the classes of modules namely dyad, input, floating link transformations and constraint module.

1. **Input modules** are the ones through which the mechanisms receive angular or linear inputs depending upon the type of pair involved. These modules contribute to the overall d.o.f. by $+1$.
2. **Dyad modules** typically consists of two links connected by a joint with two half joints to be connected at points on two distinct links whose kinematic states are known. These modules do not contribute to the overall d.o.f.
3. **Transformation modules** are the ones which determine the unknown kinematic states of points on a given link from known kinematic states of other points on the same link. These modules also do not contribute to the overall d.o.f.
4. **Constraint modules** arise when the kinematic state of a number of points belonging to a given link have been computed independently for the sake of ease of computation. Hence, in this module, consistency of the computed quantities are verified in accordance with the assumption of the rigidity of the links. These modules contribute to the overall d.o.f. by -1.

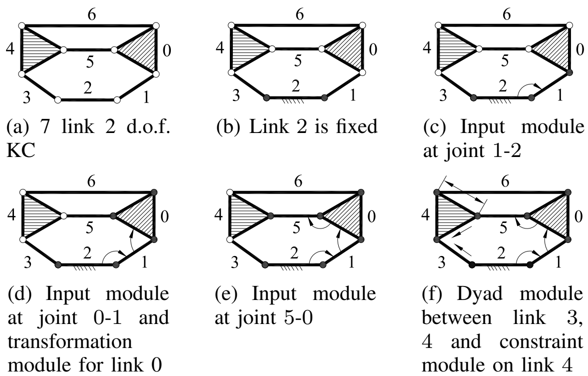
The whole mechanism is constructed in a step-by-step manner, starting from the ground or fixed link by adding one module at a time in a particular order. Thus, it involves at every step, a decision about which particular module is to be added from the modules in the set, discussed above to construct a desired part of a given mechanism. A solution to this problem of deciding which module to be added next is called a procedure or module sequence.

B. *Module sequence*

The main objective of a module sequence is to derive and represent the construction of a mechanism in terms of a set of predefined modules. The module sequence is completed when kinematic states of all the links in the mechanism are known starting from the ground link. Each step goes through the recognition, extraction and status upgradation phases. For details on how these phases occur, refer [14]

$$\begin{array}{ccccccccc} l_1\text{-}l_2 & & l_3\text{-}l_4 & & l_5 & & l_6-l_7 & & l_8 \\ | & & | & & | & & | & & | \\ m_1 & - & m_2 & - & m_3 & - & m_4 & - & m_5 \end{array}$$

Fig. 10: Symbolic representation of module sequence



(a) 7 link 2 d.o.f. KC

(b) Link 2 is fixed

(c) Input module at joint 1-2

(d) Input module at joint 0-1 and transformation module for link 0

(e) Input module at joint 5-0

(f) Dyad module between link 3, 4 and constraint module on link 4

$$\underbrace{\overset{1\text{-}2}{0} - \overset{1\text{-}2}{2}}_{\text{1 d.o.f. block}} - \underbrace{\overset{0\text{-}1}{0} - \overset{0\text{-}1}{2} - \overset{5\text{-}0}{0} - \overset{5\text{-}0}{2} - \overset{3\text{-}4}{1} - \overset{3\text{-}4}{2} - \overset{6\text{-}4}{3} - \overset{6\text{-}4}{2}}_{\text{1 d.o.f. block}}$$

(g) Overall module sequence schematically.

Fig. 11: Steps in construction of mechanism using predefined modules.

and [15]. Module sequence when applied for a mechanism is illustrated by the following example.

For the sake of clarity, a symbolic representation of the module sequence, which was introduced in [14], will be used in the rest of the paper. Input, dyad, transformation and constraint modules are represented by 0, 1, 2, and 3 respectively. The module sequence is thus represented in the form as shown in the Fig. 10, where m_1, m_2, m_3...etc are the module types and l_1, l_2, l_3...etc. are the links in the corresponding module. Module sequence for a 2-DOF, 7 link mechanism (link 2 is fixed) is determined as follows. Module sequence proceeds on the basis of updating the status of joint locations. Status of a link (in planar mechanisms) in this context can be related to the kinematic state of a body. If at least two points/joints are known on a link, then it implies that status of the same is completely known, since two points on a planar body are sufficient to know its position and orientation (or kinematic state). For details on these statuses, refer [14] and [15]. The module sequence or the procedure of constructing the mechanism does not depend on the geometric dimensions and actual values of input parameters. It depends on the fixed link and input pair definition. Same mechanism may have different module sequence depending on fixed link and input locations. This

$$\underbrace{\overset{0\text{-}3}{0} - \overset{0\text{-}3}{2} - \overset{1\text{-}2}{1} - \overset{1\text{-}2}{2}}_{\text{1 d.o.f. block}} - \underbrace{\overset{4\text{-}0}{0} - \overset{4\text{-}0}{2} - \overset{5\text{-}6}{1} - \overset{5\text{-}6}{2}}_{\text{1 d.o.f. block}}$$

Fig. 12: Module sequence for KC represented in Fig. 1(a).

implies that a KC can be constructed in any number of ways. All the known joints are represented as solid circles and remaining as empty circles. Here, as we can see that link 2 is fixed; so the location of joints associated with it i.e., 1-2 and 2-3 are known (Fig. 11(b)). Now input is given to link 1 with respect to link 2. Hence joint 1-0 is also known (Fig. 11(c)). Similarly, input is given to link 0 with respect to link 1 which gives status of joint 5-0 (Fig. 11(d)). Here two points (joints) on link 0 are known, joint 0-6 is known using transformation module. Now another input is given to link 5 with respect to link 0 which gives the status of joint 4-5. On observation, there is a dyad module that can be added at link 3 and 4 which gives the status of joint 3-4 (Fig. 11(e)). The left out joint is 4-6. Therefore, a constraint module between 5-4 and 4-6 can be introduced on link 4 which is computed independently. In this way, whole mechanism status is known. The overall module sequence for the given mechanism is represented symbolically in Fig. 11(g).

A block in a module sequence is defined as a section of the main sequence. For example, in the above module sequence the blocks are indicated by an under brace. These are single degree of freedom blocks. The d.o.f. of the mechanism/block is obtained as the number of input modules minus the number of constraint modules. So to identify the single d.o.f. blocks, first begin with right end of the module sequence and move towards left. Assign value of d.o.f. $= 0$. While moving towards left of the module sequence, if an input or transformation or dyad or a constraint module is found, add $+1$ or 0 or 0 or -1 respectively to d.o.f. until d.o.f.$= 1$. The resulting part of sequence covered till now is single d.o.f. block. It is a subchain of the main KC. Again assign value of d.o.f.$= 0$ and repeat the above steps with the next module, until whole of the module sequence is completed. Obviously, the d.o.f. of KC should be equal to number of single d.o.f. blocks in the sequence. Similarly, module sequence for the mechanism (link 3 is fixed) shown in Fig. 1(a) is obtained as in Fig. 12. The above scheme of modular kinematics has been implemented and the module sequence in the above format is obtained easily from the link-link adjacency information for any planar linkage mechanism. Thus, in rest of the paper, the module sequence for a mechanism is assumed to be readily available for the d.o.f. analysis.