Link Invariant Functions and Detection of Isomorphism and Inversions of Kinematic Chains

Jagpal Singh Bal[1], P. B. Deshmukh[2]
Department of Mechanical Engineering
FET, Shri Shankaracharya Group of Institutions
Bhilai (CG), India
[1]jagpalsinghbal@yahoo.com, [2]deshmukhpb@yahoo.com

A. Jagadeesh[3]
Department of Mechanical Engineering
GDR College of Engineering and Technology
Bhilai (CG), India
[3]jagadeesh.anne@gmail.com

***Abstract*—Detection of isomorphism and inversions are two closely associated problems encountered during structural synthesis and analysis of kinematic chains. Most of the solutions presented in the literature are based on adjacency matrices or their modification. Studies incorporating other aspects such as distance matrix and loops are limited. Present work defines link invariant functions based on distance matrix and loops of a kinematic chain. The functions generate a set of structural invariants capable of detecting distinct links of a kinematic chain and isomorphism between kinematic chains simultaneously. Proposed heuristic takes into account all aspects of kinematic chain at once, satisfying necessary and sufficient conditions. The method is simple, reliable and can easily be implemented on a computer. It has been tested successfully on known cases of kinematic chains up to 10 links and having 1, 2 and 3 degrees of freedom. An example of chains with higher number of links is also presented to demonstrate the effectiveness of the method.**

Keywords—Link invariant functions; isomorphism; inversions; kinematic chains; mechanisms

I. INTRODUCTION

Kinematic chains are building blocks for synthesis of mechanisms. Identification of isomorphism among kinematic chains is an age old problem taunting researchers over a period of 50 years since its introduction in 1960s. Unidentified isomorphism results in duplication of designers work; therefore, it is an essential step during the synthesis of mechanisms. Researchers [1] have tried to develop methods to enumerate kinematic chains without testing isomorphism. Population of chains increases with higher number of links and detection of isomorphism by visual inspection becomes impractical. Several attempts have been made to develop reliable and computationally efficient algorithms for automated test of isomorphism but each has its own shortcomings. In order to achieve greater efficiency in computer-aided mechanism design, search for simple, reliable and effective methods is of great significance.

Uicker and Raicu [2] proposed characteristic polynomial of adjacency matrix as a numerical measure of kinematic chain structure and conjectured that it is only a necessary condition, however the converse may not be true. Mruthyunjaya [3], during synthesis of kinematic chains detected non-isomorphic chains having identical characteristic polynomials. Spurred by this discovery, characteristic polynomial of vertex-vertex degree matrix was proposed by Mruthyunjaya and Balasubramanian [4]. Discovery of counter examples by Hawang and Hawang [5] led to the development of new indices. Characteristic polynomial of distance matrix was proposed by Dubey and Rao [6]. Ambekar and Agrawal suggested MAX code [7] and min code [8] as a method of identification of isomorphism. Distance based indices were proposed by Yadav, Pratap and Agrawal [9], which has found application to limited cases of kinematic chains. Rao and his co-workers utilized a concept from communication theory and several indices based on Hamming number were proposed by them [10, 11]. Chu and Cao [12] proposed link's adjacent table (ACT) and complete adjacent table (CACT) obtained by successive replacement of link labels, level wise, in a systematic manner so as to produce a unique string as a method for detection of isomorphism. However, computer implementation of the method is not clearly defined and the structure of the table is complex to grasp. Rao and Raju [13] proposed a method based on loop formations which includes all aspects of kinematic chains and can be used for detection of isomorphism, inversions and type of freedom. Ding and Huang proposed methods based on canonical perimeter topological graph [14] and characteristic adjacency matrix [15]. Application of Fuzzy Logic [16] and Genetic Algorithm [17] were introduced by Rao for detection of isomorphism and structural analysis of kinematic chains. Bedi and Sanyal [18] proposed modified joint connectivity approach as an index for testing isomorphism among kinematic chains. Link-joint connectivity table (LJCT) was proposed by Sanyal [19] for detection of distinct inversions of kinematic chains. Approaches based on Artificial Neural Network (ANN) have been reported by Kong, Li and Zhang [20] for detection of isomorphism.

In the light of literature reviewed and in view of the fact that most of the methods have included limited properties of kinematic chains, an attempt is made in the present work to incorporate all aspects of kinematic chains for detection of isomorphism and inversions. Graph theory has extensively been used in the study of kinematic chains since its introduction to kinematic structural studies by Dobrjanskyj and Freudenstein [21]. Kinematic chains are represented by graphs and two kinematic chains are said to be structurally equivalent if their graphs are isomorphic to each other. Therefore, kinematic chain isomorphism problem is reduced to finding the isomorphism between their graphs.

II. TERMINOLOGY AND DEFINITIONS

Graph theory [22, 23] has played an important role in the solution of many problems of mechanism synthesis and analysis. A number of terms relevant to the paper are presented in the paragraphs to follow.

A. Graph and Graph Properties

A *graph* consists of a set of *vertices* (or nodes or points) together with a set of *edges* (or lines). The set of vertices is connected by the set of edges satisfying the relation $E \subseteq [V]^2$. The graph can be denoted by the symbol *G*, the vertex by set *V(G)*, and the edge by set *E(G)*. The usual way to picture a graph is by drawing dot or circle for each vertex and joining two of these dots or circles by a line if the corresponding two vertices form an edge. Edges and vertices in a graph should be labelled or coloured, otherwise they are indistinguishable.

1) Degree: The degree of a vertex is defined as the number of edges incident with that vertex. A vertex of zero degree is called an isolated vertex. Vertex of degree two is called a binary vertex, a vertex of degree three a ternary vertex, and so on. For the graph shown in *Fig. 1*, the degree of vertex 2 is three, the degree of vertex 10 is one, and vertex 11 is an isolated vertex.

2) Walks and circuits (or cycles): A sequence of alternating vertices and edges, beginning and ending with a vertex, is called a walk. A walk is called a trail if all the edges are distinct and a path if all the vertices and, therefore the edges are distinct. In a path, no edge may be traversed more than once. The length of a path or distance is defined as the number of edges between the beginning and ending vertices. If each vertex appears once, except that the beginning and ending vertices are the same, the path forms a circuit or cycle. For the graph shown in *Fig. 1*, the sequence (2, e_{23}, 3, e_{34}, 4, e_{45}, 5) is a path, whereas the sequence (2, e_{23}, 3, e_{34}, 4, e_{45}, 5, e_{52}, 2) is a circuit. The circuits are referred to as loops in the field of mechanisms.

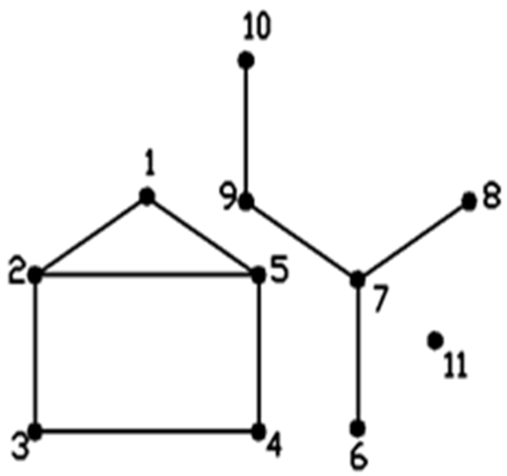


Fig. 1. A (11,10) graph

3) Graph representation of kinematic chains and mechanisms: In a graph representation of kinematic chain, links are represented by vertices and joints by edges. Four-bar chain and its graph representation is shown in *Fig. 2*.

B. Graph Isomorphism

In geometry two figures are thought of as equivalent (and called congruent) if they have identical behaviour in terms of geometric properties. Likewise, two graphs are thought of as equivalent (and called isomorphic) if they have identical behaviour in terms of graph-theoretic properties. More precisely, Two graphs G and G' are said to be isomorphic (to each other) if there is one-to-one correspondence between their vertices and between their edges such that the incidence relationship is preserved [23]. It follows that two isomorphic graphs must have the same number of vertices and the same number of edges, and the degrees of the corresponding vertices must be equal to one another.

C. Graph Invariant and Graph Certificate

A graph invariant is a function *F* such that, if applied to two isomorphic graphs *G* and *H*, then

$$F(G) = F(H) \tag{1}$$

The converse is not necessarily true. Therefore, an invariant imposes a necessary condition for isomorphism. If an invariant is both necessary and sufficient for isomorphism, it is said to be complete. A complete graph invariant is also called a certificate.

A vertex invariant assigns to every graph *G* a function f_G on the vertex set of *G*, such that

$$f_G(v)=f_{G'}[\sigma(v)] \tag{2}$$

where v is a vertex of G and $\sigma: G \rightarrow G'$ is an isomorphism. Vertex invariants can be transformed into graph invariants as follows:

Given a graph $G=(V, E)$ and vertex invariant f_G, the set

$$f_G(V):=\{f_G(v)|v \in V\} \tag{3}$$

is a graph invariant.

Examples of graph invariants include number of vertices, number of edges, degree of vertices and distance multiplicities. For each vertex, the set of distances to all other vertices is a vertex invariant.

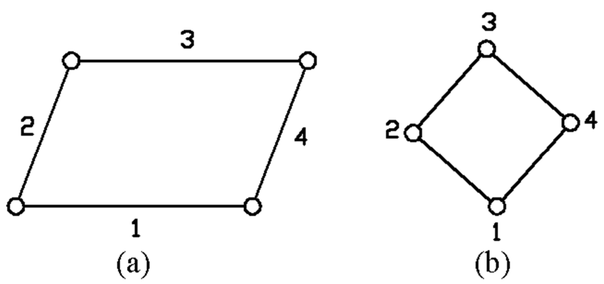


Fig. 2. (a) Four-bar chain (b) its graph representation

III. LINK INVARIANT FUNCTIONS

Vertices, edges and loops are three important parameters of a graph of kinematic chain. A function $f_G(v)$ is defined and calculated for each vertex of a graph of kinematic chain based on vertex degree, its distance from other vertices, and participation of vertex in loops.

A. Link Function

Degree of a vertex is defined as the number of edges incident with that vertex. Degree of vertex represents type of link, hence a vertex of degree two will correspond to a binary link and vertex of degree three a ternary link, and so on. *Two isomorphic graphs must have vertices of equal degree.* To account for this condition, a function (f_{Links}) to be called link function is assigned to each vertex of the graph and is given by

$$f_{Links}(v_i) = deg(v_i) \tag{4}$$

where, $deg(v_i)$ = Degree of i^{th} vertex.
The function takes degree of vertex as its value.

B. Joint Function

The failure of the earlier methods for testing isomorphism at some stage or the other, as the test sample size increases, can be attributed to the fact that almost all the earlier methods are based on direct adjacency or related matrices, which do not contain complete information regarding links at various distances i.e. other than the direct or first adjacency links. It is believed that all possible adjacency need be considered for testing isomorphism uniquely. Distance matrix of a kinematic chain contains the requisite information and is being utilized to form a function to take into account all adjacencies. However, since planar kinematic chains have non-planar graphs and existence of common distance matrix for two different kinematic chains cannot be ruled out, it is necessary to take into account all adjacencies. The distances are, in actual sense, represented by the number of edges by which the vertices are apart. Therefore, the next function f_{Joints} to be called joint function is assigned to each vertex of the graph. The value of joint function is given by

$$f_{Joints}(v_i) = \sum_{j=1}^{N} \left[d_{ij} deg(v_j)\right]^2 \tag{5}$$

where, d_{ij} = Shortest distance between vertex v_i and v_j and N = Number of vertices in the graph.

C. Loop Function

Having considered the links and joints of a kinematic chain, the next step is to assign a relation between vertices and loops. In a graph an alternating sequence of vertices without repetition is known as loop or cycle. For a graph of kinematic chain the number of basic loops (L) is determined by well-known Euler theorem, $L = E - V + 1$, where V is the number of vertices and E is the number of edges. Once the basic loops are determined, all other loops of the graph can be obtained through combination of basic loops [14].

Once all the loops are determined a function to be called loop function $\left(f_{Loops}\right)$ based on loop sizes is formed to measure participation of vertex in loops. Whether a vertex is participating in loop can be measured by assigning a number associated with the loop. Each vertex is assigned a value S if it participates in the formation of loop, where S is the size of the loop. The size of the loop indicates the number of vertices participating in the formation of loop. The value of the loop function is defined as

$$f_{Loops}(v_i) = \sum_{k=1}^{L_i} {S_k}^2 \tag{6}$$

Where, L_i = Number of loops associated with vertex v_i and S_k = Size of the k^{th} loop associated with vertex v_i.

D. Chain Link Invariant Set

Link, joint and loop functions are combined to form link invariant set *(LIS)* as

$$f_G(v) = \begin{bmatrix} f_{Links} \\ f_{Joints} \\ f_{Loops} \end{bmatrix} \tag{7}$$

Eq.(7) imposes a necessary and sufficient condition on each vertex. The set $f_G(V) := \{f_G(v) | v \in V\}$ of values calculated based on (7) to be called *link invariant set (LIS)* is an invariant for the graph of kinematic chain and summarized in a canonical form suitable for comparison. The set $f_G(V)$ is unordered and needs to be summarized in a canonical form in order to represent a kinematic chain uniquely. The value of link function can be combined to form a vertex degree listing. The vertex degree listing is defined as a list of integers representing the number of vertices of the same degree in ascending order. The first digit represents the number of vertices with degree 2, the second the number of vertices with degree 3, the third number of vertices with degree 4, and so on. Values of joint function and loop function of all the vertices are written in ascending order. The concatenation of vertex degree listing and ordered sequence of values of joint and loop function is defined as chain link invariant string (*CLIS*). The canonical form of string, *CLIS*, is used for detection of isomorphism between kinematic chains.

IV. DETECTION OF ISOMORPHISM

Based on the concept developed, a theorem for detection of isomorphism between two kinematic chains is proposed as

Theorem 1: For two kinematic chains A and B, if their link invariant set are identical, that is

CLIS(A) = CLIS(B)

the two kinematic chains are isomorphic; if not, otherwise.

The calculation may conveniently be performed by using the steps listed below.

Step 1. Computation of degree of each vertex.
Step 2. Computation of distance matrix.

Step 3. Computation of joint function for each vertex using (5).

Step 4. Determination of all possible loops associated with the graph of kinematic chain.

Step 5. Computation of loop function for each vertex.

Step 6. Combination of the values calculated in steps 1, 3 and 5 to form link invariant set *(LIS)*.

Step 7. Summary of *LIS* to form chain link invariant string *(CLIS)*.

The above computational steps are illustrated with the help of an example of 6-links, single degree of freedom kinematic chains shown in *Fig. 3*.

Example 1: A pair of non-isomorphic 1 degree of freedom kinematic chains with 6 links and 7 joints is shown in *Fig. 3*. Link invariant set needs to be calculated for identification of isomorphism between the chains and determination of distinct inversions possible from each chain. The following steps are needed for the calculation of link invariant set $f_G(V) := \{f_G(v)|v \in V\}$ as defined earlier in (4).

Step 1: Computation of degree of vertices.

Chain	v_1	v_2	v_3	v_4	v_5	v_6
Watt Chain	3	2	2	3	2	2
Stephenson Chain	3	2	3	2	2	2

Step 2: Computation of distance matrix.

	Watt Chain						Stephenson Chain					
	v_1	v_2	v_3	v_4	v_5	v_6	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	1	2	1	2	1	0	1	2	1	2	1
v_2	1	0	1	2	3	2	1	0	1	2	2	2
v_3	2	1	0	1	2	3	2	1	0	1	1	2
v_4	1	2	1	0	1	2	1	2	1	0	2	2
v_5	2	3	2	1	0	1	2	2	1	2	0	1
v_6	1	2	3	2	1	0	1	2	2	2	1	0

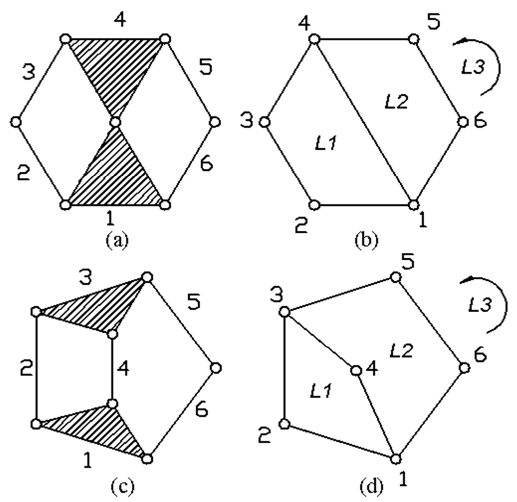


Fig. 3. (a) Watt chain (b) graph of (a) (c) Stephenson chain (d) graph of (d)

Step 3: Computation of joint function for each vertex using (5).

Chain	v_1	v_2	v_3	v_4	v_5	v_6
Watt Chain	49	101	101	49	101	101
Stephenson Chain	64	66	64	66	81	81

Step 4: Identification of basic loops and determination of all loops of the graph of kinematic chain.

Chain	List of all possible loops
Watt Chain	L_1[1,2,3,4], L_2[1,4,5,6],L_3 [1,2,3,4,5,6]
Stephenson Chain	L_1 [1,2,3,4], L_2 [1,4,3,5,6], L_3 [1,2,3,5,6]

Step 5: Calculation of loop function using (6).

Chain	v_1	v_2	v_3	v_4	v_5	v_6
Watt Chain	68	52	52	68	52	52
Stephenson Chain	66	41	66	41	50	50

Step 6: Summary of values calculated in steps 1, 3 and 5 forms link invariant set as defined in (7).

Chain	Function	v_1	v_2	v_3	v_4	v_5	v_6
Watt Chain	f_{Links}	3	2	2	3	2	2
	f_{Joints}	49	101	101	49	101	101
	f_{Loops}	68	52	52	68	52	52
Stephenson Chain	f_{Links}	3	2	3	2	2	2
	f_{Joints}	64	66	64	66	81	81
	f_{Loops}	66	41	66	41	50	50

Step 7: Chain Link Invariant String *(CLIS)*. Concatenation of sorted sequence of joint and loop values (in ascending order) along with vertex degree listing is presented as chain link invariant string.

The chain link invariant strings of Watt chain, *Fig. 3(a)*, is

[4 2 0 0] [49 49 101 101 101 101] [52 52 52 52 68 68]

The chain link invariant string of Stephenson chain, *Fig. 3(c)*, is

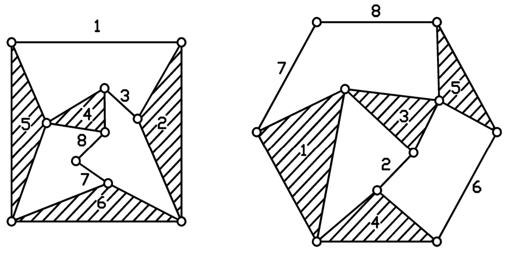
[4 2 0 0] [64 64 66 66 81 81] [41 41 50 50 66 66]

On comparing the *CLIS* of the two chains it is clear that the chains shown in *Figs. 3(a)* and *(c)* are distinct as *CLIS (A) ≠ CLIS (C)*.

Example 2: A pair of 8-link, 1 degree-of-freedom kinematic chains are shown in *Fig. 4*. The chains are isomorphic to each other; however, they have been drawn and labeled in different manner. Test for isomorphism between the two structures needs to be conducted.

CLIS of the two chains shown in *Fig. 4* are calculated following the steps of the new method developed. The *CLIS* of both the chains are found to be similar indicating that the chains are isomorphic.

CLIS (Fig. 4): [4 4 0 0] [106 106 121 141 158 178 193 218]
[150 154 174 174 204 215 215 224]



(a) (b)

Fig. 4. Eight bar 1-DOF isomorphic chains

The results also indicate that relabeling and redrawing does not affect the test.

Example 3: Two 12-bar 1-DOF kinematic chains are shown in *Figs. 5(a) and 5(b)* respectively. The chains possess identical characteristic polynomial of their degree matrices. *CLIS* for the figures are:

CLIS for *Fig. 5(a)* is

[5 6 1 0 0]
[282 356 362 416 416 466 466 482 482 528 528 612]
[644 644 720 764 764 1024 1128 1172 1172 1220 1220 1288]

CLIS for *Fig. 5(b)* is

[5 6 1 0 0]
[322 322 356 378 416 416 482 482 528 528 554 612]
[780 780 856 956 956 1208 1208 1216 1216 1264 1384 1424]

Distinct values of *CLIS* clearly reflect that the chains are uniquely identified by the method.

The method is applied to well known cases of kinematic chains up to 10-links and having one, two and three degrees of freedom. All the 16 eight bar 1-DOF chains, 40 nine-bar 2-DOF chains, 230 ten-bar 1-DOF chains and 98 ten-bar 3-DOF chains have been tested for isomorphism. All of them have yielded distinct *CLIS*. An example of chains with higher number of links cited from literature sources is also included in the present work to establish its reliability.

The method apart from detecting isomorphism between the chains, distinguishes the distinct links of the chain with little additional computational effort. This part is explained in the next section.

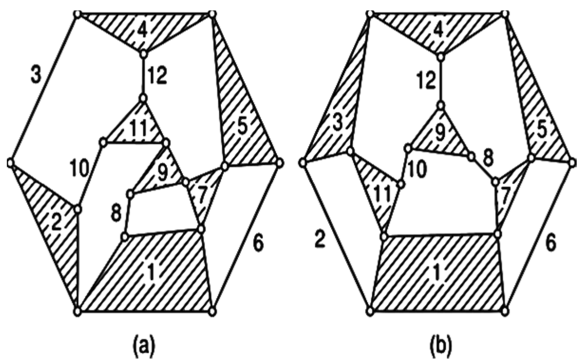


Fig. 5. A pair of 12-link, 1-DOF kinematic chains possessing identical characteristic polynomial (Degree Matrix)

V. Detection of Distinct Mechanisms

A kinematic chain is an interconnected system of links in which not a single link is fixed. Such a chain becomes a mechanism when one of the links in the chain is fixed. The fixed link is called a frame or, sometimes, a base link. The term *kinematic chain* is used to specify a particular arrangement of links and joints when it is not clear which link is to be treated as frame. When the frame link is specified, the kinematic chain is called a *mechanism*. The process of choosing different links in the chain as frames is known as kinematic inversion. In this way, for an *N*-link chain, *N* different mechanisms can be obtained. However, fixing of identical links will produce similar mechanisms. Two links of a chain are said to be identical if they are of same type and have similar disposition in the chain. For example, fixing of any of the links of a four-bar kinematic chain will result in structurally similar mechanisms, since all the links of the chain are binary and their disposition in the chain is also identical. For a designer it is necessary to avoid such duplications as the number of choices increases with chains having higher number of links. Therefore, finding the distinct mechanisms or inversions which can be derived from a chain is an important part of structural analysis of kinematic chains. In order to find the identical links it is necessary to find the disposition of frame link with respect to other links of the chain.

The link invariant set defined in previous section contains all the information necessary to identify equivalence between two links of a kinematic chain. The type of link is indicated by its degree, relation of link with other links and joints is indicated by value calculated by the joint function and participation of link in forming loops is given by the index calculated by loop function.

A. Neighborhood Joint Values

In order to measure a link's disposition in the chain it is only necessary to check its loop index and neighborhood values of joint function of adjacent links. These values are defined as neighborhood joint values (*NJV*). Mathematically, for each link (or vertex of graph of kinematic chain), it may be expressed as

$$NJV(v_i) = \sum_{k=1}^{A_v} f_{Joints}(v_k) \qquad (8)$$

where, A_v = Number of adjacent vertices associated with vertex v_i and

$NJV(v_i)$ = Neighborhood joint value of k^{th} neighbor of vertex v_i.

B. Link Neighborhood Index

Sorted sequence of *NJV*s along with values of link and loop functions is termed as link neighbourhood index (*LNI*). If any two links have the same *LNI*, they are equivalent.

$$LNI(v_i) = \left[f_{Links}, f_{Loops}, NJV(v_1), NJV(v_2), \ldots, NJV\left(v_{A_v}\right)\right] \qquad (9)$$

As an illustration, link neighborhood index *(LNI)* is calculated for the chain shown in *Fig. 6*. First, the values of loop and joint functions and neighbors for each link of the chain are determined and given below

Link No.	f_{Loops}	f_{Joints}	Neighbors
1	316	178	[2 4 5]
2	288	178	[1 3 10]
3	216	278	[2 4]
4	316	266	[1 3 9]
5	264	318	[1 6]
6	264	406	[5 7]
7	264	378	[6 10]
8	236	318	[9 10]
9	236	318	[4 8]
10	336	218	[2 7 8]

Next, neighborhood joint values *(NJV)* have been calculated as follows for each of the links using (8).

Link No.	Joint values of neighbors			*NJV*
1	178	266	318	762
2	178	278	218	674
3	178	266		444
4	178	278	318	774
5	178	406		584
6	318	378		696
7	406	218		624
8	318	218		536
9	266	318		584
10	178	378	318	874

Finally using (9), link neighborhood index *(LNI)* for *Fig. 6*, are calculated and shown in Table I.

TABLE I. LNI OF THE CHAIN OF FIG. 6.

Link No.	*LNI* values using (9)
1	[3 316 584 674 774]
2	[3 288 444 762 874]
3	[2 216 674 774]
4	[3 316 444 584 762]
5	[2 264 696 762]
6	[2 264 584 624]
7	[2 264 696 874]
8	[2 236 584 874]
9	[2 236 536 774]
10	[3 336 536 624 674]

Link neighborhood indices (*LNI*) of all the ten links have been found to be distinct. So there are ten inversions possible for the chain of *Fig. 6*, whereas the other methods [3] have reported lesser number of inversions for the same chain.

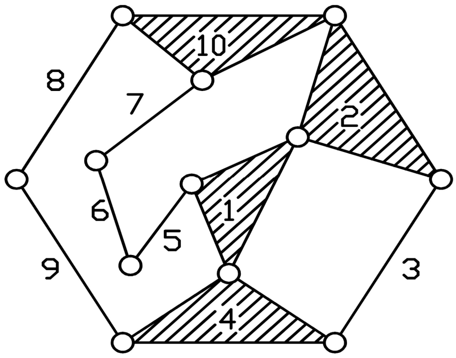


Fig. 6. A 10-bar, 3-DOF kinematic chain

The total number of inversions arrived using the *Link Neighborhood Index* method for 8-link 1-DOF, 9-link 2-DOF, 10-link 1-DOF and 10-link 3-DOF are listed in Table II and results fully agree with those available in literature.

TABLE II. LIST OF INVERSIONS

S. No.	**Type of chains**	**Total number of inversions**
1	8-link, 1-DOF	71
2	9-link, 2-DOF	254
3	10-link, 3-DOF	684
4	10-link, 1-DOF	1834

There are 71 distinct mechanisms possible from a total of 16 distinct eight-link single degree of freedom chains. These inversions were obtained based on the distinct link neighbourhood index calculated using (8) and (9). The complete set of 8-bar single degree of freedom chains along with the calculations of CLIS and inversions are appended in the paper. The set of frame links corresponding to 71 distinct mechanisms are also given.

VI. CONCLUSION

A method for detection of isomorphism and inversions among kinematic chains is presented. Method is simple, reliable and can easily be implemented on a computer. A program is written in MATLAB and entire calculations were carried out in a personal computer with core 2 duo processor and 1 GB random access memory. With the help of the present method chains can be checked for isomorphism along with distinct mechanisms that can be derived from the chains. Further work is being carried out to correlate the value of the invariant with structural properties of chains for application to creative design of mechanisms.

REFERENCES

[1] A. C. Rao and P. B. Deshmukh, "Computer aided structural synthesis of planar kinematic chains obviating the test for isomorphism," Mechanism and Machine Theory, vol. 36, pp. 489-506, 2001.

[2] J. J. Uicker and A. Raicu, "A method for the identification of and recognition of equivalence of kinematic chains," Mechanism and Machine Theory, vol. 10, pp. 375-383, 1975.

[3] T. S. Mruthyunjaya, "A computerized methodology for structural synthesis of kinematic chains - part I, II and III," Mechanism and Machine Theory, vol. 14, pp. 487-530, 1984.

[4] T. S. Mruthyunjaya and H. R. Balasubramanian, "In quest of a reliable and efficient computational test for detection of isomorphism in kinematic chains," Mechanism and Machine Theory, vol. 22, pp. 131-139, 1987.

[5] W.-M. Hwang and Y.-W. Hwang, "Computer-aided structural synthesis of planar kinematic chains with simple joints," Mechanism and Machine Theory, vol. 27, no. 2, pp. 189-199, 1992.

[6] R. K. Dubey and A. C. Rao, "New characteristic polynomial - a reliable index to detect isomorphism between kinematic chains," Journal Indian Institute of Sciences, vol. 66, pp. 227-233, 1986.

[7] A. G. Ambekar and V. P. Agrawal, "On canonical numbering of kinematic chains and isomorphism problem: max code," ASME paper no. 86-DET-169, 1986.

[8] A. G. Ambekar and V. P. Agrawal, "Canonical numbering of kinematic chains and isomorphism problem: min code," Mechanism and Machine Theory, no. 22, pp. 453-461, 1987.

[9] J. N. Yadav, C. R. Pratap and V. P. Agrawal, "Detection of isomorphism among kinematic chains using the distance concept," ASME Journal of Mechanical Design, vol. 117, pp. 607-611, 1995.

[10] A. C. Rao and D. V. Raju, "Application of the hamming number technique to detect isomorphism among kinematic chains and inversions," Mechanism and Machine Theory, no. 26, pp. 55-75, 1991.

[11] A. C. Rao and C. N. Rao, "Loop Based Pseudo Hamming Values—I. Testing Isomorphism and Rating Kinematic Chains," Mechanism and Machine Theory, vol. 28, pp. 113-127,1993.

[12] J. K. Chu and W. Q. Cao, "Identification of isomorphism among kinematic chains and inversions using link's adjacent-chain-table," Mechanism and Machine Theory, vol. 29, no. 1, pp. 53-58, 1994.

[13] A. C. Rao and V. V. N. R. P. R. Pathapati, "Loop based detection of isomorphism among chains, inversions and type of freedom in multi degree of freedom chain," ASME Journal of Mechanical Design, vol. 122, pp. 31-42, 2000.

[14] H. Ding and Z. Huang, "The establishment of the canonical perimeter topological graph of kinematic chains and isomorphism identification," ASME Journal of Mechanical Design, pp. 915-923, 2007.

[15] H. Ding and Z. Huang, "Isomorphism identification of graphs: especially for the graphs of kinematic chains," Mechanism and Machine Theory, vol. 44, pp. 122-139, 2009.

[16] A. C. Rao, "Application of fuzzy logic for the study of isomorphism, inversions, symmetry, parallelism and mobility in kinematic chains," Mechanism and Machine Theory, vol. 35, pp. 1103-1116, 2000.

[17] A. C. Rao, "A genetic algorithm for topological characteristics of kinematic chains," ASME Journal of Mechanical Design, pp. 228-231, 2000.

[18] G. S. Bedi and S. Sanyal, "Modified joint connectivity approach for identification of topological characteristics of planar kinematic chains," Journal of Mechanical Engineering Science, vol. 225, no. 11, pp. 2700-2717, 2011.

[19] S. Sanyal, "Structural identification of distinct inversions of planar kinematic chains," IIUM Engineering Journal, pp. 85-92, 2011.

[20] F. G. Kong, Q. Li and W. J. Zhang, "An artificial neural network approach to mechanism kinematic chain isomorphism identification," Mechanism and Machine Theory, pp. 271-283, 1999.

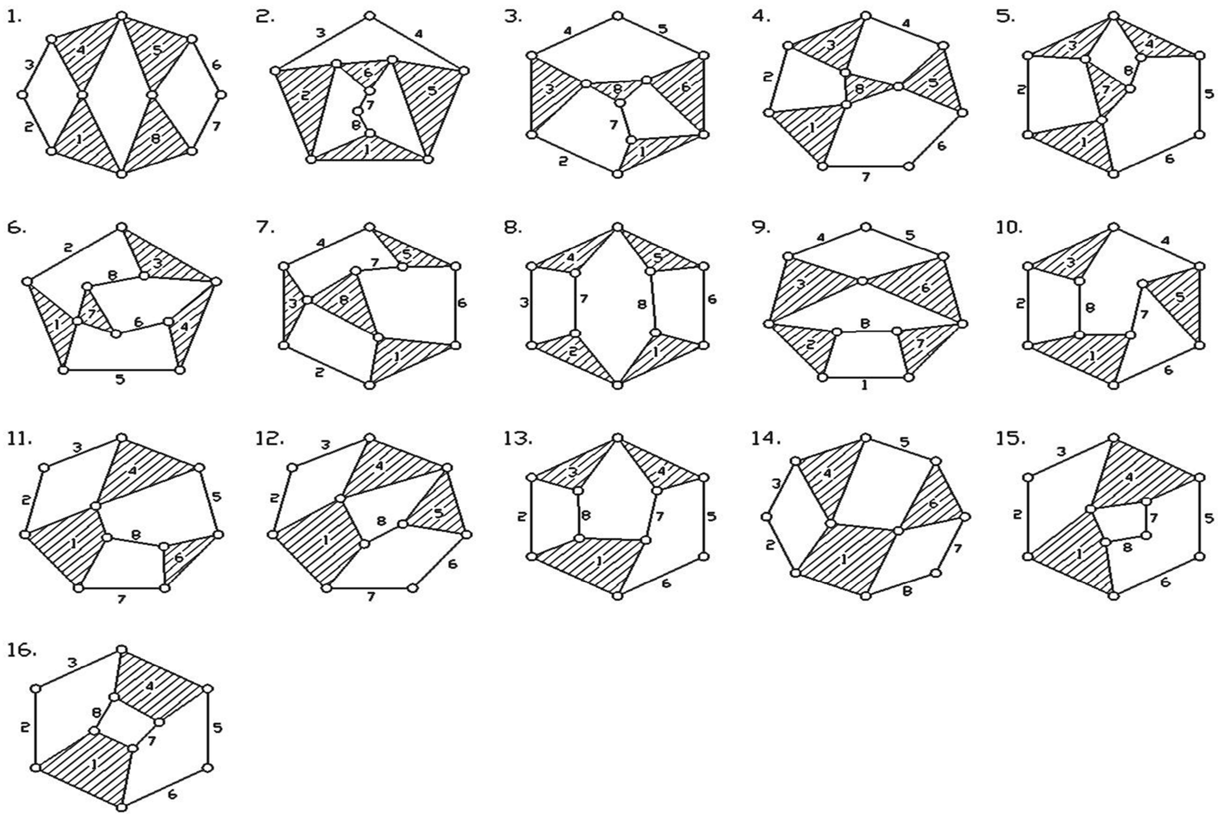
[21] L. Dobrjanskyj and F. Freudenstein, "Some application of graph theory to the structural analysis of mechanisms," ASME Journal of Engieering for Industry, pp. 153-158, 1967.

[22] F. Harary, Graph Theory, New Delhi: Narosa Publishing House, 2001.

[23] Deo, N., Graph Theory with Applications to Engineering and Computer Science. New Delhi: Prentice Hall of India Private Limited, 2007.

APPENDIX

Set of Eight Link Single Degree of Freedom Chains



Chain Link Invariant Strings and Frame Links Corresponding to 8-Link, 1-DOF Kinematic Chains

Chain No.	**Chain link invariant string *(CLIS)***	**Number of distinct mechanisms derived from the chain by link neighborhood index**	**Set of frame links**
1	[4 4 0 0][126 126 126 126 266 266 266 266] [116 116 116 116 168 168 168 168]	2	(1,4,5,8)(2,3,6,7)
2	[4 4 0 0][106 106 106 106 193 193 193 193] [178 178 178 178 219 219 219 219]	2	(1,2,5,6)(3,4,7,8)
3	[4 4 0 0][106 106 121 141 158 178 193 218] [150 154 174 174 204 215 215 224]	8	(1)(2)(3)(4)(5)(6)(7)(8)
4	[4 4 0 0][91 141 141 161 203 203 218 218] [150 150 172 172 191 215 224 224]	5	(1,5)(2,4)(3)(6,7)(8)
5	[4 4 0 0][126 126 166 166 178 178 218 218] [152 152 172 172 204 204 224 224]	4	(1,4)(2,8)(3,7)(5,6)
6	[4 4 0 0][121 121 121 121 158 158 158 158] [150 150 150 150 211 211 211 211]	2	(1,3,4,7)(2,5,6,8)
7	[4 4 0 0][106 141 141 156 158 183 183 203] [146 146 148 150 200 211 211 220]	6	(1,3)(2)(4,6)(5)(7)(8)
8	[4 4 0 0][166 166 166 166 178 178 178 178] [88 88 88 88 160 160 160 160]	2	(1,2,4,5)(3,6,7,8)
9	[4 4 0 0][106 106 141 141 178 178 238 238] [90 90 114 114 164 164 164 164]	4	(1,8)(2,7)(3,6)(4,5)
10	[5 2 1 0][124 170 170 170 170 184 184 226] [88 88 88 88 144 160 160 176]	4	(1)(2,6,7,8)(3,5)(4)
11	[5 2 1 0][89 108 145 145 166 184 205 246] [90 90 114 114 148 164 164 180]	7	(1)(2)(3)(4)(5)(6)(7,8)
12	[5 2 1 0][89 93 145 149 160 201 205 221] [77 99 99 101 101 142 151 167]	8	(1)(2)(3)(4)(5)(6)(7)(8)
13	[5 2 1 0][104 125 129 145 145 149 185 201] [75 77 77 97 97 138 147 163]	7	(1)(2,8)(3)(4)(5)(6)(7)
14	[5 2 1 0][74 128 128 180 180 186 274 274] [116 116 116 116 152 168 168 184]	5	(1)(2,8)(3,7)(4,6)(5)
15	[6 0 0 2][76 76 188 188 188 188 188 188] [88 88 88 88 88 88 156 156]	2	(1,4)(2,3,5,6,7,8)
16	[6 0 0 2][112 112 112 112 168 168 168 168] [66 66 86 86 86 86 152 152]	3	(1,4)(2,3,5,6)(7,8)
	Total	**71**	