# Optimization of Size Parameters for Interconnected Pneumatic Cylinders Positioning System

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Abstract— Servo pneumatics is a mechatronic approach that enables accurate position control of pneumatic drives with high speed. In this paper, a new method of position manipulator with two interconnected pneumatic cylinders is presented. Nonlinear mathematical model of the system comprising of mass flow rate, pressure dynamics, frictional forces and motion dynamics has been formulated. Using Matlab-Simulink software the system is simulated. The size parameters are optimized using Taguchi method for minimization of settling time. It is found that the area of the cylinders have more significance in efficient positioning of the system than length of the cylinders.

# Keywords— Servo Pneumatics; Positioning systems; Nonlinear systems; Simulation; Optimal size parameters.

## I. INTRODUCTION

Pneumatic actuators are widely used in the field of automation, robotics and manufacturing. Traditionally pneumatic cylinders are used for motion between two hard The pneumatic technology exhibits stops. manv advantages such as high speed, high force generation, better efficiency, less maintenance and low operating costs. In order to expand the capabilities of the pneumatic cylinders to be operated as multi-position actuator, servo control techniques are being used. The significant problem is that such drives are nonlinear: the pressure within the pneumatic cylinder, the frictional force, and the compressed air flow rates through the chokes of the pneumatic drive all vary in nonlinear fashion. The dead zone and time delay characteristics of the servo valve also adds to the complexity for designing controller for the system. Mathematical modelling of the system becomes very important for development of controller for the system and also to optimize the parameters in the system. Takosoglu et al [1] formulated mathematical model for servo pneumatic positioning system. Najafi et al [2] developed a mathematical model for the pneumatic positioning system also considering the cushioning in the cylinder. Miyajima et al [3] presented an explicit model for spool movement inside a proportional directional control valve. Saleem et al [4] developed mathematical model for frictional forces inside pneumatic cylinders. Hildebrandt et al [5] have optimized the system parameters for efficient positioning control.

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In Pneumatic actuators there will be always a compromise between speed and the accurate positioning. But the present industry requires the actuators having higher stroke lengths, high speed and better accuracy. Many researchers have tried different types of hybrid actuators which has one actuator to travel the course and another for accurate positioning. Chiang et al [6] have developed hybrid pneumatic-piezoelectric actuator for long range and precise positioning. Chiang [7] developed a X-Y dual-axial intelligent servo pneumatic-piezoelectric hybrid actuator for position control with high response, large stroke and nanometer accuracy. Liu et al [8] developed a hybrid actuator comprising of piezoelectric impact force coupled with differential pressure for the pneumatic positioning device. Nishioka et al [9] proposes a new control method for a multiplex pneumatic transmission constructed with special resonant valves and air tubes with a control system driven by air vibration in air tubes without electrical wires. But these hybrid actuators do not posses all the advantages of pneumatic actuators. In this paper a system comprising of two pneumatic cylinders attached to each other is proposed to solve the problem. One cylinder supplied with high pressure will undergo the course movement and another cylinder supplied with low pressure will be used to fine movement.

The size parameters of the cylinders are the important factors in determining the speed and accurate positioning of the system. So an optimal design of these cylinder size parameters is required for efficient usage of the system. In the current research an attempt is made to optimize these size parameters by using Taguchi analysis. Jeyapaul, et al. [10] have reviewed various taguchi based optimization approaches.

## II. SYSTEM DESCRIPTION

The system consists of two pneumatic cylinders A and B which are interconnected to form a series linear manipulator. The master cylinder A is fixed and the cylinder B is coupled to the rod end of the cylinder A. The cylinder is mainly for the course movement and is supplied directly from air source with comparatively high pressure. The cylinder B is for the fine movement which has reduced air pressure from the flow control valve as shown in the Fig. 1. Two proportional directional control valves V1 and

V2 are used to control the position of cylinders A and B respectively. Each cylinder is connected with a position transducer which senses the position and feeds it to the control system. The controller manipulates the voltage applied to the proportional control valves. The control system consists of two individual feedback controlled SISO system with a setpoint splitting technique.



Fig. 1. Schematic representation of positioning system using two pneumatic cylinders interconnected.

#### III. MATHEMATICAL MODELLING

The mathematical model of the system has been derived from physical laws and recent literature information. The system constitutes the nonlinearities of the dead zone, the mass flow rate, the pressure dynamics and the motion equation, that includes the friction dynamics.

In the modelling of the system, the following assumptions are made:

- The gas is perfect.
- The pressures and temperature within each chamber are homogeneous.
- Kinetic and potential energy terms are negligible.
- Tube length can be ignored when air supply is very close to valve and cylinder.
- Valve dynamics is sufficiently faster than mechanical systems dynamics.

The total force developed in the series connected manipulator is given by algebraic sum of force developed in two individual cylinders as given by (1).

$$\mathbf{F} = \mathbf{F}_{\mathbf{a}} + \mathbf{F}_{\mathbf{b}} \tag{1}$$

where F is the total force generated,  $F_a$  and  $F_b$  are forces generated in cylinders A and B respectively.

The individual forces  $F_a$  and  $F_b$  are given by (2) and (3) respectively.

$$F_{a} = A_{a1}P_{a1} - A_{a2}P_{a2} - F_{fa}$$
(2)

$$F_{b} = A_{b1}P_{b1} - A_{b2}P_{b2} - F_{fb}$$
(3)

where  $A_{a1}$  and  $A_{a2}$  are cross sectional area of two chambers of cylinder A,  $A_{b1}$  and  $A_{b2}$  are cross sectional area of two chambers of cylinder B,  $P_{a1}$  and  $P_{a2}$  are absolute pressure in two chambers of cylinder A,  $P_{b1}$  and  $P_{b2}$  are absolute pressure in two chambers of cylinder B,  $F_{fa}$  and  $F_{fb}$  are frictional forces in two cylinders.

The position of the entire linear manipulator is the sum of position values of the two individual values as given by (4).

$$\mathbf{x} = \mathbf{x}_{a} + \mathbf{x}_{b} \tag{4}$$

where x is the position of the coupled manipulator,  $x_a$  and  $x_b$  are the positions of cylinders A & B respectively. The position values of individual cylinders A and B are given as per Newton's second law of motion by (5) and (6) respectively.

$$\ddot{\mathbf{x}}_{a} = \frac{\mathbf{F}_{a}}{\mathbf{M}_{L} + \mathbf{m}_{a} + \mathbf{M}_{b} + \mathbf{m}_{b}}$$
(5)

$$\ddot{\mathbf{x}}_{\mathrm{b}} = \frac{\mathbf{F}_{\mathrm{b}}}{\mathbf{M}_{\mathrm{L}} + \mathbf{m}_{\mathrm{b}}} \tag{6}$$

where  $m_a$  and  $m_b$  are internal piston mass of cylinders A and B respectively,  $M_b$  is the total mass of cylinder B,  $M_L$  is the external load.

The absolute pressure values inside the two chambers of cylinders A and B are given by (7), (8), (9) and (10) respectively.

$$\dot{P}_{a1} = \frac{\kappa}{A_{a1}(l_{a0} + x_a)} \left( R\dot{m}_{a1} - P_{a1}A_{a1}v_a \right)$$
(7)

$$\dot{P}_{a2} = \frac{\kappa}{A_{a2} \left( l_a + l_{a0} - x_a \right)} \left( -R\dot{m}_{a2} + P_{a2}A_{a2}v_a \right) \quad (8)$$

$$\dot{P}_{b1} = \frac{\kappa}{A_{b1}(l_{b0} + x_b)} \left( R\dot{m}_{b1} - P_{b1}A_{b1}v_b \right)$$
(9)

$$\dot{P}_{b2} = \frac{\kappa}{A_{b2} \left( l_b + l_{b0} - x_b \right)} \left( -R\dot{m}_{b2} + P_{b2} A_{b2} v_b \right)$$
(10)

where  $\kappa$  is adiabatic exponent,  $l_a$  and  $l_b$  are stroke lengths of cylinders A and B respectively,  $l_{a0}$  and  $l_{b0}$  are lengths of dead zone of cylinders A and B respectively, R is specific gas constant,  $m_{a1}$  and  $m_{a2}$  are mass flow rates to two chambers of cylinder A,  $m_{b1}$  and  $m_{b2}$  are mass flow rates to two chambers of cylinder B,  $v_a$  and  $v_b$  are velocity of cylinders A and B respectively.

The mass flow rate values for two chambers of the cylinders A and B are given by (11), (12), (13) and (14) respectively.

$$\dot{\mathbf{m}}_{a1} = \partial_0 \mathbf{x}_{ra} \left( \mathbf{C}_{14} \mathbf{P}_{as} \mathbf{w}_{14} - \mathbf{C}_{45} \mathbf{P}_{a1} \mathbf{w}_{45} \right) \tag{11}$$

$$\dot{\mathbf{m}}_{a2} = \partial_0 \mathbf{x}_{ra} \left( \mathbf{C}_{23} \mathbf{P}_{a2} \mathbf{w}_{23} - \mathbf{C}_{12} \mathbf{P}_{as} \mathbf{w}_{12} \right)$$
(12)

$$\dot{\mathbf{m}}_{b1} = \partial_0 \mathbf{x}_{rb} \left( \mathbf{C}_{14} \mathbf{P}_{bs} \mathbf{w}_{14} - \mathbf{C}_{45} \mathbf{P}_{b1} \mathbf{w}_{45} \right)$$
(13)

$$\dot{\mathbf{m}}_{b2} = \partial_0 \mathbf{x}_{rb} \left( \mathbf{C}_{23} \mathbf{P}_{b2} \mathbf{w}_{23} - \mathbf{C}_{12} \mathbf{P}_{bs} \mathbf{w}_{12} \right)$$
(14)

where  $\partial_0$  is air density,  $x_{ra}$  and  $x_{rb}$  are spool movements in the proportional directional control valves connected to cylinders A and B respectively which depends on the coil

voltages  $u_a$  and  $u_b$  applied,  $P_{as}$  and  $P_{bs}$  are absolute pressure values of air supplies to cylinders A and B respectively,  $C_{14}$ ,  $C_{45}$ ,  $C_{23}$  and  $C_{12}$  are sonic conductance consistent with the standard ISO 6358-1989 for critical pressure ratio,  $w_{14}$ ,  $w_{45}$ ,  $w_{23}$  and  $w_{12}$  are nonlinear flow function (sonic flow and subsonic flow) depending on the pressure ratio and on the critical pressure ratios  $b_{14}$ ,  $b_{45}$ ,  $b_{23}$  and  $b_{12}$ .

The equations of frictional forces in cylinders A and B are given by (15) and (16) respectively.

$$F_{fa} = f_1 v_a + F_k sign(v_a) + F_{pr} e^{-\left(\frac{v_a}{v_{sa}}\right)} sign(v_a) + k_{pa}(P_{a1} - P_{a2})$$
(15)

$$F_{fb} = f_1 v_b + F_k sign(v_b) + F_{pr} e^{-\left(\frac{v_b}{v_{sb}}\right)} sign(v_b) + k_{pb}(P_{b1} - P_{b2})$$
 (16)

where  $f_l$  is viscous coefficient of friction,  $F_k$  is Kinetic coefficient of friction,  $v_{sa}$  and  $v_{sb}$  are stribeck velocities,  $F_{pr}$  is break away force and  $k_{pa}$  and  $k_{pb}$  are friction coefficients due to seals.

## IV. SIMULATION ENVIRONMENT

The simulation model of the positioning system containing two interconnected cylinders is created based on (1) to (16) using Matlab-Simulink software. The simulation model of the positioning system containing two interconnected cylinders is shown in Fig. 2. The mass flow rates through the proportional valves are formulated using equations (11) to (14) which depend on the control voltage from the controller. The pneumatic pressure ports have been designed using the equations (7) to (10). The force and position of the cylinders are obtained by using equations (1) to (6). The simulation is conducted using the parameters as shown in Table I. Proportional controllers with a gain value of 10 are used for controlling each cylinder positions to form a closed loop control system. Setpoint splitting technique is used such that 90% of the travel is done by cylinder A and remaining 10% of fine movement is produced by cylinder B. The simulation results of the positioning system comprising two cylinders, when subjected to step change in input signal from 0 mm to 100 mm at simulation time 0s is shown in Fig. 3.

TABLE I. SYSTEM PARAMETERS USED IN THE NUMERICAL SIMULATIONS

Parameter	Values
ma and mb	0.05 Kg
Mb	0.4 Kg
ML	3 Kg
K	1.4
R	288 Nm/KgK
$\partial_0$	1.225 Kg/m <sup>3</sup>
Pas	0.6 MPa
P <sub>bs</sub>	0.1 MPa
$T_0, T_{as}, T_{bs}$	298 K
$l_{a0}, l_{b0}$	0.02 m
C14, C45, C23 and C12	1.462 X 10 <sup>-8</sup> m <sup>4</sup> s/Kg
b14,b45,b23 and b12	0.28
$f_l$	250 Ns/m
F <sub>k</sub>	100 N
$v_{sa}$ and $v_{sb}$	0.1 m/s
Fpr	200N
k <sub>pa</sub> and k <sub>pb</sub>	3 N/Pa



Fig. 2. Simulation model of positioning system using two pneumatic cylinders interconnected.



Fig. 3. Simulation results of positioning system using two pneumatic cylinders interconnected when a step change in setpoint from 0 mm to 100 mm is applied at simulation time 0s.

## V. OPTIMIZATION AND RESULTS

For optimizing the size parameters of the cylinders in the positioning system for fast response, Taguchi based technique is used. The input parameters for the optimization are Area of cylinders and lengths of the cylinders. The output parameter is the settling time of the position value to the desired value. Settling time is the time required for the response curve to reach and stay in final steady state value. It is obtained from the simulation response. The various level values of the input parameters are shown in Table II.

TABLE II. VALUES OF SIZE PARAMETERS OF TWO CYLINDERS FOR VARIOUS LEVELS

Level	Parameter 1		Parameter 2	Parameter 3		Parameter 4
	$\begin{array}{c}A_{a1}\\X10^{-5}\\m^2\end{array}$	$A_{a2}$ $X 10^{-5} m^2$	l <sub>a</sub> m	$A_{b1} X 10^{-5} m^2$	$A_{b2}$ $X 10^{-5} m^2$	l <sub>b</sub> m
Level 1	25	18	0.1	25	18	0.1
Level 2	49	30	0.2	49	30	0.2
Level 3	64	45	0.3	64	45	0.3

For reducing the number of simulation tests to be carried out, the Taguchi design of experiments is used. L9 orthogonal table design has been selected based on Taguchi design of experiments which is shown in Table III.

Based on the design of experiments, nine trails are carried out and the settling time value is obtained from

simulation results. From these values the signal to noise ratio is obtained using the formula given by (17).

$$S / N Ratio = -10 \log(y^2)$$
(19)

where y is the settling time of the respective trail. The Settling time and S/N ratio for each trails based on Taguchi Design of Experiments is shown in the Table IV.

 TABLE III.
 TAGUCHI L9 ORTHOGONAL TABLE DESIGN

Trail No.	Parameter 1	Parameter 2	Parameter 3	Parameter 4
1	Level 1	Level 1	Level 1	Level 1
2	Level 1	Level 2	Level 2	Level 2
3	Level 1	Level 3	Level 3	Level 3
4	Level 2	Level 1	Level 2	Level 3
5	Level 2	Level 2	Level 3	Level 1
6	Level 2	Level 3	Level 1	Level 2
7	Level 3	Level 1	Level 3	Level 2
8	Level 3	Level 2	Level 1	Level 3
9	Level 3	Level 3	Level 2	Level 1

TABLE IV. SETTLING TIME AND S/N RATIO FOR EACH TRAILS BASED ON TAGUCHI DESIGN OF EXPERIMENTS

Trail No.	Settling Time (s)	Signal to Noise (S/N) Ratio
1	2.4	-7.60422
2	2.5	-7.9588
3	2.6	-8.29947
4	2.4	-7.60422
5	2.2	-6.84845
6	1.8	-5.10545
7	1.5	-3.52183
8	1.2	-1.58362
9	1.4	-2.92256

Table V shows the consolidated S/N ratio of all the input factors with all the levels. The values have been computed by the adding the all MRPI values for corresponding level of each process parameters. For example, Parameter 1 has Level 1 in the trails 1, 2 and3. So the cumulative MRPI value is given by average of MRPI values for trails 1, 2 and 3. In the Table 5, the maximum level value of each parameter indicates the optimal level of input parameters. So the optimal settling time is achieved when parameters 1 and 2 in level 3, parameter 3 in level 1 and parameter 4 in level 2.The minimum time response is obtained when the cylinder parameters are  $A_{a1}=64 \times 10^{-5} \text{ m}^2$ ,  $A_{a2}=45 \times 10^{-5} \text{ m}^2$ ,  $I_a = 0.3m$ ,  $A_{b1}=25 \times 10^{-5} \text{ m}^2$ ,  $A_{b2}=18 \times 10^{-5} \text{ m}^2$  and  $I_b = 0.2m$  So the minimum settling time is obtained when the size of the cylinder A is more than the size of cylinder B.

TABLE V. S/N RATIO FOR ALL POSSIBLE SOLUTIONS

Levels	Parameter 1	Parameter 2	Parameter 3	Parameter 4
Level 1	-7.95416	-6.24342	-4.76443	-5.79175
Level 2	-6.51938	-5.46363	-6.16186	-5.52869
Level 3	-2.676	-5.44249	-6.22325	-5.82911

For finding the most dominant parameter in determining the optimization, the analysis of variance (ANOVA) technique is used. Table VI Shows the ANOVA table obtained by using Minitab software. Fig. 4 shows the main effects diagram for the optimization variations of each parameter. From Table VI and Fig. 4, it is observed that that the area of the cylinders has more influence on the system performance than the length of the cylinders.

TABLE VI. ANALYSIS OF VARIANCE TABLE

Source	DF	Seq SS	Adj SS	Adj MS	F
Parameter 1	1	1.92667	1.92667	1.92667	51.3778
Parameter 2	1	0.04167	0.04167	0.04167	1.1111
Parameter 3	1	0.13500	0.13500	0.13500	3.6000
Parameter 4	1	0.00667	0.00667	0.00667	0.1778
Error	4	0.15000	0.15000	0.03750	
Total	8	2.26000			



Fig. 4. Main effects plot for Settling time using ANOVA Technique

## VI. CONCLUSION

In this paper, a new method of position manipulator with two interconnected pneumatic cylinders has been presented. One of the cylinders is used for course movement and another for fine movement. Detailed mathematical model of the system consisting of motion dynamics, pressure dynamics, mass flow rate variations and frictional forces are presented. The model is simulated using Matlab-Simulink software. Using Taguchi technique, the optimal size parameters are obtained. It is observed that the minimum settling time is obtained when the size of the cylinder A is more than the size of cylinder B. Also it has been found that area of the cylinders has more influence on the system performance than the length of the cylinders. Further research includes real time implementation and design of control system for this interconnected cylinders based positioning system.

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