

Dimensional Synthesis of Three-point Hitch Linkage System of Tractor – An Approach Based on Maximizing Mechanical Advantage

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Abstract—Non-dominated Sorting Genetic Algorithm (NSGA-II) was applied to identify the improved design solutions to three-point hitch linkage system of a tractor by formulating the optimal dimensional synthesis as a constrained multi-objective optimization problem. Objective functions were developed to maximize the mechanical advantage of the driving mechanism of the linkage system, minimize the deviation of mechanical advantage from its mean value throughout the movement range and obtain the transport pitch in the vicinity of 8° . Length of lift arm and lower link, spatial location of pivot point of lower link, and distance of lift rod connection point on the lower link from the pivot point of lower link were taken as the design variables. The values of all the design variables were initialized and constrained within the range of values existing in the commercially available tractors. The constraints imposed by the governing standards were applied. The algorithm was implemented in MATLAB with path generation program for the hitch points as the sub-function. Ten best design solutions were identified, of which 8 solutions were found to be superior to the existing design of hitch. The optimal dimensional synthesis of three-point hitch linkage system by NSGA-II was found to be fast, flexible, efficient and better than manual search of optimal solutions.

Keywords—three-point hitch linkage; optimal dimensional synthesis; four-bar linkage; Newton–Raphson solution; Non-dominated Sorting GA; heuristic search

I. INTRODUCTION

Three-point hitch linkage system for farm tractors is the fundamental device for agricultural works. In a vertical longitudinal plane, three-point hitch linkage system is a six-bar mechanism consisting of two four-bar linkages sharing two links [1-3] as shown in Fig. 1. The governing standards provide freedom to the designer to select any value for the spatial location of the pivot point of links and length of links, but the design has to fulfill certain requirements related to lower hitch point height, transport height, movement range, lower hitch point clearance, mast adjustment height positions, transport pitch, convergence distances and leveling adjustment. These parameters are called geometric performance parameters [1].

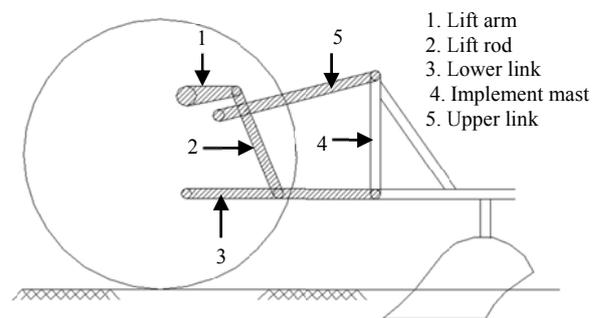


Fig. 1. Three-point hitch linkage system of tractor.

Optimal dimensional synthesis of three-point hitch linkage system is a multi-objective optimization problem. Here, design solutions are subjected to a number of constraints related to geometric performance parameters and, construction and shape of the rear part of the tractor model where hitch system is provided [1, 3, 4]. Ambike and Schmiedeler [1] reported the application of geometric programming to generate the kinematic configurations of the hitch system. Mattetti et al. [4] proposed a methodology based on interior point algorithm for the design of three-point hitch linkage system of a tractor. Kumar [3] presented a systematic manual search for the improved hitch linkage configuration of a tractor within the design space by narrowing the range of values of the design variables in each successive stages of search. In recent years, due to increasing computational speed of computers, various heuristic approaches that imitate natural phenomena are applied to dimensional synthesis problems [5]. The present work was undertaken to apply one of the heuristic approaches to dimensional synthesis of tractor three-point hitch linkage system by formulating it as a constrained multi-objective optimization problem.

II. TRACTOR THREE-POINT LINKAGE ANALYSIS

When the lower link is horizontal and implement mast is vertical as in Fig. 2, $\varphi_2 = 3\pi/2$ radians, $\varphi_3 = \pi$ radians and $\alpha = \pi/2$ radians. The values of θ , φ_1 and φ_4 can be determined using geometric relationships [3]. The value of θ is noted as $\theta_{\text{horizontal}}$. Coordinates of lower hitch point (x_5 , y_5) is determined using (1) and (2),

(i) mechanical advantage of the driving mechanism of the hitch is as high as possible, i.e., minimize (1/maximum value of m_A).

(ii) Deviation of mechanical advantage from its mean value throughout the movement range is as low as possible, i.e., minimize (standard deviation of m_A).

(iii) pitch of the mast at the highest point of movement range of hitch is as close as possible to 8° , i.e., minimize (transport pitch – 8).

Design constraints: The constraints introduced herein ensure that,

(i) the first four-bar linkage of the three-point hitch works within the full range of movement of lift arm (between $\theta_{up} = 170.63^\circ$, $\theta_{down} = 78.89^\circ$) pre-defined for the particular tractor model.

(ii) the values of design variables, L_1 , x_L , y_L and L_3/L_5 are within the specified range (mean \pm standard deviation of the existing tractors).

(iii) distance from end of power take-off to centre of lower hitch point with the lower link horizontal is within the range specified by the standards.

(iv) the values of transport height, movement range, lower hitch point clearance and highest position of mast adjustment height are above the minimum values of the corresponding parameters, and the values of lower hitch point height and lowest position of mast adjustment height are below the maximum values of the corresponding parameters specified in the standards.

(v) the vertical convergence distance is more than 0.9 times the tractor wheelbase.

(vi) the toggle positions are never encountered in the movement range of hitch.

Algorithm was developed for the determination of the values of constraint and objective functions as follows:

Having known the values of the full range of movement of lift arm, the parameters related to constraints and objective functions were determined as follows:

(i) Values of L_1 , L_3 , L_4 , L_5 , L_8 , x_A , y_A , x_L , y_L , x_U , y_U were determined from the design vector and predefined hitch parameters. Standard mast height (L_6) of 610 mm for the category-II hitch was considered.

(ii) For the position of lower hitch point at the desired value of lower hitch point height (maximum value of 200 mm specified by the standards or any other desired value) when $\theta = \theta_{down}$, length of lift rod (L_2) required was determined by geometric relationships.

(iii) Length of upper link required (L_7) and its inclination with horizontal for maintaining lower link horizontal and implement mast vertical were calculated. The value of ϕ_4 was taken equal to $3\pi/2$ – angle made by upper link with horizontal.

(iv) The value of ϕ_1 and $\theta_{horizontal}$ were determined using geometric relationships. $\phi_2 = 3\pi/2$ radians. Coordinates of lower hitch point (x_5 , y_5) were determined using equations (1) and (2).

(v) Taking $\alpha = \pi/2$ radians and $\phi_3 = \pi$ radians, coordinates of upper hitch point (x_7 , y_7) were determined using equations (5) and (6).

(vi) Value of θ was incremented by a small value and the values of ϕ_1 and ϕ_2 for the new value of θ were determined using Newton–Raphson method. The values of ϕ_1 and ϕ_2 calculated earlier were used as initial solution during iteration. Putting $\alpha = \phi_2 - \pi$, values of ϕ_3 and ϕ_4 were determined for the new value of α (or θ) in similar way as above considering the second four-bar linkage. The values of ϕ_3 and ϕ_4 calculated earlier were used as initial solution during iteration. Coordinates of lower and upper hitch points were determined for each value of θ using equations (1), (2), (5) and (6). The procedure was repeated by incrementing the value of θ till it became equal to θ_{up} . Coordinates of upper and lower hitch points at each value of θ were stored.

(vii) Step (vi) was repeated by decrementing the value of θ by a small value from the position of $\theta = \theta_{horizontal}$ till the value of θ became equal to θ_{down} . Coordinates of lower and upper hitch points were determined at each value of θ and stored.

(viii) Paths of motion of lower and upper hitch points were plotted as shown in Fig. 4.

Constraints [7] were applied as follows:

(i) Lift arm moved in the range of values between $\theta_{up} = 170.63^\circ$, $\theta_{down} = 78.89^\circ$.

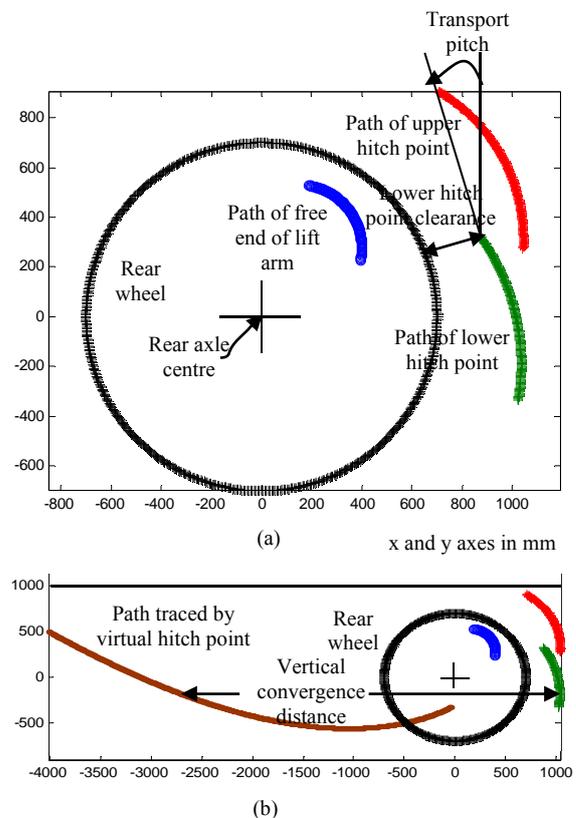


Fig. 4. (Colour online) Path of motion of lower and upper hitch points at the test settings of the selected tractor in vertical longitudinal plane. (a) Various geometric performance parameters of hitch system. (b) Path of virtual hitch point.

(ii) Range of values of design variables : $L_1 = 145$ to 347 mm, $x_L = 74$ to 246 mm, $y_L = 169$ to 275 mm, $L_3/L_5 = 0.47$ to 0.61 . The range was decided based on the mean \pm standard deviation values of the corresponding dimensions for category-II hitch as reported by Kumar [3] for the existing hitch systems of the commercially available tractors.

(iii) Length of lower link (L_5) = Horizontal distance from rear axle centre to end of power take-off (taken from test report) $- x_L + 550$ mm $+ 0$ to 75 mm.

(iv) (Lower hitch point height $- 200$ mm) ≤ 0
 (950 mm $-$ transport height) ≤ 0
 (650 mm $-$ movement range) ≤ 0
 (100 mm $-$ lower hitch point clearance) ≤ 0
 Lowest mast adjustment height = 0 or (Lowest mast adjustment height $- 200$ mm) ≤ 0
 Highest mast adjustment height = 0 or (610 mm $-$ Highest mast adjustment height) ≤ 0

(v) $\{(0.9 \times \text{wheel base of tractor}) - \text{vertical convergence distance}\} \leq 0$

(vi) The value of $(\theta - \pi - \phi_1)$ while incrementing the value of θ , and $(\theta - \phi_1)$ while decrementing the value of θ was checked in each iteration for overlapping collinear crank-rocker toggle and extended collinear crank-rocker toggle, respectively. If this value nearly approached zero, iteration was stopped and the values of θ (θ_{up} or θ_{down}) and y_5 obtained in the previous step were stored.

Values of the objective functions were calculated as follows:

Included angles between links L_1 and L_2 (ν) and between L_2 and L_3 (μ) were calculated at each value of θ using geometrical relationships. Standard deviation of m_A was determined using equation (11). Transport pitch = angle made by the line drawn between (x_5, y_5) and (x_7, y_7) with vertical when $\theta = \theta_{up}$. Value of 3rd objective function could be determined using transport pitch.

Values of objective functions: (1/maximum value of m_A), standard deviation of m_A , (transport pitch $- 8$).

GA was implemented as follows:

Dimensional synthesis of three-point hitch linkage system is a multi-objective constrained optimization problem. Hence, a single design solution will not express the best performance in all of the objectives. Therefore, a set of trade-off solutions known as Pareto or non-dominated set has to be determined. Deb et al. [8] has proposed Fast Non-dominated Sorting Genetic Algorithm (NSGA-II) to determine a set of trade-off solutions for the multi-objective optimization problems. It was applied in the present work:

(i) Population of design vectors (chromosomes) was initialized with random values and these values were within the specified range. Each chromosome consisted of design variables.

(ii) Algorithm described above was used to determine the full range of movement of lift arm and generate the path of motion of hitch points. The values of constraints were

calculated. If any chromosome failed to satisfy all the constraints, a new chromosome was randomly generated till it satisfied all the constraints. Values of objective functions for each chromosome were calculated.

(iii) The chromosomes in the initial population were sorted based on non-domination [9]. The chromosomes of the first front were assigned rank 1, the second front chromosomes were assigned rank 2 and so on. The crowding in each front was calculated.

(iv) Evolution process was started on the population. Parents were selected for the reproduction to generate the offsprings. Binary tournament selection was employed with selection based on rank and crowding distance. A lower rank and higher crowding distance was the selection criteria.

(v) Real coded simulated binary crossover and polynomial mutation were performed on the parent chromosomes to generate offspring chromosomes. Values of the constraints and objective functions were calculated for each offspring chromosome.

(vi) Intermediate population was developed by combining the chromosomes of the current population and the offspring. Non-dominated sorting of intermediate population was carried out and current population was filled with best fit chromosomes based on their rank and crowding distance.

(vii) Steps (iv), (v) and (vi) were repeated till same set of chromosomes remained in the current population continuously for 20 generations.

Selection, crossover, mutation and replacement of worst chromosomes with best fit ones constitute one generation. The best fit chromosomes of the last population were stored as the set of best design solutions for the three-point hitch linkage system of tractor.

The entire procedure was executed in MATLAB with the program developed for the generation of path of hitch points [3] as the sub-function. The following values were used for various GA parameters:

Population size : 10
 Size of mating pool for the selection : 5
 Cross over probability : 0.9
 Distribution index for crossover : 20
 Mutation probability : 0.1
 Distribution index for mutation : 20

IV. RESULTS AND DISCUSSION

In the present work, design vector has five design variables, L_1 , x_L , y_L , L_5 and L_3/L_5 . The values of design variables were initialised and constrained during the search process within the range of values already exist in commercially available tractors. Other variables can also be taken in the design solution along with these variables depending upon the requirements. The algorithm developed for the linkage analysis restricts the length of lift rod to the length required for the lower hitch point to reach either the maximum value of lower hitch point height specified in the standards or the desired value of lower hitch point height entered by the designer when $\theta =$

θ_{down} . Lower hitch point is free to lift to a height till θ becomes equal to θ_{up} . Algorithm could be modified in other ways as well by fixing the value of any other geometric performance parameters. Constraints applied to the optimization problem ensure that the design solutions meet the requirements specified in the standards. Their values can be changed if desired for the better solutions. Additional constraints related to practically feasible zones where pivot points of links can be provided can also be declared.

The non-dominated set of design solutions along with their objective function values are presented in Table 1. The geometric performance parameters of the existing linkage system of the hitch and that of the design solutions are presented in Table 2 for the purpose of comparison in the improvement of the design. All the identified design solutions had the transport height lower than that of the existing design, but it is higher than that specified by the standards (950 mm). The movement range for the new design solutions was higher than that specified by the standards (650 mm) as well as the existing design. The existing design had the transport pitch of 15.87°. During the optimization process, one of the objectives is to search for the solutions with transport pitch of 8°. Due to this, transport pitch was lower than the existing design and the highest position of mast adjustment height was higher than the existing design for all the new design solutions. Further, vertical convergence distance was higher than that recommended by the standards for all the new design solutions. Hence, all the new design solutions fulfill the requirements of the kinematic design and can be considered for the improvement of the existing hitch system.

The magnitude of force transmitted through the linkage system should be high and uniform throughout the movement range for the effective operation of the implements [4]. Except for the 5th and 6th design solutions, the minimum and maximum values of mechanical advantage were higher for all the identified design

solutions as compared to the existing design. But, the variation in mechanical advantage throughout the movement range was lowest for the existing design of hitch. Mattetti et al. [4] reported that the increase in length of lift rod (L_2) increases the load capacity and movement range, while link length L_3 has the strong influence on the uniformity of load capacity in the movement range. Length of lift rod (L_2) required for the identified design solutions were more than the existing design of hitch. Further, among the design solutions, ones with link length L_3 close to or less than the existing design of hitch (5th and 6th design solutions) had the higher uniformity in mechanical advantage. The mechanical advantage at the lower hitch point height ($\theta = \theta_{down}$) was lower for all the design solutions as compared to the existing design. However, mechanical advantage at the lower hitch point height was comparatively higher than the minimum value of mechanical advantage observed throughout the movement range for each design solution. Thus, all the new design solutions, except 5th and 6th solutions, were found to be superior to the existing design of hitch.

TABLE I. NON-DOMINATED SET OF DESIGN SOLUTIONS ALONG WITH VALUES OF OBJECTIVE FUNCTIONS

Design variables					Objective functions		
L_1 , mm	x_{L_1} , mm	y_{L_1} , mm	L_5 , mm	L_3/L_5	1	2	3
322	75	275	984	0.61	1.149	0.082	2.163
322	91	268	975	0.61	1.170	0.079	2.294
343	135	275	963	0.61	1.186	0.075	2.061
314	179	262	929	0.61	1.261	0.066	0.270
276	204	174	867	0.61	1.400	0.059	3.228
245	205	226	905	0.51	1.491	0.061	0.364
347	78	275	957	0.61	1.093	0.093	4.735
339	75	273	962	0.61	1.105	0.090	4.141
337	144	275	945	0.61	1.187	0.075	1.621
300	184	275	925	0.61	1.266	0.064	-1.576

Objective functions: 1. $(1/m_A)$ at $\theta = \theta_{up}$, 2. Standard deviation of m_A , 3. Transport pitch - 8

TABLE II. NON-DOMINATED SET OF DESIGN SOLUTIONS ALONG WITH VALUES OF OBJECTIVE FUNCTIONS

Design variables	Existing hitch	Design solutions									
		1	2	3	4	5	6	7	8	9	10
L_1 , mm	255	322	322	343	314	276	245	347	339	337	300
x_{L_1} , mm	155	75	91	135	179	204	205	78	75	144	184
y_{L_1} , mm	189	275	268	275	262	174	226	275	273	275	275
L_5 , mm	880	984	975	963	929	867	905	957	962	945	925
L_3/L_5	0.59	0.61	0.61	0.61	0.61	0.61	0.51	0.61	0.61	0.61	0.61
L_2 , mm	572	657	657	661	678	657	647	642	644	663	694
L_3 , mm	523	600	595	587	563	529	460	584	587	576	564
L_7 , mm	699	715	722	753	764	737	769	691	693	744	764
LHH, mm	367	200	200	200	200	200	200	200	200	200	200
TH, mm	1020	968	974	1017	984	960	952	1000	989	1005	952
MR, mm	653	768	774	817	784	759	752	800	789	805	752
LHCl, mm	230	235	242	249	273	289	306	195	203	241	277
HMAH, mm	756	821	820	864	867	770	824	810	807	863	896
TP, °	15.87	10.16	10.29	10.06	8.27	11.23	8.36	12.74	12.14	9.62	6.42
VCD, mm	3725	16131	12936	17012	11636	3460	6140	15589	14556	16809	17261
Min. m_A and θ (°) at which it occurs	0.552, 108.01	0.579, 96.80	0.575, 96.82	0.577, 96.78	0.558, 98.19	0.509, 90.00	0.454, 95.35	0.586, 93.15	0.584, 93.89	0.575, 96.59	0.559, 100.50
Max. m_A and θ (°) at which it occurs	0.737, 170.63	0.870, 170.63	0.855, 170.63	0.843, 170.63	0.793, 170.63	0.714, 170.63	0.671, 170.63	0.915, 170.63	0.905, 170.63	0.843, 170.63	0.790, 170.63
m_A at $\theta = \theta_{down}$	0.609	0.597	0.594	0.594	0.578	0.514	0.465	0.597	0.596	0.592	0.586
S.D. of m_A	0.049	0.082	0.079	0.075	0.066	0.059	0.061	0.093	0.090	0.075	0.064

LHH = Lower hitch point height, TH = Transport height, MR = Movement range, LHCl = Lower hitch point clearance, HMAH = Highest position of mast adjustment height, TP = Transport pitch, VCD = Vertical convergence distance, m_A = mechanical advantage, S.D. = Standard deviation

The search of optimum design solutions by one of the nature inspired techniques of optimization, NSGA-II is successful in identifying 8 best design solutions. The designer can select any one of the 8 best solutions for the improvement in the design of the existing three-point hitch system of the selected tractor considering other practical aspects in the tractor.

The procedure presented here is fast, flexible and very efficient. It can be used for the design of the three-point hitch linkage system of any new tractor by fixing the spatial locations of pivot point of lift arm and upper link or any other dimensions as felt appropriate during the design process. Further, objective functions can be modified as desired, but it should involve both kinematic design and force transmission requirements.

V. CONCLUSIONS

The present work successfully demonstrates the application NSGA-II to the identification of the improved design solutions to the three-point hitch linkage system of a tractor. Three-objective functions were developed involving quality of force transmission and kinematic design considerations. The values of various design variables initialized and constrained within the range already exist in the commercially available tractors. The constraints imposed by the standards were applied. Ten best design solutions were identified, of which 8 design solutions were found to be superior to the existing design of hitch. The designer can select any one of the superior

solutions considering practical aspects in the tractor. The procedure was found to be fast, flexible and efficient for the design of three-point hitch linkage system of tractors.

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