A Novel Approach for Detection of Isomorphism of Kinematic Chains

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***Abstract*— Countless methods are reported to check isomorphism amongst kinematic chains. A novel method for detection of isomorphism and inversions based on theoretic approach, simple to work out and reliable is recommended in this paper. The work offered here is focused on recognition of isomorphic chains with their inversions.**

Keywords— ***kinematic chains; isomorphism; inversions, least distance matrix***

I. Introduction

Structural analysis and creation of kinematic chains is a vital aspect of mechanism design. The mainly important stage in the study of kinematic structure of mechanisms is the structural synthesis or classification and listing of kinematic chains with a known number of links and degree of freedom.

A.C. Rao[1] suggested utility of fuzzy logic to inspect isomorphic chains and inversions. Fuzzy membership is assigned to each link of a kinematic chain which develops a fuzzy vector in terms of fits for every link, on the basis of its adjacency. In other words, for each link separate vectors for first adjacency, second adjacency and so on are projected. Numerical measures to evaluate the numerous individual chains with the similar number of links and degrees of freedom (d.o.f.) for chain properties like symmetry, parallelism and mobility are projected. Fuzzy entropy is applied with the intention of evaluating the chains for the mobility.

Chang et al [2] anticipated a fresh method based on Eigen vectors and Eigen values to recognize isomorphism among kinematic chains. Kinematic chains are initially represented by Adjacent Matrices. By evaluating the Eigen values and related Eigen vectors of Adjacent Matrices, the isomorphic chains can be recognized.

Cubillo and Wan [3] suggested an innovative method to identify isomorphic chains. With this new procedure, it is only important to compare Eigen values and numerous Eigen vectors of adjacent matrices of isomorphic kinematic chains to recognize the isomorphism among kinematic chains.

A. C. Rao and Raju[4] suggested a new method for detecting the isomorphism among chains, called the chain loop string is developed for a kinematic chain with simple joints to identify isomorphic chains. Another invariant called the link adjacency string is proposed, which is an outcome of the same method to detect inversions of a particular chain. The proposed method is also significant to know the type of freedom of a chain in case of multi degree of freedom chains.

S.C. Sarkar & Khare[5] proposed a hypothetical study for detecting isomorphism and outcome of uncertainty in 10 bar kinematic chain, by means of the theory of directed graph. In this case, flow of motion among links is estimated considering all possible paths for motion conduction as a substitute of only the closest path.

Srinath and Rao [6] offered a technique based on correlation conception for detection of isomorphic chains with their inversions. Correlation among two links illustrates the number of links typically connected to them. The method uses the adjacency matrix to obtain the correlation matrix.

Rao et al [7] introduced the concept of Hamming distance to the structural synthesis of kinematic chains. The Hamming code of the link is the row of the adjacency matrix related to a link. The number of places where the Hamming codes for the two links differ is known as the Hamming distance between two links. The Hamming matrix is the matrix of the same size as the adjacency matrix when the i j^{th} entry corresponds to the Hamming distance between the links i and j. The sum of the parallel row of the Hamming matrix is called the link Hamming number. The sum of the complete link Hamming numbers is called the chain Hamming number. The link Hamming string is defined as the concatenation of the link Hamming number with the frequency of the rate of all the integers from n down to zero. The concatenation of the chain

Hamming value and the entire link Hamming strings arranged in decreasing order is called the chain Hamming string.

Mruthyunjaya and Balasubramanian [8] anticipated a vertex-vertex degree matrix whose ijth entry is the sum of degrees of links i and j if i and j are adjacent and is equal to 1 otherwise. The characteristic polynomial of this matrix effectively recognized all the 10-link kinematic chains with up to 3 degrees of freedom.

According to Ambekar and Agrawal [9] two canonical forms of any adjacency are possible – one yielding a maximum code – Max Code and the other yield a minimum code – Min Code. The Min Code is used as a canonical number to identify the kinematic chains. The method is used in Watt's and Stephenson's chain.

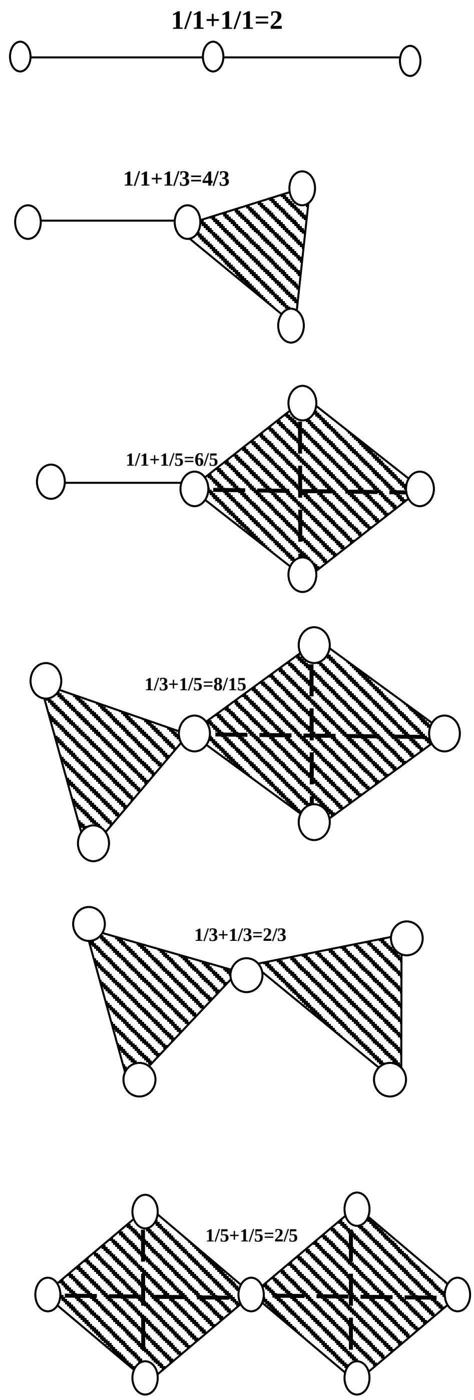
A new method to classify the discrete mechanisms from a given kinematic chain was projected by Ali et al **[**10**].** The Kinematic chains are characterized in the form of Joint-Joint matrices. Two structural invariants, which are the addition of the absolute quality polynomial coefficients and maximum absolute value of the characteristic polynomial coefficients, are used as the complex identification number of a kinematic chain and mechanisms. The method is capable of detecting isomorphism in all types of kinematic chains.

Wen-Miin Hwang et al [11] proposed a straight forward approach for the computer-aided structural arrangement of planar kinematic chains with simple joints, which consists of efficient generation of possible slender link adjacency matrices, recognition of degenerate chains and detection of isomorphism among kinematic chains. Based on the planned algorithm, a computer program is developed such that the catalogues of planar kinematic chains with the known number of links and degrees of freedom can be synthesized.

Huafeng Ding et al [12] proposed some fresh concepts, such as, the maximum perimeter degree sequence, the perimeter topological graph as well as the perimeter loop, in addition to the method for obtaining the perimeter loop is also involved. Then, based on the perimeter topological graph and some policy for relabeling its vertices canonically, a one-to-one explanatory technique, the canonical adjacency matrix set of kinematic chains, is projected. One more characteristic of the method is that in the canonical adjacency matrix set the element number is compacted, usually to only one. Subsequently, an efficient method to identify isomorphic chains is specified.

II. TERMINOLOGY

1) NODE VALUE:-A node value is defined as a numeric value assigned to the particular node of a link. Node value is assigned as the ratio of order of link to number of parameters and the joint value is sum of node values connected at the joint.



2) LEAST DISTANCE:-It is defined as shortest distance (path) in terms of joint value between the two links.

3) LEAST DISTANCE MATRIX (LDM):-It is a square matrix. It is formed on the basis of sum of least distance between node values of the two links.

4) RELATIVE LEAST DISPOSITION MATRIX (RLDM):- It refers to sum of absolute differences of the elements of particular rows of LDM.

5) RELATIVE LEAST DISPOSITION STRING (RLDS):- It refers to elements of a particular row (or column) of RLDM taken in ascending order.

6) RELATIVE LEAST DISPOSITION LINK VALUE (RLDLV):- The sum of the elements of each row of RLDM will give a particular value. This value is termed as RLDLV for that particular link.

7) RELATIVE LEAST DISPOSITION CHAIN STRING (RLDCS):-The sum of all the RLDLV's of a particular chain along with all the elements of the matrix will give the RLDCS for that particular chain.

III. METHODOLOGY

The method involves a novel approach for detection of isomorphism and inversions. The method is explained using Watt's and Stephenson's chain.

A least distance matrix (LDM) is formed using the values on the basis of the parameter. Watt chain is shown in fig. 1.1. Link A & D are ternary link and link B, C, D & E are binary link. Each node of binary link is assigned a joint value "1"and for ternary link joint value "1/3" is assigned. Also quaternary link and pent nary link had the node values 1/5 and 1/7 respectively.

For simplicity of computation a non fractional value is assigned to each joint value on the basis of various links connectivity.

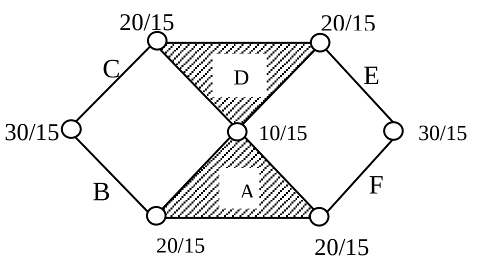
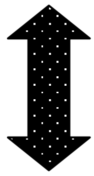


Fig 1.1 (WATT chain)

Thus at a node joining a binary link and ternary link, a joint value will be assigned as 4/3(20/15) (1/3, joint value of ternary link and 1, joint value of binary link) as shown in figure.

	A	B	C	D	E	F
A	0	20	30	10	30	20
B	20	0	30	30	50	40
C	30	30	0	20	40	50
D	10	30	20	0	20	30
E	30	50	40	20	0	30
F	20	40	50	30	30	0



	A	B	C	D	E	F
A	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}
B	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}
C	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}
D	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}
E	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}
F	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	d_{66}

As shown in the matrix, for Watt chain, ternary link A is connected to binary link B and the node value is assigned "20". For connectivity between link A and link C a shortest distance is considered i.e., 20+10=30. Similarly for connectivity between link A and link D the joint value will be "10" and so on. A 6*6 matrix will be formed for a 6 link watt chain.

For interconnectivity between the links a new matrix i.e., RELATIVE LEAST DISPOSITION MATRIX (RLDM) is introduced.

The formula used for RLDM is

$$\sum_{i=j=0}^{k=n} d_{ij} - d_{kj}$$

Where n is 6 for six link kinematic chain 8 for eight link one degree of freedom kinematic chain. The value of i and j varies from 1 to 6 for watt & Stephenson chain.

The RELATIVE LEAST DISPOSITION MATRIX (RLDM) for Watt chain is given below:-

	A	B	C	D	E	F
A	0	100	120	60	120	100
B	100	0	100	120	140	120
C	120	100	0	100	120	140
D	60	120	100	0	100	120
E	120	140	120	100	0	100
F	100	120	140	120	100	0

As shown in the matrix, the LDM is transformed into RLDM using the formula.

The element in first row, first column i.e., AA (0) is obtained by

$\mathbf{d_{AA}}=(|(d_{11}-d_{11})|+|(d_{12}-d_{12})|+|(d_{13}-d_{13})|+|(d_{14}-d_{14})|+|(d_{15}-d_{15})|+|(d_{16}-d_{16})|$

"|" (modulus) is used for absolute value of the difference of the respective element. Similarly for element in first row, second column i.e., AB (100) is obtained by

$\mathbf{d_{AB}}=(|(d_{21}-d_{11})|+|(d_{22}-d_{12})|+|(d_{23}-d_{13})|+|(d_{24}-d_{14})+|(d_{25}-d_{15})|+|(d_{26}-d_{16}))|$

For all other elements in first column, the expanded formula is given by:-

$\mathbf{d_{AC}}=|(d_{31}-d_{11})|+|(d_{32}-d_{12})|+(d_{33}-d_{13})|+|(d_{34}-d_{14})|+|(d_{35}-d_{15})|+|(d_{36}-d_{16})|)=120$

$\mathbf{d_{AD}}=(|(d_{41}-d_{11})|+|(d_{42}-d_{12})|+|(d_{43}-d_{13})|+|(d_{44}-d_{14})|+|(d_{45}-d_{15})|+|(d_{46}-d_{16})|)=60$

$\mathbf{d_{AE}}=(|(d_{51}-d_{11})|+|(d_{52}-d_{12})|+|(d_{53}-d_{13})|+|(d_{54}-d_{14})|+|(d_{55}-d_{15})|+|(d_{56}-d_{16})|)=120$

$\mathbf{d_{AF}}=(|(d_{61}-d_{11})|+|(d_{62}-d_{12})|+|(d_{63}-d_{13})|+|(d_{64}-d_{14})|+|(d_{65}-d_{15})|+|(d_{66}-d_{16})|)=100$

RLDS for link A, B, C, D, E & F are: - [0, 60, 2(100), 2(120)] , [0, 2(100), 2(120), 140] , [0, 2(100), 2(120), 140] ,[0, 60, 2(100), 2(120)] , [0, 2(100), 2(120), 140] & [0, 2(100), 2(120), 140] respectively.

From above values of strings it is clear that link A & D will have the same inversions and link B, C, E & F will have the same inversions.

RLDLV for link A, B, C, D, E & F are: - 500, 580, 580, 500, 580, and 580 respectively & RLDCS for Watt chain are 3320, 2 [60, 6(100), 6(120), 2(140)].

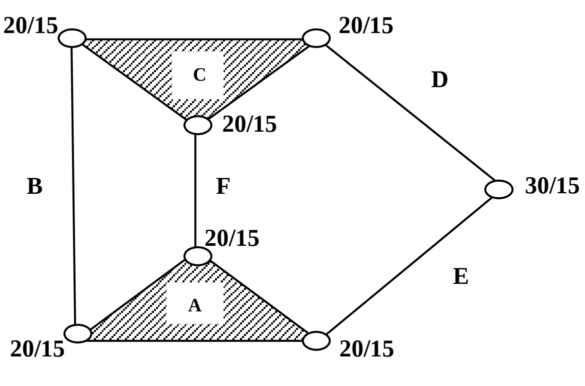


Fig 1.2 (STEPHENSON chain)

A least distance matrix (LDM) & The RELATIVE LEAST DISPOSITION MATRIX (RLDM) for Stephenson chain is given below:-

	A	B	C	D	E	F
A	0	20	40	50	20	20
B	20	0	20	40	40	40
C	40	20	0	20	50	20
D	50	40	20	0	30	40
E	20	40	50	30	0	40
F	20	40	20	40	40	0

	A	B	C	D	E	F
A	0	110	140	170	110	110
B	110	0	110	120	120	80
C	140	110	0	110	170	110
D	170	120	110	0	120	120
E	110	120	170	120	0	120
F	110	80	110	120	120	0

RLDS for link A, B, C , D , E & F are: - [0, 3(110), 140, 170] , [0, 80, 2(110), 2(120)] , [0, 3(110), 140, 170], [0, 110, 3(120), 170] , [0, 110, 3(120), 170], [0, 80, 2(110), 2(120)] respectively.

From above values of strings it is clear that link A & C will have the same inversion, link B & F will have the same inversions, link D & E will have the same inversions.

RLDLV for link A, B, C, D, E & F are: - 640, 540, 640, 640, 640 and 540 respectively. Also the sum of all the RLDLV's of a particular chain will give the RELATIVE LEAST DISPOSITION CHAIN STRING (RLDCS) for that particular chain. Hence RLDCS for Stephenson chain is 3640, 2[80, 6(110), 5(120), 140, 2(170)].

IV. ISOMORPHISM

A comparative study of Watt & Stephenson chain shows that they are non isomorphic since the RLDCS for both of the chains are different 3320, 2 [60, 6(100), 6(120), 2(140)] for Watt chain and 3640, 2[80, 6(110), 5(120), 140, 2(170)] for Stephenson chain.

RELATIVE LEAST DISPOSITION CHAIN STRING (RLDCS) is a ultimate test for isomorphism among chains. When RLDCS is different, chains will be non isomorphic. It is different for Watt & Stephenson's chain.

V. INVERSIONS

In order to know the dissimilar inversions of a chain, it is necessary to put side by side the RELATIVE LEAST DISPOSITION STRING (RLDS) of all the links. If the strings are the same, the corresponding inversions are identical, otherwise different.

A comparative study of Watt & Stephenson chain shows that Watt chain has 2 inversions and Stephenson chain has 3 inversions.

Counter examples of 10 link 1 DOF reported in [8] are tested by this method is found non isomorphic.

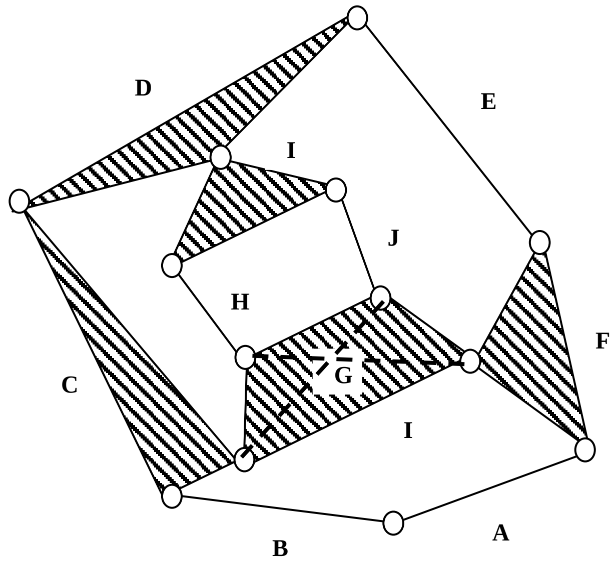


Fig-1.3

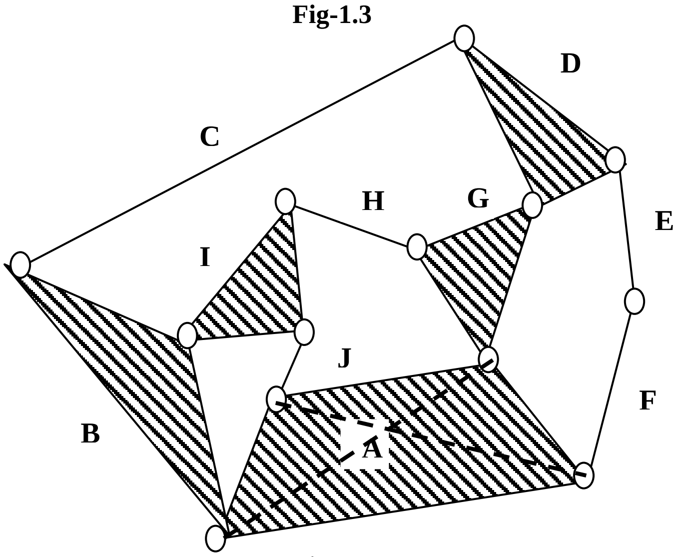


Fig-1.4

Above are two kinematic chains containing ten bars, 13 joints, and single degree-of-freedom (Fig 1.3 & fig 1.4)

RELATIVE LEAST DISPOSITION CHAIN STRING (RLDCS) for chain shown in **Fig-1.3** is 15672, 2[72, 80, 84, 102, 2(116), 2(148), 150, 2(152), 2(154), 156, 2(158), 162, 2(164), 3(168), 170, 4(174), 178, 182, 2(186), 3(190), 200, 204, 212, 214, 2(226), 228, 236, 252, 270, 306].

Inversions for the chain are (A) (B) (C) (D) (E) (F) (G) (H, J) (I).

RLDCS (RELATIVE LEAST DISPOSITION CHAIN STRING) for chain shown in **Fig-1.4** is 15744, 2[74, 88, 100, 114, 120, 126, 130, 134, 148, 152, 3(154), 2(156), 2(158), 160, 162, 2(166), 170, 174, 2(176), 178, 180, 182, 2(184), 4(188), 2(190), 210, 212, 228, 230, 234, 240, 270, 272, 310].

Inversions for the chain are (A) (B) (C) (D) (E) (F) (G) (H) (I) (J).

The method reports that chains in the fig 1.3 & 1.4 are non-isomorphic as the RLDCS (RELATIVE LEAST DISPOSITION CHAIN STRING) are different for both the kinematic chain. Also Chain in the figure 1.3 & 1.4 has 9 & 10 inversions respectively. (On the basis of RELATIVE LEAST DISPOSITION STRING (RLDS).

Note that, according to T.S. Mruthyunjaya, H.R. Balasubramaniam 1987[8], these chains are isomorphic but after that many researchers proved that these chains are non-isomorphic.

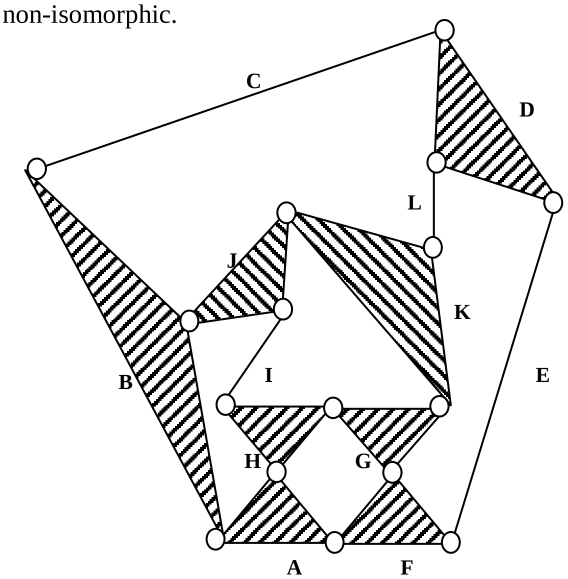


Fig 1.5

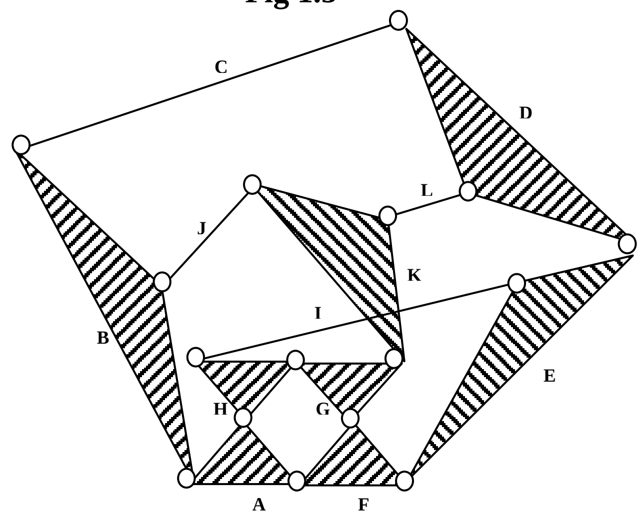


Fig 1.6

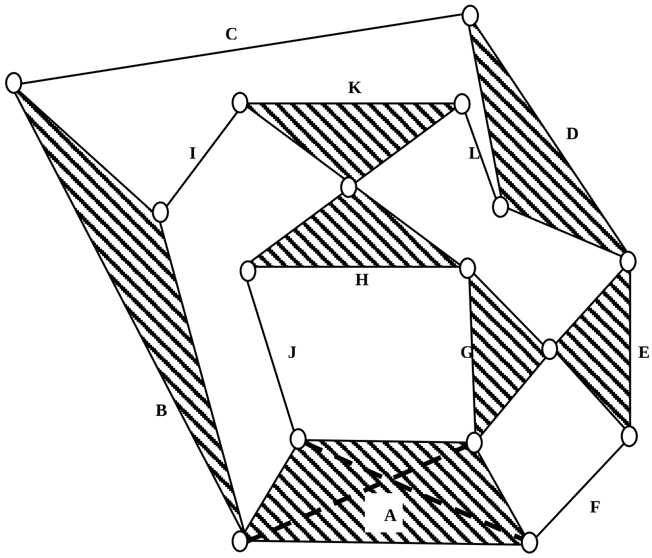


Fig 1.7

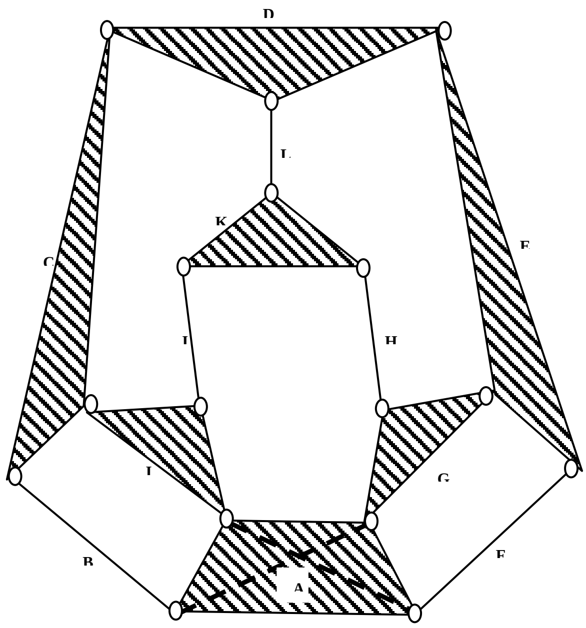


Fig 1.8

Fig 1.5, 1.6, 1.7, 1.8 are four kinematic chains with 12 bars, single degree-of-freedom.

RLDCS for chain shown in **Fig-1.5** is 28320, 2[3(110), 4(120), 6(130), 2(140), 3(160), 6(170), 2(180), 190, 7(200), 2(210), 4(230), 3(240), 7(250), 4(260), 2(280), 2(290), 330, 2(340), 350, 3(370), 380].
Inversions for the chain are (A, G) (B, K) (C, L) (D) (E) (F) (H) (I) (J).

RLDCS for chain shown in **Fig-1.6** is 27280, 2[90, 2(110), 7(120), 2(150), 3(160), 170, 180, 5(190), 6(200), 6(210), 5(220), 6(230), 6(240), 6(250), 2(260), 270, 6(290)].
Inversions for the chain are (A, G) (B, K) (C, L) (D) (E) (F) (H) (I) (J) .

RLDCS for chain shown in **Fig-1.7** is 26144, 2[2(96), 2(112), 2(120), 3(136), 4(144), 2(168), 2(172), 4(176), 9(184), 6(200), 6(216), 3(224), 2(228), 4(232), 2(236), 2(240), 2(248), 2(252), 5(256), 2(304)].
Inversions for the chain are (A) (B) (C, I) (D, K) (E, H) (F, J) (G) (L)

RLDCS for chain shown in **Fig-1.8** is 27408, 2[3(96), 2(108), 104, 2(112), 2(132), 3(136), 3(144), 2(156), 4(168), 172, 2(176), 180, 2(184), 2(188), 2(192), 2(196), 200, 2(204), 2(220), 2(224), 2(228), 2(232), 2(236), 2(244), 2(252), 2(256), 4(260), 308, 332, 2(344), 4(356), 376].
Inversions for the chain are (A) (B, F) (C, E) (D) (H, J) (G, I) (K) (L).

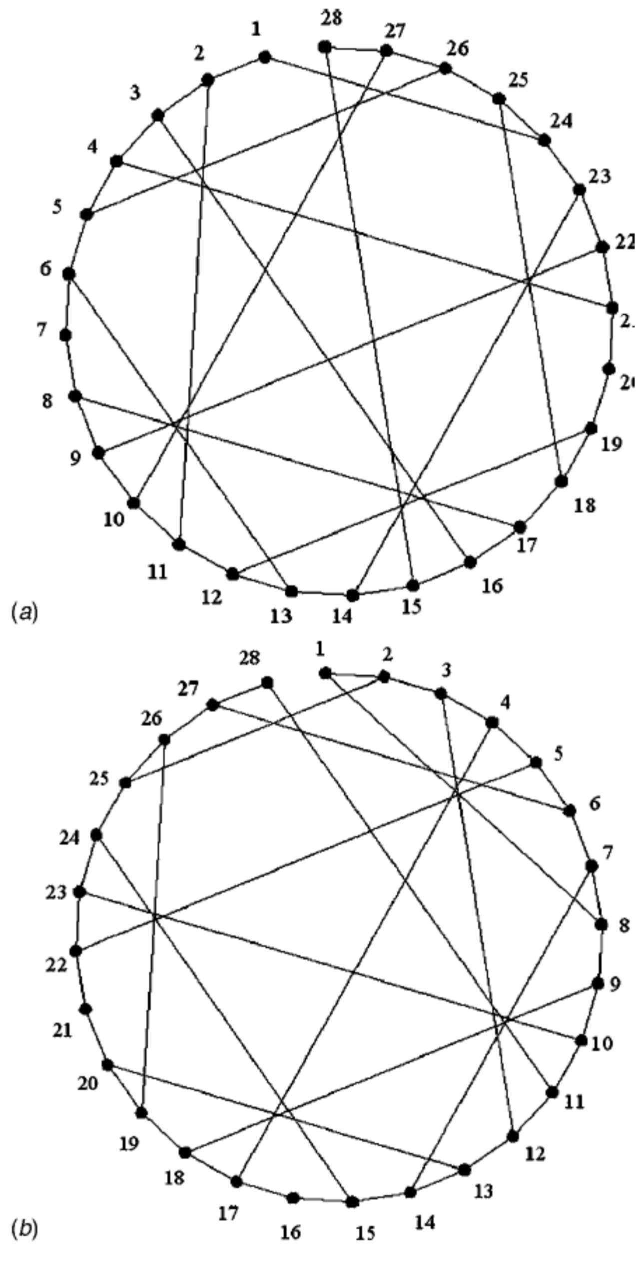


Fig 1.9

Fig 1.9 shows two graphs of 28 links kinematic chain [12]. By using proposed method on these graphs they are found isomorphic.

RLDCS for both the graphs shown in Fig 1.9 is 298240, 2 [40(240), 8(520), 44(320), 16(300), 88(360), 16(340), 32(380), 32(400), 48(540), 24(560), 8(280), 6(480), 16(620)

RESULT AND CONCLUSION:-

The method is victorious for all the chains of 8 links and the counter examples of 10 and 12 links 1 DOF chains. This method can be used for multi degree of freedom kinematic chains.

A trouble-free, well-organized and reliable method to identify isomorphism is developed when it is applied to counter examples of 10 and 12 links 1 DOF as shown in fig 1.3, 1.4, 1.5, 1.6, 1.7 &1.8 found valid.

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Chain no	Inversions	Links to be fixed
1	2	(1,2)(3,4,5,6,7,8)
2	3	(1,2)(3,4,7,8)(5,6)
3	5	(1),(2,3)(4)(5,7)(6,8)
4	8	(1)(2)(3)(4)(5)(6)(7)(8)
5	7	(1)(2)(3)(4)(5,6)(7)(8)
6	7	(1)(2)(3)(4)(5,6)(7)(8)
7	4	(1)(2,3)(4)(5,6,7,8)
8	2	(1,2,3,4)(5,6,7,8)
9	2	(1,2,3,4)(5,6,7,8)
10	8	(1)(2)(3)(4)(5)(6)(7)(8)
11	4	(1,4)(2,3)(5,6)(7,8)
12	4	(1,4)(2,3)(5,6)(7,8)
13	5	(1,4)(2)(3)(5,6)(7,8)
14	6	(1,4)(2)(3)(5,6)(7)(8)
15	2	(1,2,3,4)(5,6,7,8)
16	2	(1,2,3,4)(5,6,7,8)