Infinitesimal Mechanisms Analogy for Immobilization

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***Abstract*—This paper presents an analogy between a planar immobilized object with point contacts and a platform type first-order infinitesimal planar mechanism. It is proposed that a smooth immobilized object has similar mobility characteristics with the immobilized platform in a first-order infinitesimal mechanism. It is shown that through the classical higher pair lower pair replacement, a first-order infinitesimal mechanism is generated from second-order form-closed pair. Procedure to synthesize immobilized objects from a first-order infinitesimal mechanism is detailed and implementation is shown for two examples. It is observed that for a given first-order infinitesimal mechanism, multiple families of second-order form-closed pairs can be synthesized. A geometrical technique based upon the first-order contact of workspace boundaries of sub-chains of the linkage is used for determining the first-order infinitesimal mechanism configurations.**

Keywords – infinitesimal mechanism; form-closure; singularity; immobilization

I. INTRODUCTION

Mobility of a closed-loop kinematic linkage at an assembled configuration can be seen as its ability to remain assembled when at least one of its joint variables change by an arbitrarily small value from the initial ones. If the kinematic linkage possesses mobility over a finite neighborhood around the joint values of the assembled configuration, then the linkage has finite mobility and it is called a 'mechanism'. If the mobility is limited to an infinitesimally small neighborhood, then the linkage has infinitesimal mobility. Mobility analysis of the four-bar linkage with variable coupler length reveals that for the coupler length in-between certain maximum and minimum limits, the linkage has finite mobility [1]. At the limiting values of the coupler length, the four-bar linkage can be assembled, but its finite movability is lost (Fig.1). However, the linkage can move *infinitesimally*. Such a class of mechanisms with infinitesimal mobility are more generally called 'Infinitesimal Mechanisms' and are available in the domain of structural engineering [2]. An 'Infinitesimal Mechanism' is an assemblage of bars and pins which can undergo an infinitesimal motion such that the every bar has a deformation only above a certain order. First-

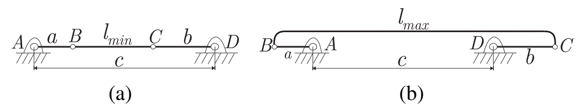


Fig. 1. Configurations of the four-bar linkage with limiting coupler lengths. (a) minimum length: $l_{min} = c - a - b$, (b) maximum length: $l_{max} = a + b + c$.

order infinitesimal mechanism is associated with second-order changes of bar lengths [3]. For another school of rigidity theorists, first-order infinitesimal mechanism is a *second-order rigid framework* and has instantaneous first-order mobility but does not have any higher-order mobility [4]. It is well known that these configurations of linkages are *kinematically singular*. From the traditional applied kinematics point of view, an infinitesimal mechanism is in a singular configuration and does not have higher-order mobility. The linkage can only be just assembled to be in a singular configuration. So, in this paper, we refer a first-order infinitesimal mechanism as *Just Assemblable Singular Configuration* (JASC) of a linkage assembly. The two linkages shown in Fig. 1 are JASC of the four-bar linkage. JASC from the well known Cayley diagram for cognate linkages is shown in Fig. 2. The duality of statics and kinematics results in an analogy between the equilibrium of structures and compatibility of mechanisms. Statically it means that at a JASC it is *possible* to have stable state of self-stress. Lack of finite motion capability is the reason for their less popularity among kinematicians. Wohlhart [5] first reported them in mechanisms literature through the study of the "degrees of shakiness" of the platform type linkages. Using this terminology, *a linkage in a JASC has first-degree of shakiness*. Infinitesimal mechanisms traditionally contain only revolute and prismatic joints; those containing higher pairs are not available in literature. This paper presents a study on first-order infinitesimal mechanisms having only higher pairs which appear in the context of immobilization.

The problem of immobilization reported in [6], [7], [8], [9], [10] deals with immobilizing smooth shapes with point contacts; in the analysis, the curvatures of the contacting objects are also required to be considered. Researchers in grasp

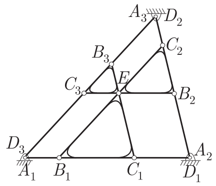


Fig. 2. Fixing the vertices of the outer triangle in the Cayley diagram results in a linkage with only first-order mobility.

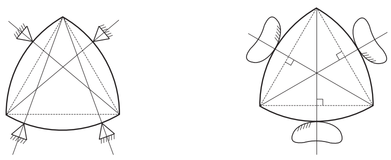


Fig. 3. Form-closure of Reuleaux triangle. (a) first-order form-closure with four contacts, (b) second-order form-closure with three contacts whose contact normal lines are concurrent at the geometric center of the triangle.

and fixture design dealt with first-order form-closure (simply form-closure) [11] even before immobilization conjectures were proposed. In the first-order form-closure analysis of objects, only contact normal information is required at a contact. Hence, in literature and this paper, contacts are shown with knife edge geometry for first-order analysis. Fig. 3(a) shows a first-order form-closed Reuleaux triangle using four contacts. Researchers in the grasp and fixture design are increasingly using the theory of second-order form-closure to study the effect of curvature on immobilization [12], [13]. The mathematical notion of immobilization corresponds to the second-order form-closure in mechanical engineering. Hence, we use both the terms interchangeably in this paper. In Fig. 3(b), the same Reuleaux triangle is second-order form-closed with only three contacts. However, the triangle has first-order mobility about the point of intersection of the three contact normal lines. At the so called '*Second-order Form Closed Configuration*' (SFCC), the movable object has first-order mobility but no higher-order mobility; a similarity it shares with JASC. This motivated us to explore the relationships between the two seemingly distinct areas of applied kinematics. In this paper, as in the second-order form-closure literature, we consider only the *restraint against second-order rotation about any point on the plane and translation in all directions*.

We show that JASC of a planar platform type linkage can be used to synthesize SFCC through the lower pair to higher pair replacement rule and that indeed such a replacement is possible by properly selecting the curvatures at the point of contact. In section 2, we present a geometrical synthesis technique for JASC. The curvature based mobility analysis and form-closure are explained in section 3. In section 4, we explain the kinematic reciprocity that connects JASC and SFCC, and derive certain generalized propositions. A procedure to synthesize SFCC from JASC and the design examples form the section 5.

II. Geometrical Determination of JASC

It is reported in literature that at a JASC, the constraint Jacobian matrix loses rank [14]. Constraint Jacobian matrix losing rank constitutes a singular configuration of the linkage which falls under the category of configuration space singularity [15]. Also, the configuration space of the linkage in JASC is just an isolated point as the linkage can be assembled only at a single configuration. Therefore, constraint Jacobian matrix losing rank is not a sufficient condition for JASC. A sufficiency criterion based on the definiteness of a particular linear combination of Hessians of the constraint equations is available in [14]. But this characterization does not help much in the synthesis of a linkage with JASC. In the following, a geometrical determination technique based upon the concept of workspace boundaries is presented.

The geometrical aspect of JASC can be well understood form the ideas of workspaces of manipulators. In a generic closed loop linkage, we choose a *non-pivoted joint* of interest as the end effector point. Let $\mathcal{W}$ be the corresponding workspace for the linkage. The linkage can be cut at this joint (say cut-joint) into k pivoted sub-chains. Let $\mathcal{W}_i$ be the workspace of the i-th sub-chain. The net workspace $\mathcal{W}$ can be obtained by the intersection of the individual workspaces of these sub-chains; i.e., $\mathcal{W} = \bigcap_{i=1}^{k} \mathcal{W}_i$. We restrict in this paper only to linkages involving two sub-chains at the cut-joint; their workspaces are $\mathcal{W}_1$ and $\mathcal{W}_2$. The workspace of any sub-chain is either a curve (say c-type) or a region (say r-type) accordingly if the sub-chain is a 1-d.o.f. or a multi-d.o.f. ($\geq$ 2-d.o.f.) linkage. *JASC requires that $\mathcal{W}$ is just a single isolated point and the linkage has first-order mobility*.

In the case when both $\mathcal{W}_1$ and $\mathcal{W}_2$ are of c-type, they can intersect to give a finite number of points. Assembling the two sub-chains at such points gives an isolated configuration, but does not give any first-order mobility as the sub-chains have 1-d.o.f. Therefore, it is further required that the two workspace curves are locally tangential in the first-order of tangency at the common point to provide first-order mobility and no higher-order mobility.

Next, consider the scenario in which at least one of the workspaces is of r-type. Since the intersection of the workspaces has to be a single isolated point, such a point should belong to the boundary of the r-type workspace. The sub-chain corresponding to r-type workspace is a multi-d.o.f. linkage. It is known in literature that a multi-d.o.f. linkage at its workspace boundary is in a singular configuration. Now, if the cut-joint's motion is frozen, this sub-chain will become a closed linkage. This closed linkage too is in a singular configuration and gains d.o.f.. So, a sub-chain with r-type workspace will have an inherent first-order mobility. Hence, in this case, it is not necessary that the two workspace boundaries are tangential at the common point. The tangent to one or both the workspace boundaries can be ill-defined at the common point as shown in Fig. 4. Such points of the r-type workspace are called cusp singularities [16]. Further, for

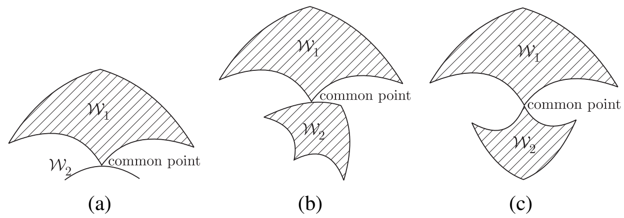


Fig. 4. Cusp singularities of r-type workspace's boundary provide locations where it can share only a single point with the other workspace's boundary without being tangential to it.

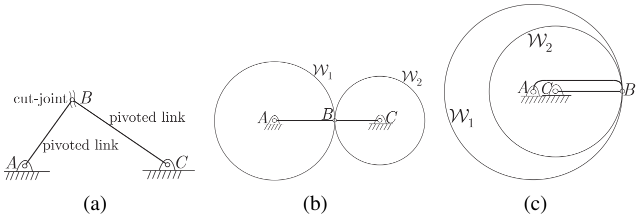


Fig. 5. Triangular truss and its JASCs. (a) truss ABC cut at joint B, (b) external tangency of circles, (c) internal tangency of circles.

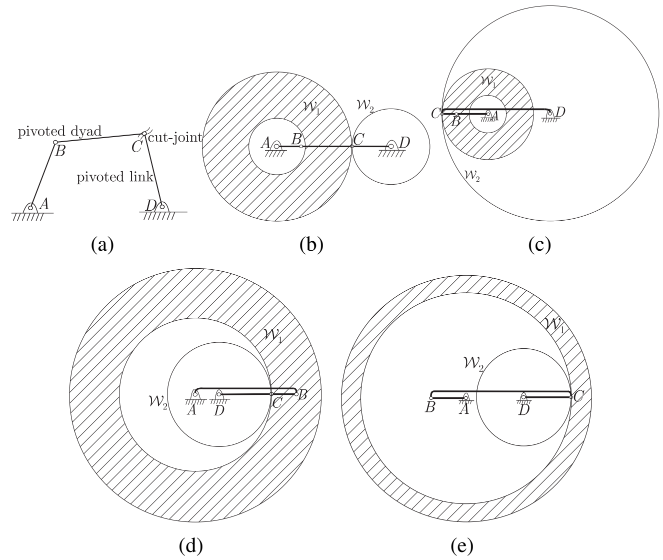


Fig. 6. Four-bar linkage and its JASCs. (a) four-bar linkage $ABCD$ cut at the joint C, (b) outer boundary circle and link circle are externally tangent, (c) outer boundary circle and link circle are internally tangent, (d) inner boundary circle and link circle are internally tangent and dyad's longer link is pivoted, (e) inner boundary circle are internally tangent and dyad's shorter link is pivoted.

a sub-chain having r-type workspace, we have to make sure that the inherent gained d.o.f. do not result in second-order mobility. For this, freeze the original cut-joint and again use the same concept of workspace boundaries at a new *cut-joint in the sub-chain*.

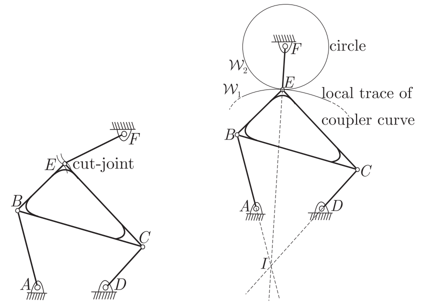
Summarizing, it is derived that if none of $\mathcal{W}_1$ and $\mathcal{W}_2$ is of r-type, they have to contact in first-order of tangency for a JASC. If at least one of them is of r-type, it is sufficient that they share only a common point on their workspace boundaries. Further, if the common point is an ordinary point for both the workspace boundaries, then $\mathcal{W}_1$ and $\mathcal{W}_2$ have to contact in first-order of tangency. Using this technique we now determine JASC for a few linkages. The selection of the initial cut-joint in a given linkage is such that the two sub-chains are familiar linkages whose workspace boundary geometries are easily derivable.

(i) Triangular truss: In Fig. 5(a), the truss ABC is cut at the joint B to give two pivoted links. The workspace of each of them is a circle whose center is at the pivot point and radius same as link length. JASC occurs when the two circles are either externally or internally tangential as shown in Fig. 5(b) and (c) respectively.

(ii) Four-bar linkage: The four-bar linkage $ABCD$ in Fig. 6(a) can be cut at the joint C to give a pivoted dyad ABC and a pivoted link DC. The workspace of the dyad is an annular region defined by two bounding circles, viz., inner boundary circle and outer boundary circle. The link's workspace is the link circle. JASCs occur in the four tangency scenarios of the two workspaces, shown in Fig. 6. In all the four cases, if the cut-joint's motion is frozen, the dyad still has only inherent first-order mobility and no second-order mobility. It is only a matter of choice that joint C is chosen as the cut-joint. In the case of the four-bar linkage, it can be easily verified that the results are same for the other cut-joint B.

(iii) 3-RR platform linkage: The linkage $ABCDEF$ shown in Fig. 7(a) can be decomposed into the four-bar linkage $ABCDE$ and the pivoted link FE, both joined at the coupler point E of four-bar linkage. The point-path of the coupler point is the coupler curve. At a JASC, the circle traced by the pivoted link's free end is tangential to the coupler curve in the first-order of tangency (Fig. 7(b)). If the pivot point F coincides with the center of curvature of the coupler curve of the four-bar linkage, then the above circle has second-order tangency with the coupler curve. Then the linkage loses second-order rigidity. This issue is also mentioned in [5] which has been derived purely through kinematic analysis. Fig. 16(a) in the Appendix shows the location of the center of curvature of the point-path of E of the same four-bar linkage $ABCDE$ obtained by geometric construction. The pivot F in Fig. 7(b) is different from the location of the center of curvature; thus the linkage under consideration is indeed in a JASC. In decomposing the linkage we could have cut the chain at joints B or C either. This raises an issue whether the check for second-order mobility and consequently the synthesis of JASC is dependent upon the four-bar sub-chain chosen. In the Appendix, it is argued through geometric constructions that the second-order mobility check is independent of the cut-joint.

Now consider the linkage shown in Fig. 8 where the three line-of-joints of the binary links 1, 3, and 5 are concurrent



(a) (b)

Fig. 7. 3-RR platform linkage and a JASC. (a) linkage $ABCDEF$ cut at the joint E, (b) coupler curve and the circle have first-order contact.

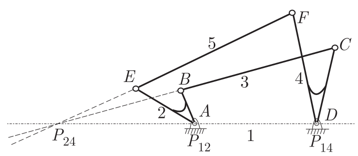


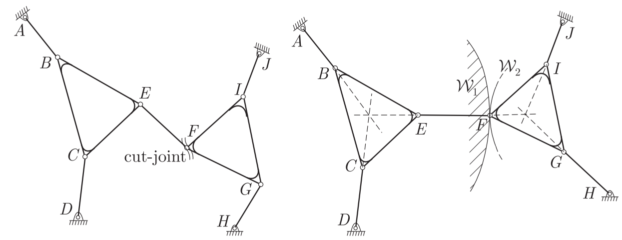
Fig. 8. Four-bar type linkage with two coupler links in a singular configuration.

at point P_{24}. Suppose that either one of the links 3 or 5 is removed. Then, for the resulting four-bar linkage, P_{24} is the velocity pole for the relative motion between links 2 and 4, in spite of which ever link is deleted. So, the simultaneous presence of the links 3 and 5 does not inhibit the linkage in having first-order mobility. Now, using the concept of workspaces of sub-chains it is difficult to answer whether this linkage is in a JASC. But recognizing that this linkage is a kinematic inversion of a 3-RR platform linkage with a binary link fixation, makes it convenient to verify whether that 3-RR platform linkage is in a JASC.

(iv) Two platform linkage: The linkage $ABCDEFGHIJ$ shown in Fig. 9(a) can be cut at the joint F to give sub-chains $ABCDEF$ and $HGIJF$. Sub-chain $ABCDEF$ is a four-bar linkage with a dangling link on coupler and its workspace is of r-type. Sub-chain $HGIJF$ is a four-bar linkage whose workspace is of c-type. In Fig. 9(b), at an ordinary workspace boundary point of linkage $ABCDEF$, the two workspace boundaries of the sub-chains are tangential in the first-order of tangency. Further, freezing the motion of joint F, the resulting 3-RR platform linkage $ABCDEF$ does not have second-order mobility which is verified by the method employed in the previous example.

III. Relationship Between Curvature and Mobility

In this section, the results of the curvature based mobility analysis are reproduced from [13] and contextualized. The three types of contacting geometries are shown in Fig. 10. These three types belong to two classes, viz., convex (Fig. 10(a)) and concave class (Fig. 10(b) and (c)). The movable and fixed curve are labeled *m*-curve and f-curve respectively.



(a) (b)

Fig. 9. (a) Two platform linkage, (b) tangency of workspaces.

The centers of curvature of f-curve and *m*-curve are C_f and C_m respectively. The contact normal line is referred to as n-line and the direction of positive y_f-axis along the n-line is referred to as n-direction. *The n-direction is in the direction away from the material side of f-curve at a contact. For a convex contact the n-direction is from C_f towards C_m and for a concave contact it is the opposite.*

For any point of contact, the plane is partitioned into four regions (Fig. 10) based on the mobility characteristics of each point on the plane for a second-order rotation about that point. For a point in region 1, both clockwise (CW) and counterclockwise (CCW) rotations are blocked. Region 2 contains points for which there is penetration for CW rotation and separation for CCW rotation. In mobility region 3, the situation is opposite. In region 4, there is no penetration for both directions of rotation. Region 4 contains points as rotation centers with bidirectional separation mobility as well as those with bidirectional contact preserving mobility. While regions 1 and 4 are the partitions of the n-line; regions 2 and 3 are the two half-planes on either side of the n-line. The region delimited by the centers of curvature is either region 1 or region 4 depending upon the geometry of the curves at the point of contact. The complementary region on the n-line is of the alternate type. If the rotation center coincides with either C_f or C_m, the contact between the curves will be preserved for the second-order rotation about this point; thus *m*-curve remains adhered to the f-curve. Translation is considered as the asymptotic rotation about the point at infinity along a line orthogonal to the direction of translation and the four mobility regions unambiguously characterize this asymptotic point. The regions 2 and 3 are similar to the unidirectional mobility regions partitioned by the n-line in Reuleaux's first-order analysis [17]. Second-order mobility analysis partitions the contact persistence velocity poles along the n-line into regions 1 and 4.

In a multiple contacts scenario, the net mobility is the intersection of the mobility obtained for the point with respect to each contact. Thus, a contact configuration for which region 1 covers the whole plane corresponds to a SFCC and the object is immobilized. Synthesis of such contact configurations have been dealt with in [13]. Fig. 11(a) illustrates a two-contact immobilization. In Fig. 11(b) and (c), concurrent n-lines of three concave and convex

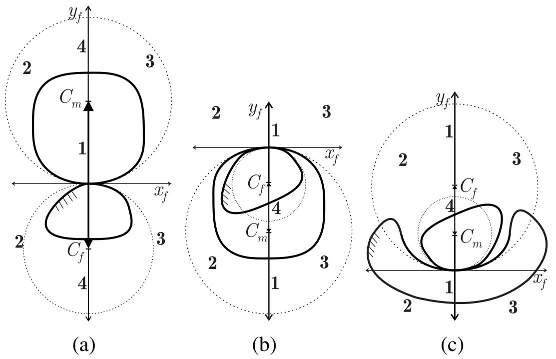


Fig. 10. Second-order mobility regions. Regions 1: Every rotation blocked, 2: CW rotation blocked, 3: CCW rotation blocked, 4: No rotation blocked. (a) convex(m)-convex(f) contact, (b) concave(m)-convex(f) contact, (c) convex(m)-concave(f) contact.

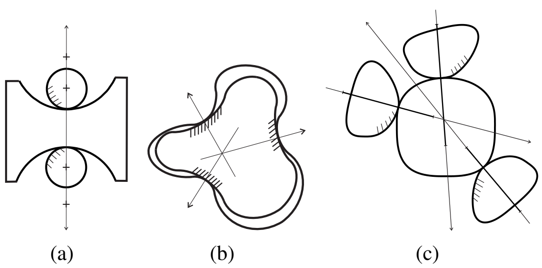


Fig. 11. SFCC pairs. (a) immobilization with two concave contacts, (b) immobilization with three concave contacts, (c) immobilization with three convex contacts.

contacts immobilize the object.

IV. Kinematic Reciprocity

It is well-known in mechanisms analysis about the replacement of a higher kinematic pair with lower kinematic pair and vice-versa to give kinematically equivalent systems. Such substitute pairs are called reciprocal pairs [18] with the reciprocity being either transitory up to certain order or full-cycle. Particularly in planar mechanisms, often a higher pair consisting of two contacting profile curves is replaced with two revolute pairs and an additional binary link is introduced connecting the centres of curvature of either of the contacting profiles. The revolute joints are at the respective centres of curvature. Such equivalence generally holds good up to second-order, i.e., up to acceleration analysis. It can be noted that such mutual replacement is possible only when there is a persistence of contact in the higher pair possibly by external means; for example, a retaining spring provides such persistence in a cam-follower mechanism. In the context of grasping and fixture design such persistence is not assumed and the constraints are unilateral. Thus, the binary link substitution concept does not apply in general at a contact.

However, for a SFCC, it is observed that *simultaneous substitution at all the contacts* gives a linkage in a JASC and the object corresponds to the platform in that linkage. The

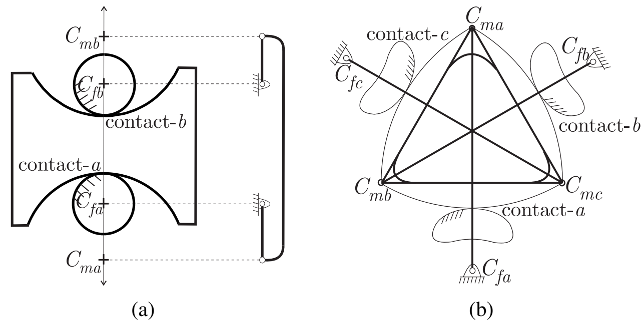


Fig. 12. Substitute-linkage for SFCC is in a JASC. (a) Two contact immobilized pair gives a four-bar linkage, (b) immobilized Reuleaux triangle gives a 3-RR platform linkage.

platform in JASC too does not have second-order rotation and translation mobility. For example, consider the two concave contact fixture in Fig. 12(a). Performing the binary link substitutions, one gets a JASC of the four-bar linkage same as the one in Fig. 6(e). In Fig. 12(b), the binary link substitution for the immobilized Reuleaux triangle gives a 3-RR platform linkage in a singular configuration. It is verified using the decomposition technique that this 3-RR platform linkage lacks second-order mobility; thus it is in a JASC. Hence, we obtain the following proposition.

Proposition 1: *At a SFCC, classical higher pair to lower pair replacement at the contacts results in a JASC of a platform type linkage.*

Now, let us consider the inverse substitution problem of replacing binary links with higher pairs in a single platform linkage which is in a JASC. In Fig. 13(a), the platform's velocity pole is located at point I, the intersection point of the lines containing the legs of the platform. Let each of the legs is to be replaced with a point contact. Each line containing the platform's leg will be the n-line for a contact and all the n-lines intersect in I. The n-direction for each n-line can be chosen in two ways. Let the n-directions for the three n-lines constitute a set. There are only two sets of n-directions such that the n-directions in each set can positively span all the directions at the point of intersection I. These two sets are named set-1 and set-2, and are shown in Fig. 13(b) and (c) respectively. Performing the first-order mobility analysis of the platforms with these two sets of unilateral constraints by Reuleaux's method [17] shows that both the platforms can have their velocity pole only at the point of intersection of the three n-lines and that pole has *contact persistence characteristic with respect to each contact*. Thus, the two platforms are kinematically equivalent to the platform of the JASC up to the first-order kinematics. The n-directions in the above two sets are such that they completely annihilate the unidirectional mobility regions and first-order mobility is possible only at the point of intersection of the three n-lines. These are the only sets which satisfy this condition. Other configurations of the n-lines that occur in the JASC of platform type linkage are : two coincident n-lines in the case of two legs lying

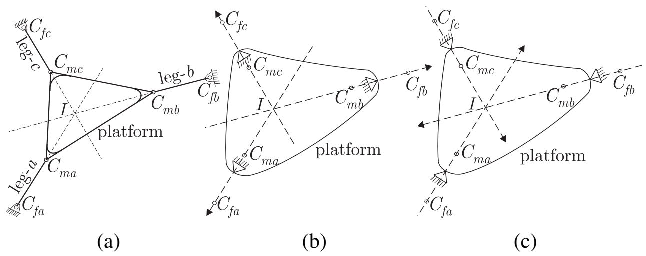


Fig. 13. First-order kinematically equivalent platforms. (a) platform of a 3-RR linkage in a JASC, (b) platform with set-1 n-directions, (c) platform with set-2 n-directions.

on the same line and three parallel n-lines in the case of three parallel legs. For two coincident n-lines, the two n-directions are chosen to be oppositely oriented to annihilate the unidirectional mobility regions and the velocity pole can lie only on the n-line. Here too, we get two sets of n-directions. For similar annihilation in the case of three parallel n-lines, the n-direction of the middle n-line should be opposite to that of the other two; meaning the two non-middle n-lines have the same n-direction. Even in this case only two sets of n-directions are possible. Hence, in all the cases discussed, there are only two sets of n-directions. The next proposition summarizes this replacement of bilateral constraints with unilateral constraints for a certain kind of equivalence in a particular case.

Proposition 2: *At a JASC of a planar platform type kinematic linkage, all the lower pair joints connecting the platform to the base through legs can be replaced with point contacts with the platform by choosing the contact directions in such a way that the first-order kinematic equivalence is maintained and the contacts persist for such equivalence without any external means.*

Now coming to the full contact geometry, each n-direction with two centers of curvature along the n-line give the contact class. Accordingly, all the contacts in Fig. 13(b) and (c) are required to be concave and convex class respectively. It is shown in [13] that for three concurrent n-lines with n-directions like the ones in Fig. 13(b) and (c), the first and second-order mobility analysis differ only about the mobility characteristic of the point *I*. So, we are sure that the second-order rotation about any point of the plane except point *I* is blocked. The second-order mobility analysis reveals that the bidirectional rotation about *I* is allowed for the platform with the n-directions and centers of curvature as shown in Fig. 13(c), but it is blocked for the platform in Fig. 13(b). The platform in Fig. 13(a) too does not have any second-order rotation about any point on the plane. Generalizing the above observations, the following proposition can be stated.

Proposition 3: *For an equivalent system mentioned in proposition 2, with centers of curvature of contacting curves at the erstwhile joint locations, and the contact class being defined by the centers of curvature and the n-direction, if the second-order rotation about point/s of intersection of the n-lines is blocked, then the contacting curves form a SFCC.*

In the next section, we give in detail a procedure to synthesize SFCC from JASC based upon the above propositions.

V. Synthesis of SFCC from JASC

This section deals with the systematic procedure of synthesizing SFCC from a JASC of single platform type linkage. The synthesis steps are given below.

A: Draw the n-lines

Locate the platform and identify the legs (binary links) connecting it to the base. The two hinge points of a binary link will be the centers of curvature (C_m and C_f) for the curves at the contact which is to be introduced in place of that link. The line joining C_m and C_f gives the corresponding n-line.

B: Choose the directions for n-lines

This step is based on proposition 2. If two n-lines are coincident, choose mutually opposite directions for the two n-directions. For three concurrent n-lines, choose the n-directions such that they positively span all the directions. Finally, in the case of three parallel n-lines, the two non-middle n-lines should have same n-direction and the middle one has the opposite direction. At this stage one gets two sets of n-directions.

C: Determine the class of each contact

For each of the n-direction in a particular set obtained from step-*B*, the class of the contact (convex or concave) is decided by the orientation of the n-direction w.r.t. C_m and C_f. For a convex contact the n-direction is from C_f towards C_m and for a concave contact it is the opposite.

D: Check whether the point/s of intersection of the n-lines belong to region 1 of second-order mobility w.r.t. at least one of the n-lines

Once the contact classes for all the n-directions are obtained, *check whether the point/s of intersection of the n-lines (single point or points on coincident lines or a point at infinity) belong to region 1 of second-order mobility analysis for at least one of the n-lines*. If the check fails, then repeat this step for another set of n-directions.

E: Choose the contact points along the n-lines

For a concave class contact, the point of contact should not belong to the line segment delimited by the centers of curvature; it is the opposite for convex contact. Once the contact point is chosen, the local geometry is defined by the two centers of curvature.

F: Connect the individual curve pieces to get closed *m*-curve and *f*-curve

It is quite possible that for a certain feasible set of n-directions in step-*B*, one might not get a feasible SFCC in step-*D*. All the designs having same nature of contact geometry for each corresponding n-line, but different locations of contact points, can be put into a single family. *If the contact class for a given n-direction is concave, then there are two possibilities for the type of contact geometry. So, every concave class contact doubles the number of families of design solutions for a given JASC*. Also, by mere selection of different location of contact points while keeping the contact types same, it may so happen that the connected *m*-curve may lie entirely inside or outside the *f*-curve. This further results in a separate family. Hence, it is possible that at the end of step-*F*, one might obtain multiple families of SFCC designs for a single set of contact-directions obtained from step-*B*. The above procedure is applied to two examples given below.

Example 1: Four-bar linkage

Let us synthesize SFCC from the linkage shown in Fig. 6(b). The steps are shown in Fig. 14. Step-*B* yields two sets of n-directions. In step-*C*, the n-directions with the given centers of curvature imply that the both the contacts in set-1 and set-2 are concave and convex class respectively. Since the two line segments, delimited by the centers of curvature, do not overlap, the only way that the entire line containing these two segments belongs to region 1 is that neither of the contacts should be of convex class. So SFCC cannot be synthesized with the n-directions of set-2. Only set-1 yields SFCC which are shown in Fig. 14(c). Since the two contacts have to be concave, minimum four (2×2) design families are readily possible. But additional designs are obtained by choosing the contact locations differently. Thus the number of distinct families of designs is seven. In Fig. 14(c), the corresponding contacts in designs (i) and (iv) have same nature of contact geometry but different contact locations. Same is the case with (v) and (vi), (ii) and (vii).

Example 2: 3-RR platform linkage

For the linkage in Fig. 13(a), step-*B* gives two sets of n-directions shown in Fig. 13(b) and (c). In step-*C*, all the contacts for n-directions of set-1 and set-2 are to be concave and convex respectively. From step-*D*, it is obtained that point *I* belongs to region 1 w.r.t every contact for a concave contact scenario but belongs to region 4 in the case of convex contact. So only set-1 generates SFCC and the other set does not. A representative set of the design families is shown in Fig. 15.

The above examples show that for a given JASC more than one family of SFCC are obtained. In other words, multiple families of SFCC give the same JASC. In this sense *JASC of linkage provides a central concept in grouping and distinguishing various SFCC apart from their synthesis*.

VI. Conclusion

SFCC can be seen as platform type first-order infinitesimal mechanism with only higher pairs. There is a symbiotic relationship between the first-order infinitesimal mechanisms and second-order form closed configurations which has been hitherto unexplored in the mechanisms literature. JASC offers a route to the designer for synthesizing complex SFCC configurations. A geometrical technique is presented for obtaining JASC of a linkage assembly. The concept of equivalent JASC is useful in grouping various SFCC designs.

Appendix

Geometric proof for independence of cut-joint in 3-RR platform linkage: For the same linkage in Fig. 7(b), consider now the scenario when F coincides with the center of curvature of coupler path of point E belonging to the four-bar sub-chain $ABCDE$. In Fig. 16(a), point F is now located using Aronhold's construction for path-curvature [18] for the same four-bar linkage. For this location of pivot F, consider the four-bar linkage $FECDB$ with B as coupler point (Fig. 16(b)). Again using the same construction, the center of curvature of the point-path of coupler point B is located at A_o. The construction steps in the second case trace back the original construction lines and angles. So, we have $P_o = Q$ and $Q_o = P$, which results in $A_o = A$. Hence, the center of curvature of the point-path of coupler point B is located at A. A similar result can be derived for the four-bar chain $ABEFC$. This shows that *if the analysis of the decomposed sub-chains at a cut-joint results in the second-order mobility, then decomposition at any another cut-joint too will result in the same*. Now using the transposition logic, a similar statement can be made about absence of second-order mobility.

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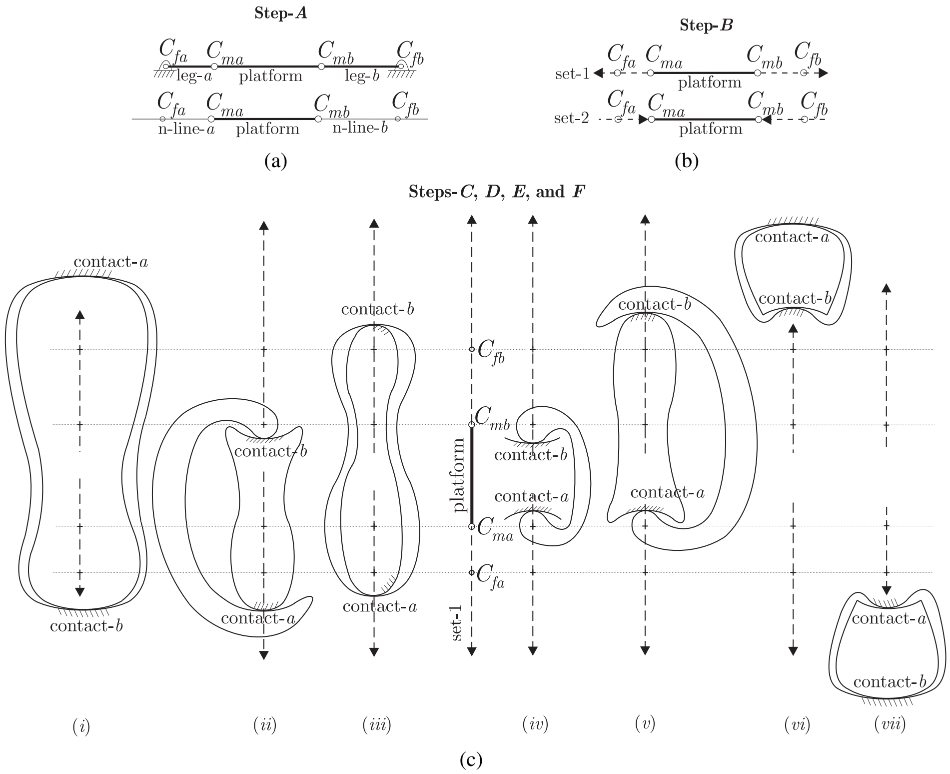

Fig. 14. SFCC from JASC of a four-bar linkage. (a) legs replaced with n-lines, (b) two sets of n-directions, (c) SFCCs for set-1.

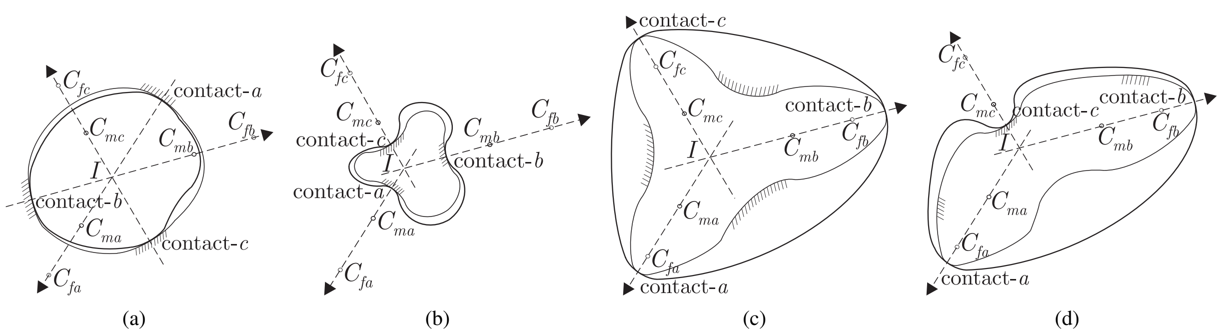

Fig. 15. SFCC designs from JASC of a 3-RR platform linkage.

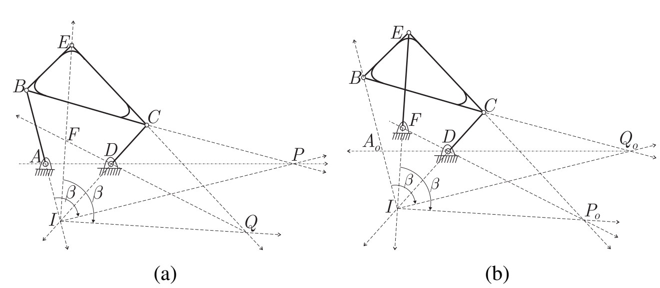

Fig. 16. Geometric construction for locating the center of curvature of point-path of the coupler point. (a) center of curvature, point F, is located for the path of coupler point E of the four-bar linkage $ABCDE$, (b) previous construction lines and angles are traced back for locating center of curvature with B as coupler point of the linkage $FECDB$.

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