Comparative Evaluation of Particle Swarm Optimization Algorithms for the Optimal Dimensional Synthesis of Planar Four-bar Mechanism

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***Abstract*—This paper presents the application of Particle Swarm Optimization (PSO) and its variants to the dimensional synthesis of 5 coupler paths described by 11 to 25 precision points. Minimization of the structural error and degree of constraint violation were taken as the objective functions. In addition to basic PSO, stretched PSO (S-PSO), near neighborhood information based PSO (NNI-PSO), gregarious PSO (G-PSO) and hybrid PSO with differential evolution operator (DE-PSO) were also applied. The results revealed that the performance of DE-PSO is superior to basic PSO and all other variants for the dimensional synthesis. Comparison of the results with other soft computing technique for the dimensional synthesis indicated that the DE-PSO can be effectively used for the dimensional synthesis of four-bar mechanism.**

Keywords—*particle swarm optimization; four-bar mechanism; dimensional synthesis; structural error; constraint violation; success rate*

I. INTRODUCTION

One of the simple, but practically important classes of mechanisms employed in most of the machines is the planar four-bar mechanism. A four bar mechanism (Fig. 1) consists of four links, viz., a fixed link (L_4), a crank (L_1), a coupler (L_2) and a follower (L_3). The required path of motion is traced by the extension (L_5) link of the coupler. The fixed link forms the base of the mechanism. The crank is the input link and it is rotated by a power source. The path traced by the coupler extension point, M for one complete rotation of the crank is shown in Fig. 1. By varying the dimensions of the links (L_1, L_2, L_3, L_4, L_5), coupler extension angle (β), angle of the fixed link with horizontal (φ) and location of the mechanism (x_A, y_A), various paths of motion of the coupler extension point can be obtained. If the desired trajectory of the coupler extension point is known, dimensional synthesis of the mechanism needs to be carried out to generate this path.

In recent years, due to increasing computational speed of computers, metaheuristics approaches that imitate natural phenomena are applied to dimensional synthesis problems [1]. Various heuristic algorithms for path synthesis have already been reported in the literature such as Genetic Algorithm [2-3], Differential Evolution [4-5], Ant Colony Optimization [6-7] and Particle Swarm Optimization [8]. Dimensional synthesis of four bar mechanism based on these stochastic search algorithms has outperformed many classical methods of mechanism synthesis.

A relatively new stochastic optimization method is the Particle Swarm Optimizer (PSO), which was introduced by Kennedy and Eberhart [9]. Many variants of PSO algorithms were developed over the years and applied to solve the various optimization problems. However, very limited attempts have been made to solve the dimensional synthesis problem using PSO algorithm. Sedlaczek and Eberhard [8] reported the PSO with augmented Lagrange multiplier method in combination with an advanced non-stationary penalty function approach and used it successfully for the dimensional synthesis of slider crank mechanism with workspace constraints. In this paper, 4 variants of PSO are applied to solve dimensional synthesis problems. A detailed performance analysis of the PSO algorithms has been carried out based on statistical analysis.

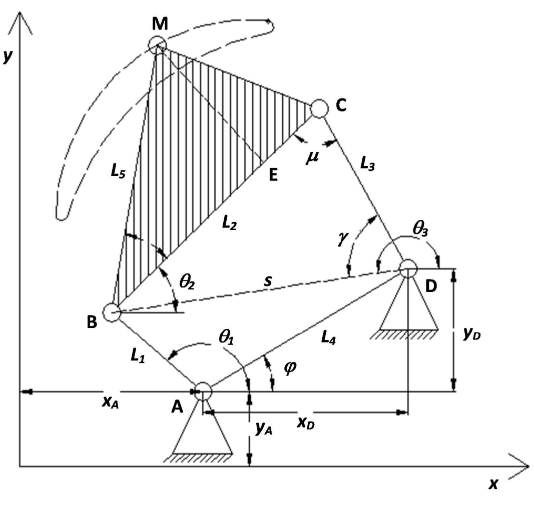


Fig. 1. Geometry of a four-bar mechanism

II. ANALYSIS OF FOUR-BAR MECHANISM

The relevant parameters defining the planar four-bar mechanism geometry are shown in Fig. 1. The position of the point B is defined by the expressions:

$$x_B = x_A + L_1 \cos\theta_1, \quad y_B = y_A + L_1 \sin\theta_1 \tag{1}$$

where θ_1 is the angle which defines the current position of the mechanism. The distance BD = s is defined by the expression

$$s = \sqrt{(x_D - L_1 \cos\theta_1)^2 + (y_D - L_1 \sin\theta_1)^2} \tag{2}$$

and it creates the angle θ_3 ($-\pi \le \theta_3 \le \pi$) with the positive direction x-axis:

$$\theta_3 = L_1 \tan\left(\frac{y_D - L_1 \sin\theta_1}{x_D - L_1 \cos\theta_1}\right) \tag{3}$$

The position of the follower (the member CD = L_3) is double valued and it depends on whether the point C is below or above the radius vector BD. The coordinates of the point C are determined by the expressions:

$$x_C = x_A + x_D + L_3 \cos(\theta_3 - t.\gamma)$$
$$y_C = y_A + y_D + L_3 \sin(\theta_3 - t.\gamma) \tag{4}$$

where t is the coefficient whose value is $t = 1$, for the case when the point C is above the line segment s (Fig. 1), and $t = -1$, for the case when the point C is below the line segment s (crossed mechanism). γ is the angle created between the follower and the line segment s. It takes the value ($0 \le \gamma \le \pi$) and is determined by the expression:

$$\gamma = \cos^{-1}\left(\frac{s^2 + {L_3}^2 - {L_2}^2}{2\, s\, L_3}\right) \tag{5}$$

The angle θ_2 created between the coupler BC and the positive part of the x-axis is determined by the expression

$$\theta_2 = L_1 \tan\left(\frac{y_C - y_B}{x_C - x_B}\right) \tag{6}$$

Finally, the position of the point M of the coupler, the point moving along the desired path, is given by the following equations:

$$x_M = x_A + L_1 \cos\theta_1 + L_5 \cos(\theta_2 + \beta)$$
$$y_M = x_A + L_1 \sin\theta_1 + L_5 \sin(\theta_2 + \beta) \tag{7}$$

III. DIMENSIONAL SYNTHESIS PROBLEM FORMULATION

Find optimum dimensions of the links of mechanism for the desired rectilinear path traced by the point M (Fig. 1) of the coupler of the four-bar mechanism so that the objective function has the minimum value. Thus, defined optimization problem can be given the following general mathematical formulation:

$$\text{minimize } f(X), \text{ subject to } \quad g_j(X) \le 0, \; j = 1, \ldots, n_g,$$

$f(X)$ is the objective function, $g_j(X) \le 0$ represent the constraints defined by the search space, n_g is the total number of constraints. $X = [x_1, \ldots, x_D]^T$ represents the design vector consisting of D design variables. The design variables are the values which should be defined during the optimization procedure. Each design variable is defined by its lower and upper boundaries. For the case of the four-bar mechanism (Fig. 1), the design vector is

$$X = [x_a, y_a, L_1, L_2, L_3, L_4, L_5, \beta, \varphi]^T.$$

The design objective is to determine the optimal link lengths such that the motion generated by the mechanism is as close as possible to the desired trajectory. The structural error is defined as

$$F_{Error} = \sum_{i=1}^{n} \left[\left(x_{id} - x_{ig}\right)^2 + \left(y_{id} - y_{ig}\right)^2 \right]$$

where, n is the number of synthesis points of interest. (x_{id}, y_{id}) and (x_{ig}, y_{ig}) are the coordinates of the coupler point. Subscripts g and d denote a generated parameter and a desired value, respectively. The objective function is minimized under the condition that the generated solution satisfies a set of constraints. The constraints introduced herein ensure that the mechanism is assembleable, link dimensions fall within a desired range, positions are generated in the desired order and the mechanism is moveable. Dimensions of the links were limited within upper and lower bounds in the PSO algorithm itself. In order to ensure that the final solution honors the desired order of points, coordinates of the coupler point for the considered set of design variables were determined by providing a certain angle of increment to the crank rotation angle, θ_1 to obtain the increasing angle of rotation of crank. To ensure that the four-bar mechanism is moveable and the drive link is a crank, the dimensions of the links must satisfy the Grashof's condition. Let L_s, L_L and L_a and L_b be the lengths of the shortest link, longest link, and the other two links, respectively. To ensure that the drive link is a crank, the following constraint must be satisfied:

$$(L_s + L_L) < (L_a + L_b)$$

According to Deb [10], the single objective constrained optimization problem can be solved by formulating two objective functions. One objective is the original objective function and other is the degree of violation of constraint.

$$f_1(x) = F_{Error} \qquad f_2(x) = \max\{0, (L_s + L_L) - (L_a + L_b)\}$$

If x satisfies all the constraints, $f_2(x) = 0$. Thus single objective dimensional synthesis problem can be transformed into the following two objective optimization problem:

$$\text{Minimize } \{ f_1(x), f_2(x) \}$$

During the process of selection of the best individual, the following selection scheme is used:

1. If the second objective values of two particles are equal to zero (feasible solutions), select the one with the smaller first objective value.

2. If the second objective values of two particles are both nonzero (infeasible solutions), select the one with the smaller second objective value (smaller constraint violation).

3. If the second objective value of one particle is zero (feasible solution), and that of the other is nonzero (infeasible solution), choose the one with the zero second objective value.

IV. PSO AND ITS VARIANTS

PSO is developed through simulation of bird flocking in multi-dimensional space. The position of i^{th} particle is represented in d^{th} dimension of the multi-dimensional space with position vector, X_i^d and velocity vector, V_i^d. Modification of the particle position is realized by the position and velocity information. Each particle tries to modify its position by considering current position $(X_i^d)^k$, current velocity $(V_i^d)^k$, the individual intelligence (*pbest*), and group intelligence (*gbest*). The following equations are utilized in computing the new position and velocity for the i^{th} particle in d^{th} dimension:

$$(V_i^d)^{k+1} = \omega (V_i^d)^k + c_1 * rand1_i^d * (pbest_i^d - (X_i^d)^k) + c_2 * rand2_i^d * (gbest^d - (X_i^d)^k) \quad (8)$$

$$(X_i^d)^{k+1} = (X_i^d)^k + (V_i^d)^{k+1} \quad (9)$$

where, $(V_i^d)^{k+1}$ is the velocity at $(k+1)^{th}$ iteration of i^{th} particle, $(V_i^d)^k$ the velocity at the k^{th} iteration of i^{th} particle, ω the inertial weight, c_1, c_2 are the cognitive and social acceleration coefficients, $rand1_i^d$ and $rand1_2^d$ are the random numbers selected between 0 and 1, $pbest_i^d$ is the best position of the i^{th} particle, $gbest^d$ the best position among the particles (group best) and $(X_i^d)^k$ is the position of the i^{th} particle at k^{th} iteration. The inertia weight 'ω' is modified using (10) to enable quick convergence.

$$\omega = (\omega_1 - \omega_2)\ [(iter_{max} - iter) / iter_{max}] + \omega_2 \quad (10)$$

where, ω_1 is the initial weight (ω_1 = 0.9), ω_2 the final weight (ω_2 = 0.4), *iter* the current iteration number and $iter_{max}$ is the maximum iteration number.

Several variations of this basic PSO scheme have been proposed in the literature for solving continuous, multi-dimensional and multi-modal optimization problems [11 - 12]. They mainly aim to prevent the solution from reaching the local optima, avoid premature convergence, maintain diversity in the swarm and reduce computation effort. The modified techniques are called (1) Stretched PSO (S-PSO) [13], (2) Near Neighborhood Interaction based PSO (NNI-PSO) [14], (3) Gregarious PSO (G-PSO) [15] and (4) Hybrid PSO with Differential Evolution operator (DE-PSO) [16].

A. S-PSO

In order to escape from the local minima, two-stage transformation equations on the original fitness function *f(x)* is used. This can be applied immediately after a local minimum of the function *f(x)* has been detected. This transformation is defined as follows:

$$G(x) = f(x) + \gamma_1 \|x - \bar{x}\| \big(sign\big(f(x) - f(\bar{x})\big) + 1\big)$$

$$H(x) = G(x) + \gamma_2 \frac{sign\big(f(x) - f(\bar{x})\big)}{\tanh\Big(\mu\big(G(x) - G(\bar{x})\big)\Big)}$$

where, γ_1, γ_2 and μ are arbitrary chosen positive constants and $sign(\cdot)$ defines the well known triple valued sign function.

B. NNI-PSO

In order to improve the local exploitation capability, global exploration capability and convergence speed, velocity updating equation (8) is modified as follows:

$$(V_i^d)^{k+1} = \omega (V_i^d)^k + c_1 * rand1_i^d * (pbest_i^d - (X_i^d)^k) + c_2 * rand2_i^d * (gbest^d - (X_i^d)^k + c_3 * rand3_i^d * (p_i^{d\#} - (X_i^d)^k)$$

where, $p_i^{d\#}$ is the *pbest* in the near neighborhood. To enhance the efficiency of PSO, equation (10) is changed by introducing a power factor (*m*).

$$\omega = (\omega_1 - \omega_2)\ [(iter_{max} - iter) / iter_{max}]^m + \omega_1$$

C. G-PSO

If the Euclidean distance between its current position and the global best position is less than ε (a small value), the particle's velocity is re-initialized in the range, [$-V_{max}$, V_{max}]. Otherwise, the particle will take a step along the direction towards the global best position as follows :

$$V_i^d = \gamma * rand_i^d * (gbest^d - X_i^d)$$

where, γ is the step size. The value of step size is linearly adjusted at the end of every iteration as follows:

If the fitness value of the best position of the present iteration is less than that of the previous iteration,

$$\gamma = max(\gamma - \delta, \gamma_{min}), \quad else,\ \gamma = min(\gamma + \delta, \gamma_{max})$$

where, δ is a constant. As the particles do not memorize their previous search history, there is no update of personal best in G-PSO.

D. DE-PSO

In this method, equations (8) and (9) are used at the odd iterations and equation (11) at the even iterations. The DE mutation operator is defined over the particle's best positions $pbest_i$ with a trial point $T_i = pbest_i$ which for the particles d^{th} dimension is derived as

$$If\ (rand < CR\ or\ d = k)\ then\ T_i^d = gbest^d + \delta_2^d \quad (11)$$

where, k is a random integer value within [1, number of dimensions] which ensures the mutation in at least one dimension, CR is a crossover constant (CR ≤ 1) and δ_2 is the case of $N = 2$ for the general difference vector

$$\delta_N = \frac{1}{N} \sum_{I}^{N} \Delta$$

where, Δ is the difference between two elements randomly chosen in the set. If the fitness value of T_i is better than the one for $pbest_i$, then T_i will replace $pbest_i$. After the DE operator is applied to all the particles' individual best values, the *gbest* value is chosen among the *pbest* set providing the social learning capability, which might speed up the convergence.

V. IMPLEMENTATION OF PSO AND ITS VARIANTS

Coordinates of the following four different paths available in the literature [17-19] were considered:

Example 1: (4.04, 4.29), (4.24, 4.01), (4.23, 3.46), (4.21, 3.05), (3.89, 2.98), (3.67, 3.20), (3.47, 3.63), (3.35, 4.09), (3.34, 4.48), (3.53, 4.58), (3.77, 4.53) : Total 11 precision points.

Example 2: (4.15, 2.21), (4.50, 2.18), (4.53, 1.83), (4.13, 1.68), (3.67, 1.58), (2.96, 1.33), (2.67, 1.06), (2.63, 0.82), (2.92, 0.81), (3.23, 1.07), (3.49, 1.45), (3.76, 1.87) : Total 12 precision points.

Example 3: (0.50, 1.10), (0.40, 1.10), (0.30, 1.10), (0.20, 1.00), (0.10, 0.90), (0.005, 0.750), (0.02, 0.60), (0.00, 0.50), (0.00, 0.40), (0.03, 0.30), (0.10, 0.25), (0.15, 0.20), (0.20, 0.30), (0.30, 0.40), (0.40, 0.50), (0.50, 0.70), (0.60, 0.90), (0.60, 1.00) : Total 18 precision points.

Example 4: (7.03, 5.99), (6.95, 5.45), (6.77, 5.03), (6.40, 4.60), (5.91, 4.03), (5.43, 3.56), (4.93, 2.94), (4.67, 2.60), (4.38, 2.20), (4.04, 1.67), (3.76, 1.22), (3.76, 1.97), (3.76, 2.78), (3.76, 3.56), (3.76, 4.34), (3.76, 4.91), (3.76, 5.47), (3.80, 5.98), (4.07, 6.40), (4.53, 6.75), (5.07, 6.85), (5.05, 6.84), (5.89, 6.83), (6.41, 6.80), (6.92, 6.58) : Total 25 precision points.

There are nine independent design variables for the optimal synthesis of four bar mechanism. The bounding interval of the design variables is given below:

x_a: -5.00 to 5.00; y_a: -5.00 to 5.00
L_1: 0.10 to 5.00, L_2: 1.00 to 10.00; L_3: 1.00 to 10.00
L_4: 1.00 to 10.00; L_5: 1.00 to 15.00
β: -50° to 350 °; φ: -90° to 90°

Initial particles were selected randomly within this range. A swarm size of 60 was considered for running all the variants of PSO. The parameter values of different variants of PSO are presented below:

Basic PSO : Initial velocity = -0.5 to 0.5, ω = 0.9 to 0.4, c_1 = 2.0, c_2 = 2.0.
S-PSO : Initial velocity = -0.5 to 0.5, ω = 1.0 to 0.4, c_1 = 0.5, c_2 = 0.5, γ_1 = 5000, γ_2 = 0.5, $\mu = 10^{-10}$.
NNI-PSO : Initial velocity = -0.5 to 0.5, ω = 0.9 to 0.2, c_1 = 1.0, c_2 = 1.0, c_3 = 2.0, m = 1.2.
G-PSO : Initial γ = 3.0, γ = 2.0 to 4.0, δ = 0.5, $\varepsilon = 10^{-8}$.
DE-PSO : Initial velocity = -0.5 to 0.5, ω = 0.4, c_1 = 2.0, c_2 = 2.0, CR = 0.9.

During the process of implementation, the velocity components of particle V_i^d are limited to a maximum allowable modulus V_{max}, as follows:

$$V_i^d = \begin{cases} -V_{max}, & if\ V_i^d < V_{max} \\ V_{max}, & if\ V_i^d > V_{max} \\ V_i^d, & otherwise \end{cases}$$

The value of V_{max} is defined as one half of the total search range in each dimension. The position of particles in each dimension beyond the specified bounding interval was adjusted as follows:

$$X_i^d = \begin{cases} X_{min}^d, & if\ X_i^d < X_{min} \\ X_{max}^d, & if\ X_i^d > X_{max} \\ X_i^d, & otherwise \end{cases}$$

All the PSO variants were coded in MATLAB. They are allowed to run for a maximum of 10000 iterations. The set of design variables which resulted in the best objective function value was used for generating the trajectory. The data of objective function value by each PSO variant were subjected to t-test to determine significant difference in the performance of PSO variants.

VI. RESULTS AND DISCUSSION

The optimal values of the design variables of the four-bar mechanism obtained using variants of PSO for generating 4 different trajectories along with the value of structural error is presented in Tables 1-4. In general, the structural error for the variants of PSO is less than that of basic PSO indicating the better search of solution by the variants of PSO. The PSO variants, S-PSO, NNI-PSO, G-PSO and DE-PSO found the best solution almost in the same area of the design space. Except for the examples 1 and 2, they resulted in more or less same values of design variables. The path traced by the optimal values of design variables are shown in Fig. 2-5. The difference between the desired path and the path generated by the design variables obtained through the basic PSO is higher than that obtained through its variants.

TABLE I. OPTIMAL VALUES OF THE DESIGN VARIABLES OBTAINED FROM VARIANTS OF PSO FOR THE EXAMPLE 1

Design variables	Basic PSO	S-PSO	NNI-PSO	G-PSO	DE-PSO
x_A	-1.11	-0.73	-2.45	-1.79	-4.88
y_A	-5.00	-1.11	-2.73	-1.67	-3.77
L_1	0.56	0.54	0.55	0.52	0.51
L_2	5.56	3.34	4.70	4.08	5.79
L_3	6.17	4.78	5.47	3.72	4.10
L_4	5.55	6.88	8.88	6.89	8.81
L_5	10.28	6.78	9.11	7.90	11.57
β (deg.)	-50.00	2.31	13.35	17.61	17.02
Φ (deg.)	43.80	6.79	1.92	1.34	2.21
Error	0.41	0.07	0.05	0.05	0.04

TABLE II. OPTIMAL VALUES OF THE DESIGN VARIABLES OBTAINED FROM VARIANTS OF PSO FOR THE EXAMPLE 2

Design variables	Basic PSO	S-PSO	NNI-PSO	G-PSO	DE-PSO
x_A	4.29	4.18	4.11	3.80	4.02
y_A	-4.54	-2.83	-2.22	-2.59	-2.59
L_1	0.43	0.50	0.51	0.59	0.54
L_2	4.20	2.88	2.48	3.39	3.34
L_3	4.44	4.82	4.17	5.38	7.25
L_4	8.11	7.12	6.09	8.14	10.00
L_5	6.29	4.44	3.84	4.17	4.18
β (deg.)	114.04	108.30	105.35	103.18	110.20
Φ (deg.)	-37.20	37.81	-35.50	-35.40	-40.17
Error	0.89	0.16	0.15	0.16	0.14

TABLE III. OPTIMAL VALUES OF THE DESIGN VARIABLES OBTAINED FROM VARIANTS OF PSO FOR THE EXAMPLE 3

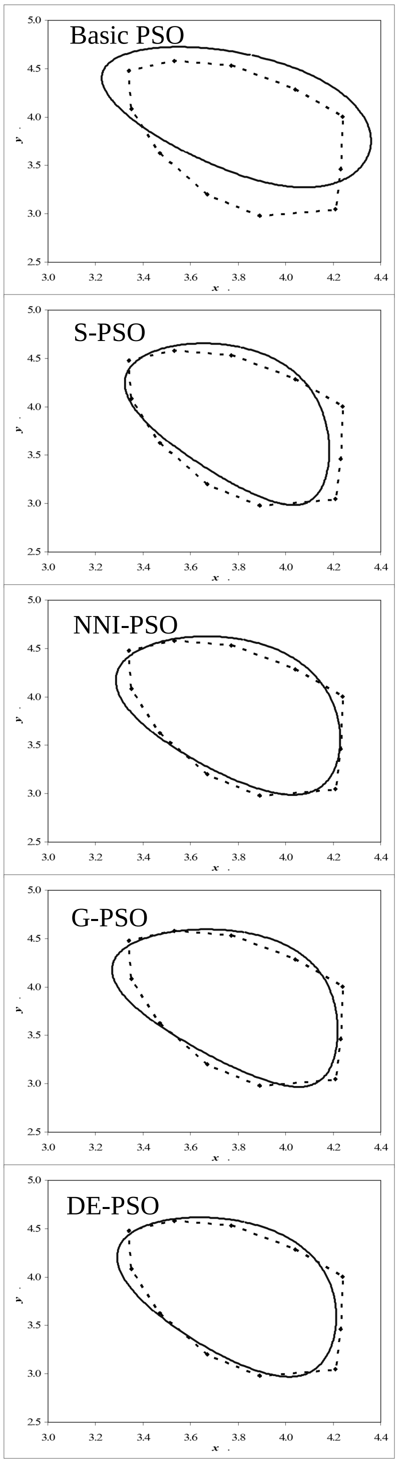
Design variables	Basic PSO	S-PSO	NNI-PSO	G-PSO	DE-PSO
x_A	-0.42	0.89	1.05	1.36	1.23
y_A	-1.01	-0.45	0.05	0.58	0.24
L_1	0.45	0.32	0.24	0.27	0.22
L_2	5.26	7.35	7.93	7.88	8.61
L_3	6.01	6.86	7.46	6.16	7.84
L_4	10.00	1.62	1.06	2.06	1.16
L_5	1.83	1.30	1.03	1.13	1.09
β (deg.)	33.37	13.02	60.67	157.33	102.29
Φ (deg.)	4.78	40.54	22.73	-9.66	9.73
Error	0.08	0.03	0.02	0.03	0.02

TABLE IV. OPTIMAL VALUES OF THE DESIGN VARIABLES OBTAINED FROM VARIANTS OF PSO FOR THE EXAMPLE 4

Design variables	Basic PSO	S-PSO	NNI-PSO	G-PSO	DE-PSO
x_A	-0.69	-5.00	-5.00	-5.00	-5.00
y_A	0.78	0.00	-0.02	-0.04	-0.01
L_1	2.03	2.10	2.10	2.10	2.10
L_2	4.32	6.52	6.52	6.55	6.53
L_3	4.80	4.31	4.32	4.36	4.31
L_4	6.81	8.51	8.51	8.58	8.52
L_5	7.32	11.35	11.35	11.37	11.35
β (deg.)	-3.77	-1.44	-1.50	-1.10	-1.42
Φ (deg.)	-2.75	-0.11	-0.06	-0.36	-0.11
Error	4.64	1.54	1.54	1.55	1.54

The experimental results in terms of the mean objective function value, the best objective function value, the standard deviation, number of iterations and the CPU time are summarized in Table 5. The basic PSO and its variants were ranked based on the best objective function value. For all the examples, the results in terms of the best objective function value of the DE-PSO are much better than those of other methods. Also, the mean objective function value and the standard deviation are much better for most of the examples, which means that the searched solutions are more stable. The DE-PSO requires less number of iterations than other methods thanks to its better searching ability. For all the examples, the basic PSO got stuck in the first local minima it encountered during the search process. As DE-PSO has the mutation operation in the PSO, it generated a long jump using the mutation operator and avoided local minima. The performance of DE-PSO was followed by NNI-PSO. The near neighborhood information of the NNI-PSO helped it to find the better solution but it required more number of iterations than DE-PSO and its next best PSO variant. G-PSO searched better solution than that of S-PSO for the examples 1 and 2, but for the rest, solution searched by the S-PSO was better than that of G-PSO. The G-PSO required more number of iterations than S-PSO for all the examples.

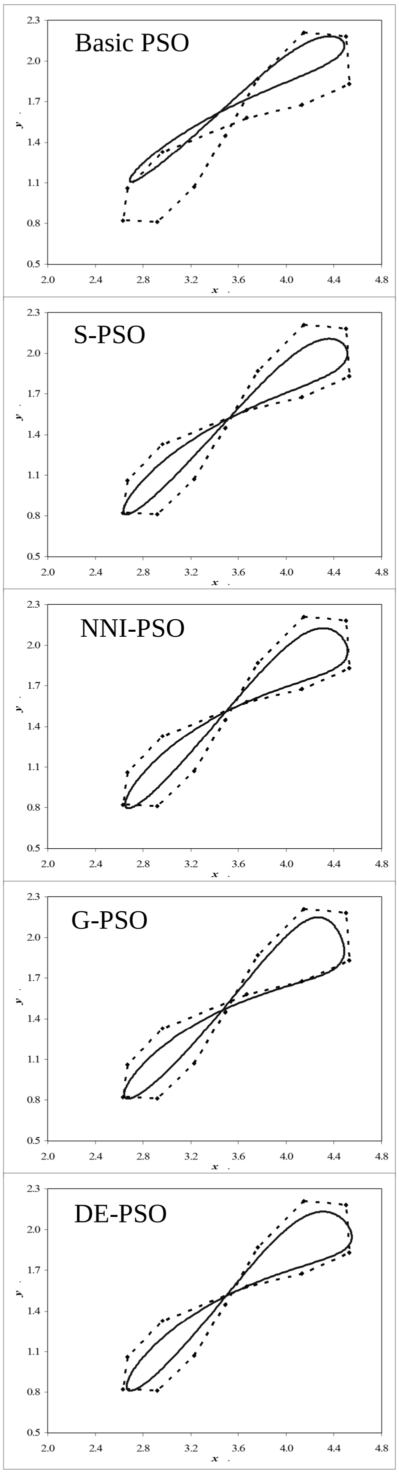
Table 6 indicates that DE-PSO is significantly better than all other variants of PSO for the dimensional synthesis of four-bar mechanism. Eventhough, NNI-PSO was found to be better than DE-PSO for the examples 1 and 4, the t-values are non-significant. Variation in the best objective function value obtained through NNI-PSO, G-PSO and DE-PSO for the example 4 was found to be non-significant. This indicates that for the trajectories described by higher number of precision points, use of NNI-PSO and G-PSO methods is on par with DE-PSO. However, use of NNI-PSO for the dimensional synthesis of the example 4 requires 9.6 times higher number of iterations and 4.7 times higher CPU time than that of DE-PSO (Table 5). Similarly, use of G-PSO for the dimensional synthesis of the example 4 requires 13.9 times higher number of iterations and 5.9 times higher CPU time than that of DE-PSO. Therefore, the DE-PSO method is superior to other PSO methods for the dimensional synthesis of four-bar mechanism. The success rate of DE-PSO for the examples 1, 2, 3 and 4 was found to be 10, 18, 18 and 74 % respectively. DE-PSO needs to be improved for success rate and stability.



---- Desired path --- Generated path

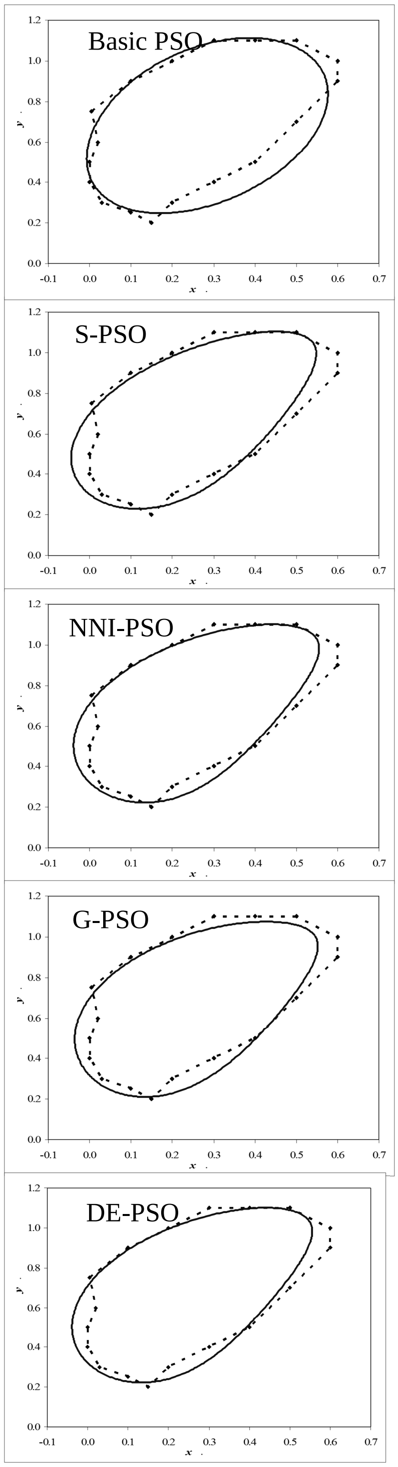
Fig. 1. Path traced by the optimized link dimensions by variants of PSO for the example 1.

Results obtained through DE-PSO were compared with the best results obtained through other techniques like GA [17], GA-FL [18] and ant-gradient [7] (Table 7). It shows that the structural error for other techniques is less than 0.04. Further, the optimized design parameters were found to be in different area of the design space may be due to certain additional constraints imposed on the objective function. However, DE-PSO is a simple technique and has lower computational cost as compared to other techniques. Therefore, it can be effectively used for the approximate dimensional synthesis of four-bar mechanism.



---- Desired path --- Generated path

Fig. 2. Path traced by the optimized link dimensions by variants of PSO for the example 2.

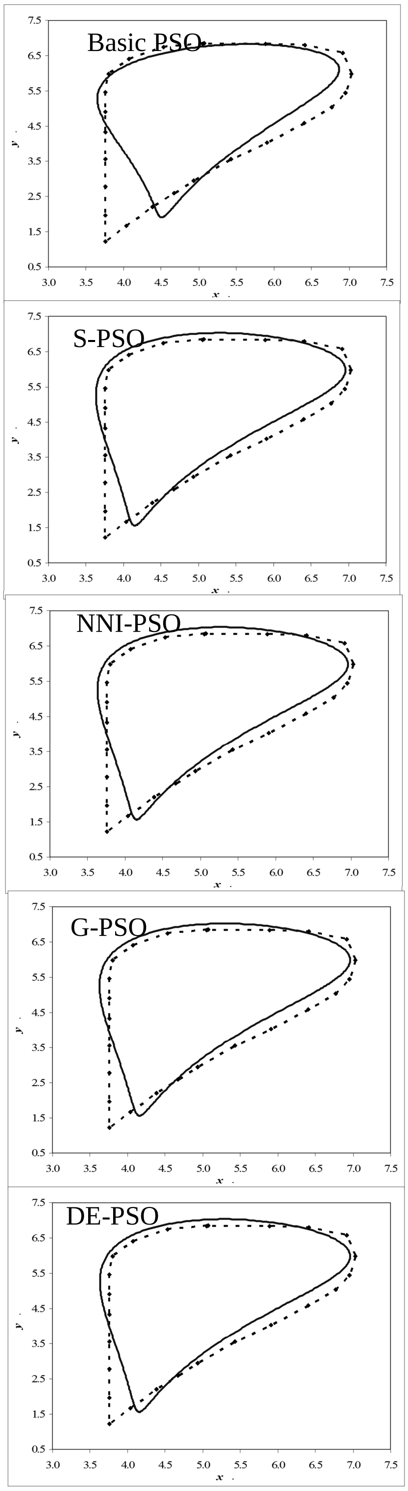


---- Desired path --- Generated path

Fig. 3. Path traced by the optimized link dimensions by variants of PSO for the example 3.

VII. CONCLUSIONS

The performance evaluation of the basic PSO and other promising PSO variants for the dimensional synthesis of four-bar mechanism was studied. The comparative evaluation shows that for the 4 examples considered in this paper, all the variants of PSO found the optimum solution in the same area of the design space. For the trajectories described by higher number of precision points, there was negligible difference in the solutions obtained through DE-PSO, NNI-PSO and G-PSO. In terms of best objective function value and stability, performance of DE-PSO was superior to all other PSO variants. Therefore, DE-PSO technique can be effectively used for the approximate dimensional synthesis of four-bar mechanism.



---- Desired path --- Generated path

Fig. 4. Path traced by the optimized link dimensions by variants of PSO for the example 4.

TABLE V. COMPARISON BERWEEN VARIANTS OF PSO FOR DIMENSIONAL SYNTHESIS OF FOUR-BAR MECHANISM

Criteria	Basic PSO	S-PSO	NNI-PSO	G-PSO	DE-PSO
			***Example* 1**		
Mean	3.930	2.175	0.337	0.793	0.411
Best	0.410	0.070	0.045	0.048	0.037
Std. dev.	0.965	1.449	0.787	1.288	0.767
Iterations	137	520	784	875	363
CPU time	25.18	105.06	162.11	137.29	125.36
Rank	5	4	2	3	1
			Example 2		
Mean	5.791	3.675	2.389	2.937	1.432
Best	0.885	0.156	0.145	0.153	0.135
Std. dev.	1.523	2.433	1.808	2.327	1.539
Iterations	324	737	3638	830	168
CPU time	68.92	147.72	797.05	160.57	64.77
Rank	5	4	2	3	1
			Example 3		
Mean	1.167	0.753	0.067	0.160	0.049
Best	0.083	0.031	0.024	0.032	0.023
Std. dev.	0.563	0.621	0.049	0.180	0.042
Iterations	87	415	466	789	323
CPU time	21.19	107.93	116.22	175.77	153.39
Rank	5	3	2	4	1
			Example 4		
Mean	36.420	10.569	1.847	4.241	2.575
Best	4.642	1.544	1.544	1.545	1.544
Std. dev.	23.826	19.421	0.709	12.569	6.548
Iterations	151	1247	3033	4391	315
CPU time	54.20	382.34	951.33	1204.26	202.40
Rank	5	3	2	4	1
Overall ranking	5	3	2	3	1
(Average ranking number)	(5.0)	(3.5)	(2.0)	(3.5)	(1.0)

TABLE VI. T-VALUE BETWEEN BASIC PSO AND OTHER VARIANTS

t-value between	Example 1	Example 2	Example 3	Example 4
		Basic PSO and		
S-PSO	-7.13	-5.21	-3.49	-5.95
NNI-PSO	-20.39	-10.17	-13.75	-10.26
G-PSO	-13.78	-7.26	-12.04	-8.45
DE-PSO	-20.18	-14.23	-14.01	-9.69
		S-PSO and		
NNI-PSO	-7.88	-3.00	-7.77	-3.17
G-PSO	-5.04	-1.55^{NS}	-6.47	-1.93
DE-PSO	-7.61	-5.51	-7.99	-2.76
		NNI-PSO and		
G-PSO	2.14	1.31^{NS}	3.50	1.34^{NS}
DE-PSO	0.48^{NS}	-2.85	-2.04	0.78^{NS}
		G-PSO and		
DE-PSO	-1.80	-3.82	-4.25	-0.83^{NS}

TABLE VII. T-VALUE BETWEEN BASIC PSO AND OTHER VARIANTS

Design variables	Example 1	Example 2	Example 3	Example 4
	GA [18]	*GA [17]*	*GA-FL [18]*	*Ant-gradient [7]*
x_A	-2.99	0.77	-3.06	-8.79
y_A	-1.62	0.29	-1.30	-1.20
L_1	0.48	1.13	0.42	1.89
L_2	3.37	4.45	2.32	8.41
L_3	6.00	3.62	3.36	6.75
L_4	6.00	3.02	4.07	13.08
L_5	8.71	3.06	3.90	14.45
β (deg.)	328.96	300.96	-15.60	11.15
Φ (deg.)	-4.45	29.74	-9.10	-21.86
Error	0.02	0.04	0.006	0.02

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