Kinematic Analysis and 3D Workspace Development of 3DOF Parallel Manipulator with a Rotary Base

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***Abstract*— This paper presents the systematic methods to deal with kinematic analysis of 3-PRS configuration parallel manipulator involving non-linear simultaneous equations. Using the loop closure constraints, three coupled equations are formulated using configuration of the manipulator. Bezout's resultant and Sylvester method are used to determine roots of three non-linear equations. The tilt of the moving platform is measured with respect to fixed base for single prismatic joint actuation or any combination of these three joints actuations. The maximum tilt of moving platform for the proposed configuration is also computed for limited range of joints. Tool tip coordinates are determined using vector approach with reference to established coordinate system on the base platform. Jacobian matrices are derived for active and passive variables for singularity determination. Using tool tip coordinates for various combinations of linear actuations as point cloud, workspace for 3 – DOF parallel manipulator is developed.**

Keywords— Parallel manipulators, Bezout's resultant, Jacobian, Singularity, Workspace

I. Introduction

Even though Stewart platform and Hexapod offers 6-DOF as parallel manipulators (PMs) for some applications, the practical usage of 3 DOF PMs due to less complex configuration and power requirements for many industrial applications may not be neglected. The development of parallel manipulators can be dated back to the early 1960s, when Gough and Whitehall [1] first devised a six-linear jack system for the purpose of a universal tire testing machine. Later, Stewart [2] developed a platform manipulator for use as a flight simulator. Since 1980, there has been an increasing interest in development of parallel manipulators. Potential applications of parallel manipulators include mining machines [3], walking machines [4] and pointing devices [5]. Parallel Manipulators (PMs) are widely used due to many inherent characteristics over the serial manipulator. These configurations accuracy are very high due to non-cumulative joint errors as compared to serial manipulators. Its payload-to-weight ratio and structural rigidity is relatively high because load is carried out by several links in parallel. In parallel manipulator the effort is designing one kinematic chain which is usually repeated symmetrically for whole robot. In conventional machine tools like conventional drilling machine, inclined holes are difficult to be drilled with higher accuracy without orienting the work piece. Moreover, machining on the inclined surfaces is also difficult conventionally. The machining on prismatic surfaces of work part with machine orientation is possible using parallel configuration as one of good alternative. Parallel manipulator survey for industrial applications, space explorations, food industries, medical surgery equipments, mining and many more is as reported in [6]. The exponential growth of publication on parallel robots in the last five years points to the potential embedded in this structure that has not yet been fully exploited. The parallel architecture can also be used for earth quake motion simulator [7]. The inverse kinematics of new 3-PRS configuration was carried out and same was simulated using ADAMS software. The kinematic analysis results were compared and reported [8]. Simulation of 3-RPR, 3-UPS and 3-RPS was carried out to determine the torque requirement at time of machining for single and two links linear actuation simultaneously [9]. Isotropic or singularity condition using the variation of kinematic condition index (KCI) from condition number of Jacobian matrix (J) was determined after deriving the kinematic equations of parallel manipulators [10]. Singularity analysis of multi loop platform was investigated and geometric condition based on the concept of common tangent was highlighted by D. Basu and A. Ghosal [11]. Forward and inverse kinematic, dexterity characteristics is investigated and reachable workspace is generated for the proposed three degree of freedom 3-PRC (Prismatic-Revolute- Cylindrical) parallel manipulator by Yangmin Li and Qingsong Xu [12]. More recently, direct kinematics closed form solution of a 4PUS + 1PS parallel manipulator using dialytic elimination method to solve univariate eight degree polynomial [13]. The size and shape of workspace of 3-PPSP 6 DOF parallel mechanism was investigated [14] by Whee-kuk Kim et.al. The motivation behind this work stems from the fact that the work part positioning and orientation is not supportive due to geometric limitation for any processing purpose then only alternative left in direction of design change of machine or robotic tool. Here, effort is put up for such processing operations to be performed with parallel manipulator instead of serial manipulation even though existence of closed loop may complicate its design, analysis and control at all level. In this paper parallel manipulator with three degree of freedom is considered for its kinematic investigation.

II. 3-PRS PARALLEL CONFIGURATION

A. Manipulator's Architecture

A TRIPOD consists of different parts like rotary base, fixed base, square cushion rigidly fixed with base, recirculating ball screws, connecting links, pins for joints, movable platform with attached tool. In each limb of the manipulator, ball screw joint with translational motion acts as prismatic joint and generated link motion transmission through revolute joint and spherical joint at last gives a pose for a particular tool attached with moving platform. The coupled motion of each limb for the 3-PRS configuration is responsible for final pose of the tool, which is useful for various manufacturing operations as well as in medical science for surgery. The degree of freedom (DOF) of the 3- PRS mechanism is calculated by using Grübler–Kutzbach criterion:

$$\mathrm{DOF} = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i \tag{1}$$

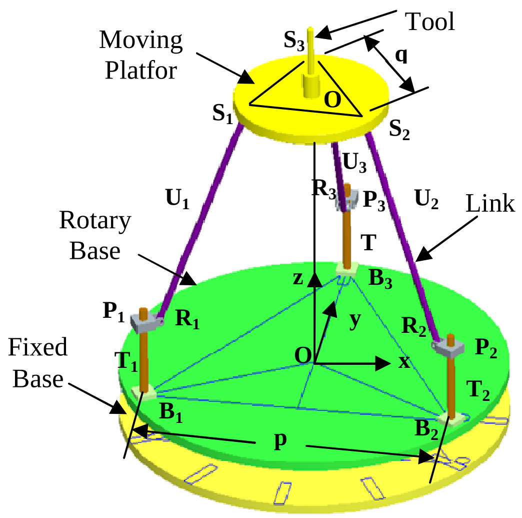

Fig. 1 Tripod with 3-PRS configuration and rotary base

When no relative motion between fixed and rotary base,

$= 6(8 - 9 - 1) + (1 \times 3 + 1 \times 3 + 3 \times 3) = 3$

Here, λ is the degree of freedom of the space utilized for mechanism while in operation. In the above configuration for spatial motion $\lambda = 6$, n is the number of links of mechanism; j is the number of joints of mechanism for; f_i is the degree of freedom of i^{th} joints. Therefore, the moving platform has instantaneously two rotational degrees of freedom and one translational degree of freedom along z-axis. The mechanism with active rotary base has 4-DOF as $n = 9$, (3 recirculating ball screws, 3 connecting links, 1 moving platform, 1 rotary base and 1 fixed base for this parallel architecture) and $j = 10$. Spatial 3 – DOF parallel manipulator as shown in fig.1 consists of a top, moving platform connected to a bottom base platform with three 'legs'. Each limb of symmetric parallel configuration consists of a prismatic, rotary and a spherical joint (P - R - S). Recirculating ball screws are utilized as prismatic joints for the manipulator. Three servo motors are attached for these screw joints actuation and all other joints are considered passive. It may be noted that this is a fully parallel manipulator, since each limb has only one actuated joint. By actuating the recirculating ball screws, the tilting motion of the top platform can be obtained which is connected through three spherical joints. For kinematic investigation of a manipulator, the rotary base is considered as a passive joint but the same is active for work space generation.

B. Formulations of loop closure equations

Because the motions of the links are constrained by the revolute joints, the connecting links R_1S_1, R_2S_2 and R_3S_3 can only rotate on their corresponding fixed planes defined by set of points $\{B_1, R_1, S_1\}, \{B_2, R_2, S_2\}$ and $\{B_3, R_3, S_3\}$.

Three loop closure equations formed for symmetric parallel architecture as shown in fig. 1 are for the loops $OO_1S_1R_1P_1B_1O$, $OO_1S_2R_2P_2B_2O$ and $OO_1S_3R_3P_3B_3O$. Consider the plane $OO_1S_1R_1P_1B_1O$, in which a loop 1 is investigated for the kinematic analysis point of view.

On XY-plane of the rotary base as shown in fig. 3,

$$\overrightarrow{R_1S_1} = -\frac{\sqrt{3}}{2}cos\theta_1\,\hat{\imath} + \frac{1}{2}cos\theta_1\hat{\jmath} - sin\theta_1\hat{k} \tag{2}$$

$$\overrightarrow{R_2S_2} = \frac{\sqrt{3}}{2}cos\theta_2\,\hat{\imath} + \frac{1}{2}cos\theta_2\hat{\jmath} - sin\theta_2\hat{k} \tag{3}$$

$$\overrightarrow{R_3S_3} = -cos\,\theta_3\,\hat{\imath} - sin\theta_3\,\hat{k} \tag{4}$$

Where, i, j and k represents the unit vectors along X, Y and Z axes respectively. The right handed Cartesian coordinate system is placed on the base at a centre of equilateral triangle. θ_i is the inclined angle with the XY plane and U_i as shown in fig. 1. φ is the angle of bisection of base platform with a magnitude of 30° due to equilateral triangle configuration between legs of the tripod. When the feeds of the screws (the position of the sliders) are given, the direct kinematics problem is defined to find the position and orientation of the moving platform. The actuated joints are the prismatic (P) joints. The linear displacement of nut is represented by T_i. Hence, $T = [T_1\ T_2\ T_3]^T$ be the translational vector of three actuated joint variables in the direction of z- axis only. The position vectors OP_1, OP_2 and OP_3 can be expressed as:

$$\overrightarrow{OP_1} = -\frac{p}{\sqrt{3}}cos30^\circ\hat{\imath} - \frac{p}{\sqrt{3}}sin30^\circ\hat{\jmath} + T_1\hat{k} \tag{5}$$

$$\overrightarrow{OP_2} = \frac{p}{\sqrt{3}}cos30^\circ\hat{\imath} - \frac{p}{\sqrt{3}}sin30^\circ\hat{\jmath} + T_2\hat{k} \tag{6}$$

$$\overrightarrow{OP_3} = \frac{p}{\sqrt{3}}\hat{\jmath} + T_3\hat{k} \tag{7}$$

Where, p is the distance between two prismatic joints on fixed base. The constraint equations of the 3-RPS configuration with a rotary base should satisfy the following three constraint equations invariably with the fixed length L of the link R_iS_i.

$$\overrightarrow{OO_1} + \overrightarrow{O_1S_i} = \overrightarrow{OB_i} + \overrightarrow{B_iP_i} + \overrightarrow{P_iR_i} + \overrightarrow{R_iS_i} = \overrightarrow{X_i} \tag{8}$$

Where, $i = 1, 2, 3$

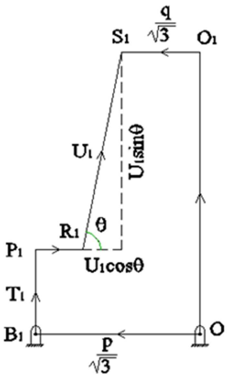

Fig. 2 Loop - 1 on plane $OB_1P_1R_1S_1O_1O$

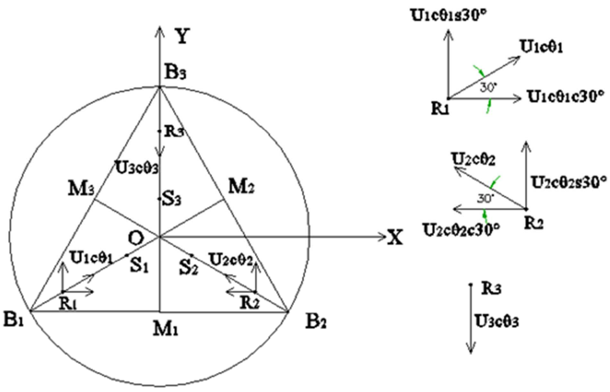

Fig. 3 Resolving the link length R_iS_i on XY-plane

With reference to configuration as shown in the fig. 2 and fig. 3,

$$|OB_1| = |OB_2| = |OB_3| = \frac{p}{\sqrt{3}}$$

$$|OM_1| = |OM_2| = |OM_3| = \frac{p}{2\sqrt{3}}$$

The equation (8) is represented in matrix form as,

$$\overrightarrow{X_1} = \begin{bmatrix} -\frac{p}{2}+\frac{\sqrt{3}}{2}b+\frac{\sqrt{3}}{2}U\cos\theta_1 \\ -\frac{p}{2\sqrt{3}}+\frac{b}{2}+\frac{1}{2}U\cos\theta_1 \\ T_1+U\sin\theta_1 \end{bmatrix}$$

$$\overrightarrow{X_2} = \begin{bmatrix} \frac{p}{2}-\frac{\sqrt{3}}{2}b-\frac{\sqrt{3}}{2}U\cos\theta_2 \\ -\frac{p}{2\sqrt{3}}+\frac{b}{2}+\frac{1}{2}U\cos\theta_2 \\ T_2+U\sin\theta_2 \end{bmatrix}$$

$$\overrightarrow{X_3} = \begin{bmatrix} 0 \\ \frac{p}{\sqrt{3}} - b - Ucos\theta_3 \\ T_3 + Usin\theta_3 \end{bmatrix}$$

In other words,

$$\left|\overrightarrow{OO_1} + \overrightarrow{O_1S_i}\right|^2 = \left|\overrightarrow{X_i}\right|^2 \quad (9)$$

The distance between any two centers of spherical joints at moving platform is equal to q. At last, the constraint equations can be rewritten in following form using (9),

$$\left|\overrightarrow{S_1S_2}\right|^2 = \left|\overrightarrow{O_1S_1} - \overrightarrow{O_1S_2}\right|^2 = \left|\overrightarrow{X_1} - \overrightarrow{X_2}\right|^2 = q^2 \quad (10a)$$

$$\left|\overrightarrow{S_2S_3}\right|^2 = \left|\overrightarrow{O_1S_2} - \overrightarrow{O_1S_3}\right|^2 = \left|\overrightarrow{X_2} - \overrightarrow{X_3}\right|^2 = q^2 \quad (10b)$$

$$\left|\overrightarrow{S_3S_1}\right|^2 = \left|\overrightarrow{O_1S_3} - \overrightarrow{O_1S_1}\right|^2 = \left|\overrightarrow{X_3} - \overrightarrow{X_1}\right|^2 = q^2 \quad (10c)$$

Substituting (8) into (10a), the following form of the equations can be derived:

$$T_1^2\text{-}2T_1T_2+2T_1U\sin\theta_1\text{-}2T_1U\sin\theta_2+T_2^2\text{-}2T_2U\sin\theta_1+ 2T_2U\sin\theta_2+U^2\cos^2\theta_1+U^2\cos\theta_1\cos\theta_2+U^2\cos^2\theta_2+ U^2\sin^2\theta_1\text{-}2U^2\sin\theta_1\sin\theta_2+U^2\sin^2\theta_2+3bU\cos\theta_1+ 3bU\cos\theta_2\text{-}\sqrt{3}Up\cos\theta_1\text{-}\sqrt{3}Up\cos\theta_2+3b^2\text{-}2\sqrt{3}bp+p^2=q^2 \quad (11a)$$

$$T_2^2\text{-}2T_2T_3+2T_2U\sin\theta_2\text{-}2T_2U\sin\theta_3+T_3^2\text{-}2T_3U\sin\theta_2+ 2T_3U\sin\theta_3+U^2\cos^2\theta_2+U^2\cos\theta_2\cos\theta_3+U^2\cos^2\theta_3+ U^2\sin^2\theta_2\text{-}2U^2\sin\theta_2\sin\theta_3+U^2\sin^2\theta_3+3bU\cos\theta_2+ 3bU\cos\theta_3\text{-}\sqrt{3}Up\cos\theta_2\text{-}\sqrt{3}Up\cos\theta_3+3b^2 - 2\sqrt{3}bp + p^2 = q^2 \quad (11b)$$

$$T_1^2\text{-}2T_1T_3+2T_1U\sin\theta_1\text{-}2T_1U\sin\theta_3+T_3^2\text{-}2T_3U\sin\theta_1+ 2T_3U\sin\theta_3+U^2\cos^2\theta_1+U^2\cos\theta_1\cos\theta_3+U^2\cos^2\theta_3+ U^2\sin^2\theta_1\text{-}2U^2\sin\theta_1\sin\theta_3+U^2\sin^2\theta_3+3bU\cos\theta_1+ 3bU\cos\theta_3\text{-}\sqrt{3}Up\cos\theta_1\text{-}\sqrt{3}Up\cos\theta_3+3b^2\text{-}2\sqrt{3}bp+p^2=q^2 \quad (11c)$$

Equation (11) can be rewritten as the following form, the derived equations represents more general case as reported by Meng-Shiun Tsai et al. [15]:

$$C_{11}\, c\theta_1 + C_{12}c\theta_2 + C_{13}s\theta_1 + C_{14}s\theta_2 + C_{15}c\theta_1c\theta_2 + C_{16}s\theta_1s\theta_2 + C_{17} = 0 \quad (12a)$$

$$C_{21}\, c\theta_2 + C_{22}c\theta_3 + C_{23}s\theta_2 + C_{24}s\theta_3 + C_{25}c\theta_2c\theta_3 + C_{26}s\theta_2s\theta_3 + C_{27} = 0 \quad (12b)$$

$$C_{31}\, c\theta_1 + C_{32}c\theta_3 + C_{33}s\theta_1 + C_{34}s\theta_3 + C_{35}c\theta_1c\theta_3 + C_{36}s\theta_1s\theta_3 + C_{37} = 0 \quad (12c)$$

Using trigonometric formulations and substituting them in (12a-12c),

$$\left.\begin{aligned} c\theta_i &= \frac{1-x_i^2}{1+x_i^2} \\ s\theta_i &= \frac{2x_i}{1+x_i^2} \end{aligned}\right\}, \text{Where } x_i = \tan(\theta_i/2) \quad (13)$$

$$(L_1 x_2^2+L_2x_2+L_3)x_1^2+(L_4 x_2^2+L_5x_2+L_6)x_1+L_7 x_2^2 +L_8 x_2+L_9=0 \quad (14a)$$

$$(M_1 x_3^2+M_2x_3+M_3)x_2^2+(M_4 x_3^2+M_5x_3+M_6)x_2+M_7 x_3^2 +M_8 x_3+M_9=0 \quad (14b)$$

$$(N_1 x_3^2 + N_2x_3 + N_3)x_1^2 + (N_4 x_3^2 + N_5x_3 + N_6)x_1 + N_7 x_3^2 + N_8 x_3 + N_9 = 0 \quad (14c)$$

III. BEZOUT'S RESULTANT

Suppose, for the given two uni-variate polynomials $f, g \in R(x)\backslash\{0\}$ with $\deg(f) = \mathrm{m}$ and $\deg(g) = \mathrm{n}$. Assuming $\mathrm{m} \geq \mathrm{n}$,

$$f(x) = u_m x^m + u_{m-1}x^{m-1} + \cdots + u_1 x + u_0, \quad u_m \neq 0$$

$$g(x) = v_n x^n + v_{n-1}x^{n-1} + \cdots + v_1 x + v_0, \qquad v_n \neq 0 \tag{15}$$

Bezout matrix [14, 15] of f and g is defined by $\mathrm{n} \times \mathrm{n}$ symmetric matrix $\widehat{\mathrm{B}}(f, g) = \left(\widehat{\mathrm{b}}_{\mathrm{ij}}\right)_{\mathrm{i,j=0}}^{\mathrm{n-1}}$ is expressed as

$$\widehat{\mathrm{b}}_{\mathrm{ij}} = \left|\mathrm{u_0 v_{i+j-1}}\right| + \left|\mathrm{u_1 v_{i+j-2}}\right| + \cdots + \left|\mathrm{u_k v_{i+j+k-1}}\right|$$

Where, $\left|\mathrm{u_p v_q}\right| = \mathrm{u_p v_q - u_q v_p}, \quad \mathrm{k = \min(i-1, j-1)}$) and $\mathrm{v_p} = 0$, if $\mathrm{p} > n$

OR

The Bezout's resultant $R(f, g) = R_{i,j}(f, g)$ is the symmetric $\mathrm{n} \times \mathrm{n}$ matrix is defined by,

$$R_{ij}(f, g) = \sum Det \begin{vmatrix} u_p & u_q \\ v_p & v_q \end{vmatrix}$$

Here, i , j=0, 1, ….n-1 and summation on right side of the resultant is taken over all for p and q for which $p \geq \max(n-i, n-j)$ and $p + q = 2n - i - j - 1$.

The equations (14a) and (14c) can be represented as,

$$u_2 {x_1}^2 + u_1 x_1 + u_0 = 0$$

$$v_2 {x_1}^2 + v_1 x_1 + v_0 = 0 \tag{16}$$

Eliminating x_1 using Bezout's approach for $\mathrm{m} = \mathrm{n} = 2$,

$$\mathrm{B}_2(f, g) = \begin{bmatrix} u_{[2v1]} & u_{[2v0]} \\ u_{[2v0]} & u_{[1v0]} \end{bmatrix} \tag{17}$$

$$= \begin{bmatrix} u_2 v_1 - v_2 u_1 & u_2 v_0 - v_2 u_0 \\ u_2 v_0 - v_2 u_0 & u_1 v_0 - v_1 u_0 \end{bmatrix} = \begin{bmatrix} \mathrm{y_1} & \mathrm{y_2} \\ \mathrm{y_2} & \mathrm{y_3} \end{bmatrix}$$

Where,

$$u_2 = \mathrm{L_1\, x_2}^2 + \mathrm{L_2 x_2} + \mathrm{L_3}\ , u_1 = \mathrm{L_4\, x_2}^2 + \mathrm{L_5 x_2} + \mathrm{L_6}\ ,$$

$$u_0 = \mathrm{L_7\, x_2}^2 + \mathrm{L_8\, x_2} + \mathrm{L_9}\ , v_2 = \mathrm{N_1\, x_3}^2 + \mathrm{N_2 x_3} + \mathrm{N_3}\ ,$$

$$v_1 = \mathrm{N_4\, x_3}^2 + \mathrm{N_5 x_3} + \mathrm{N_6}\ , v_0 = \mathrm{N_7\, x_3}^2 + \mathrm{N_8\, x_3} + \mathrm{N_9}$$

Using formulation (17),

$$y_1 = A_1 {x_2}^2 {x_3}^2 + A_2 {x_2}^2 x_3 + A_3 {x_2}^2 + A_4 x_2 {x_3}^2 + A_5 x_2 x_3 + A_6 x_2 + A_7 {x_3}^2 + A_8 x_3 + A_9 \tag{18a}$$

$$y_2 = B_1 {x_2}^2 {x_3}^2 + B_2 {x_2}^2 x_3 + B_3 {x_2}^2 + B_4 x_2 {x_3}^2 + B_5 x_2 x_3 + B_6 x_2 + B_7 {x_3}^2 + B_8 x_3 + B_9 \tag{18b}$$

$$y_3 = C_1 {x_2}^2 {x_3}^2 + C_2 {x_2}^2 x_3 + C_3 {x_2}^2 + C_4 x_2 {x_3}^2 + C_5 x_2 x_3 + C_6 x_2 + C_7 {x_3}^2 + C_8 x_3 + C_9 \tag{18c}$$

Using the determinant of the 2×2 Bezout's matrix of (17), the resulting equation becomes (19) and using it with (14b),

$$I_5 {x_2}^4 + I_4 {x_2}^3 + I_3 {x_2}^2 + I_2 x_2 + I_1 = 0 \tag{19}$$

$$J_3 {x_2}^2 + J_2 x_2 + J_1 = 0 \tag{14b}$$

Where, $J_3 = M_1\, {x_3}^2 + M_2 x_3 + M_3$

$$J_2 = M_4\, {x_3}^2 + M_5 x_3 + M_6$$

$$J_1 = M_7\, {x_3}^2 + M_8\, x_3 + M_9$$

Bezout's matrix for these non linear simultaneous polynomial equations with coefficients I and J with different degrees $\mathrm{m} = 4$ and $\mathrm{n} = 2$ is represented in following matrix, the resultant is determinant of the matrix,

$$\begin{bmatrix} I_5 J_1 - I_3 J_3 & I_4 J_1 - I_3 J_2 - I_2 J_3 & -I_2 J_2 - I_1 J_3 & -I_1 J_2 \\ I_5 J_2 - I_4 J_3 & I_5 J_1 - I_3 J_3 & -I_2 J_3 & -I_1 J_3 \\ J_3 & J_2 & J_1 & 0 \\ 0 & J_3 & J_2 & J_1 \end{bmatrix} \tag{20}$$

$${I_1}^2 {J_3}^4 - \left(I_1 I_2 J_2 + 2 I_1 I_3 J_1 - {I_2}^2 J_1\right) {J_3}^3 + \left(I_1 I_3 {J_2}^2 + 3 I_1 I_4 J_1 J_2 + 2 I_1 I_5 {J_1}^2 - I_2 I_3 J_1 J_2 - 2 I_2 I_4 {J_1}^2 + {I_3}^2 {J_1}^2\right) {J_3}^2 - \left(4 I_1 I_5 J_1 {J_2}^2 - I_2 I_4 J_1 {J_2}^2 - 3 I_2 I_5 {J_1}^2 J_2 - I_3 I_4 {J_1}^2 J_2 - 2 I_3 I_5 {J_1}^3 + I_1 I_4 {J_2}^3 + {I_4}^2 {J_1}^3\right) J_3 + I_1 I_5 {J_2}^4 - I_2 I_5 J_1 {J_2}^3 + I_3 I_5 {J_1}^2 {J_2}^2 - I_4 I_5 {J_1}^3 J_2 + {I_5}^2 {J_1}^4 = 0 \tag{21}$$

The above equation is 16th order equation containing only a variable x_3. Solution of the equation gives 16 roots containing positive, negative and imaginary roots. The imaginary and negative values of roots are discarded as the physical configurations are not possible. Two values of the variable x_2 *and* x_1 is computed for the corresponding value of x_3. There are 16 values of x_3 variables, out of which 4 are real positives, which are considered for further computation. Using expression (16) and (19), the value of x_1 and x_2 is calculated respectively.

IV. SINGULARITY ANALYSIS

In general, the three loop closure equations (11a) to (11c) is represented as,

$$\mathrm{g_i}(\mathrm{T_1}, \mathrm{T_2}, \mathrm{T_3}, \theta_1, \theta_1, \theta_3) = 0, \quad \mathrm{i} = 1,2,3 \tag{22}$$

Where, $\mathrm{T_1}, \mathrm{T_2}$ and $\mathrm{T_3}$ are active and $\theta_1, \theta_1, \theta_3$ are passive joint variables

By differentiating loop closure equations, the joint rates relationship is determined.

$$\frac{\partial g_i}{\partial T_i}\frac{\partial T_i}{\partial t} + \frac{\partial g_i}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} = 0 \tag{23}$$

In matrix form the relation is represented as,

$$\begin{bmatrix} \frac{\partial g_1}{\partial T_1} & \frac{\partial g_1}{\partial T_2} & \frac{\partial g_1}{\partial T_3} \\ \frac{\partial g_2}{\partial T_1} & \frac{\partial g_2}{\partial T_2} & \frac{\partial g_2}{\partial T_3} \\ \frac{\partial g_3}{\partial T_1} & \frac{\partial g_3}{\partial T_2} & \frac{\partial g_3}{\partial T_3} \end{bmatrix} \begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \end{bmatrix} + \begin{bmatrix} \frac{\partial g_1}{\partial \theta_1} & \frac{\partial g_1}{\partial \theta_2} & \frac{\partial g_1}{\partial \theta_3} \\ \frac{\partial g_2}{\partial \theta_1} & \frac{\partial g_2}{\partial \theta_2} & \frac{\partial g_2}{\partial \theta_3} \\ \frac{\partial g_3}{\partial \theta_1} & \frac{\partial g_3}{\partial \theta_2} & \frac{\partial g_3}{\partial \theta_3} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = 0 \tag{24}$$

The above relation is expressed as,

$$[J][\dot{T}] + [J^*][\dot{\theta}] = 0 \tag{25}$$

Where, $[J]$ is jacobian matrix of dimension 3×3. $[J^*]$ is also a square matrix of the same dimension for the configuration under consideration. The jacobian matrix $[J]$

is determined by differentiating three equations (11a) to (11c) by active joint variables T_i,

$$[J] = \begin{bmatrix} A_{11} & A_{12} & 0 \\ 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} \end{bmatrix} \qquad (26a)$$

Where,

$$A_{11} = 2T_1 - 2T_2 + 2Us\theta_1 - 2Us\theta_2$$
$$A_{12} = 2T_2 - 2T_1 - 2Us\theta_1 + 2Us\theta_2$$
$$A_{22} = 2T_2 - 2T_3 + 2Us\theta_2 - 2Us\theta_3$$
$$A_{23} = 2T_3 - 2T_2 - 2Us\theta_2 + 2Us\theta_3$$
$$A_{31} = 2T_1 - 2T_3 + 2Us\theta_1 - 2Us\theta_3$$
$$A_{32} = 2T_3 - 2T_1 - 2Us\theta_1 + 2Us\theta_3$$

By differentiating loop closure equations (12a to 12c) with respect to passive joint variables θ_i, the three equations can be written in matrix form as follows and the matrix is denoted by $[J^*]$

$$[J^*] = \begin{bmatrix} B_{11} & B_{12} & 0 \\ 0 & B_{22} & B_{23} \\ B_{31} & 0 & B_{33} \end{bmatrix} \qquad (26b)$$

Where,

$$B_{11} = C_{13}\,c\theta_1 - C_{11}s\theta_1 - C_{15}c\theta_2 s\theta_1 + C_{16}c\theta_1 s\theta_2$$
$$B_{12} = C_{14}c\theta_2 - C_{12}s\theta_2 - C_{15}c\theta_1 s\theta_2 + C_{16}c\theta_2 s\theta_1$$
$$B_{22} = C_{23}c\theta_2 - C_{21}s\theta_2 - C_{25}c\theta_3 s\theta_2 + C_{26}c\theta_2 s\theta_3$$
$$B_{23} = C_{24}c\theta_3 - C_{22}s\theta_3 - C_{25}c\theta_2 s\theta_3 + C_{26}c\theta_3 s\theta_2$$
$$B_{31} = C_{33}c\theta_1 - C_{31}s\theta_1 - C_{35}c\theta_3 s\theta_1 + C_{36}c\theta_1 s\theta_3$$
$$B_{33} = C_{34}c\theta_3 - C_{32}s\theta_3 - C_{35}c\theta_1 s\theta_3 + C_{36}c\theta_3 s\theta_1$$

The obtained jacobian matrix $[J^*]$ is same as reported by D. Basu and A. Goshal [11]. By analyzing the Jacobian matrix of a manipulator, the singular configurations of the robot are determined. The determinant of jacobian matrix equal to zero represents the manipulator with a singular configuration. Usually, the mobility of a manipulator is reduced near a singularity.

Case studies:

Assumed configuration parameters are: p=0.75; U=0.482; q=0.3; b=0.041; T_1=0.16; T_2=0.16; T_3=0.16; TL=0.175

Case 1: Only one screw is actuated by 0.02 units *which is T_1=0.14 unit from base* (Refer fig. 1)

The resultant 16th order equation is,

$$23.4602x_3^{16}+4.2382\,x_3^{15}-0.5525x_3^{14}-2.2488x_3^{13}$$
$$-14.5661x_3^{12}-0.3318x_3^{11}+4.7297x_3^{10}+0.2469x_3^{9}$$
$$+0.3063x_3^{8}+0.0063x_3^{7}-0.1409\,x_3^{6}-0.0053x_3^{5}$$
$$-0.0129x_3^{4}-0.0004x_3^{3}=0 \qquad (27)$$

TABLE 1: PASSIVE JOINT ANGLES FOR SINGLE ACTUATION

θ_1	θ_2	θ_3	det_J	det_J*
62.8949	***63.0021***	***63.0021***	***0***	***0.0221***
23.5081	20.8027	63.0021	0	0.0031
62.8950	63.0021	63.0021	0	0.0221
23.5080	20.8027	63.0021	0	0.0031
64.2304	64.2340	61.7091	0	0.0236
22.1054	19.4738	61.7091	0	0.0032
64.2304	61.7091	19.4738	0	-0.0238
-11.0281	-13.6969	19.4738	0	-0.0025

Case 2*: All screws are actuated with different magnitudes of 0.3, 0.4 and 0.2 units from assumed level.*

$$23.4870\,x_3^{16}+6.3613\,x_3^{15}-0.2221\,x_3^{14}-3.3540x_3^{13}$$
$$-14.7671x_3^{12}-0.5088x_3^{11}+4.7399x_3^{10}+0.3713x_3^{9}$$
$$+0.3135x_3^{8}+0.00096\,x_3^{7}-0.1410x_3^{6}-0.0079x_3^{5}$$
$$-0.0130x_3^{4}-0.0007x_3^{3}=0 \qquad (28)$$

TABLE 2: PASSIVE JOINT ANGLES FOR MULTIPLE ACTUATIONS WITH DIFFERENT MAGNITUDES

θ_1	θ_2	θ_3	det_J	det_J*
63.0278	62.9491	62.9521	0	0.0221
22.0871	23.4520	62.9521	0	0.0030
63.6114	63.5577	62.3767	0	0.0229
21.4711	22.8173	62.3767	0	0.0031
64.2480	64.2208	61.7189	0	0.0236
20.7884	22.1156	61.7189	0	0.0032
62.3615	63.6265	18.8347	0	-0.0240
-12.9452	-11.6105	18.8347	0	-0.0025

Tool is attached with the moving platform at its centre. Using vector approach, the coordinates of the tool tip is computed for the known value of linear actuation and tool length (TL).

For case 1: *Determination of tool tip coordinates*

The computed angles using Bezout's approach are, $\theta_1 = 62.8949^\circ, \theta_2 = 63.0021^\circ$ and $\theta_3 = 63.0021^\circ$.

Using equation (8), the values of vectors

$\overrightarrow{X_1} = (-0.1493, -0.0862, 0.5691)$,

$\overrightarrow{X_2} = (0.15, -0.0866, 0.5895)$ and

$\overrightarrow{X_3} = (0, 0.1732, 0.5895)$. Each vector represents point lies on the plane or plane parallel to the tilted moving platform. The tilt of the moving platform with reference to base can be computed from the angle between normal vector of moving and base platform. The normal vector is the cross product between the two vectors lies in that plane. The normalized normal vector is $-0.0680\,\vec{\imath} - 0.0393\,\vec{\jmath} + 0.9969\,\vec{k}$. Tool tip coordinates are determined using centre point coordinates, tool length and normalized normal vector. As shown in fig. 4,

$$A_2 = \frac{\overrightarrow{X_2} + \overrightarrow{X_3}}{2}, B_2 = \frac{\overrightarrow{X_1} + \overrightarrow{X_3}}{2}$$

$$nA = [(\vec{B}_2 - \vec{B}_1) \times (\vec{A}_1 - \vec{B}_1)] \cdot [(\vec{A}_2 - \vec{A}_1) \times (\vec{B}_2 - \vec{B}_1)]$$
$$nB = [(\vec{A}_2 - \vec{A}_1) \times (\vec{A}_1 - \vec{B}_1)] \cdot [(\vec{A}_2 - \vec{A}_1) \times (\vec{B}_2 - \vec{B}_1)]$$
$$d = [(\vec{A}_2 - \vec{A}_1) \times (\vec{B}_2 - \vec{B}_1)] \cdot [(\vec{A}_2 - \vec{A}_1) \times (\vec{B}_2 - \vec{B}_1)]$$

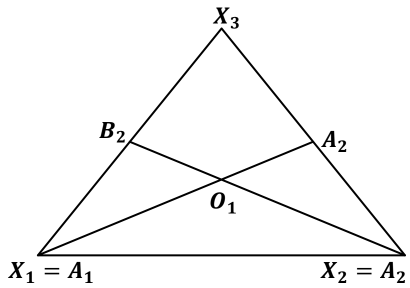

Fig. 4 Top view of moving platform

The Moving platform centre coordinates are,

$$O_1 = \vec{A}_1 + (nA/d)(\vec{A}_2 - \vec{A}_1) \tag{30}$$

Tool tip coordinates are found out using tool length, centre coordinates of moving platform and normalized normal vector of moving platform as,

$$\vec{O}_{tip} = \vec{O}_1 + TL\left(\frac{\overrightarrow{X_1X_2} \times \overrightarrow{X_1X_3}}{|\overrightarrow{X_1X_2} \times \overrightarrow{X_1X_3}|}\right) \tag{31}$$

Tool tip coordinates (-0.0117, -0.0067, 0.7571) are determined using the coordinates of centre point O_1, for a specified tool length and direction cosines of the normal vector N_2. The another simple approach is determination of centre of a circle inscribed in equilateral triangle,

$$O_1 = \frac{\vec{s_1} + \vec{s_2} + \vec{s_3}}{3} \tag{32}$$

Workspace is developed using point cloud of such tool tip coordinates subsequently. Synchronized tool tip coordinates for its position and orientation is important during the machining while working on prismatic surface with contouring.

V. SYLVESTER'S METHOD

Two different non-linear simultaneous equations with degree $m = 4$ and $n = 2$, the corresponding Sylvester's matrix is:

$$S_{p,q} = \begin{bmatrix} I_5 & I_4 & I_3 & I_2 & I_1 & 0 \\ 0 & I_5 & I_4 & I_3 & I_2 & I_1 \\ J_3 & J_2 & J_1 & 0 & 0 & 0 \\ 0 & J_3 & J_2 & J_1 & 0 & 0 \\ 0 & 0 & J_3 & J_2 & J_1 & 0 \\ 0 & 0 & 0 & J_3 & J_2 & J_1 \end{bmatrix} \tag{33}$$

Here, one can easily see that in Bezout's approach the size of the matrix is 4×4, but in Sylvester's theory the size of the matrix is 6×6. So the computation process for solving Sylvester's matrix is too lengthy and time consuming compared to that of the Bezout's matrix. The result obtained through this method is also same as Bezout's resultant.

VI. WORKSPACE DEVELOPMENT

Parallel manipulators have smaller workspace compared to serial manipulator as found from various literatures [12, 14, 17]. The workspace means set of all spatial coordinates of the centre of the moving platform or tool tip positions for entire working range of active joints actuation. The workspace analysis is always imperative to avoid singular configurations. Moreover, many facts can be observed to enhance the parallel manipulator configuration further. The generated workspace is always constrained due to joint-angle limitations, link-length limitations with regard to structural bending, space constraints or any joint-interference. It is always desirable to analyze the shape and volume of the workspace for the particular application requirements point of view. It is difficult to express complete workspace as it does not reveal actual tool tip orientation information of the machine tool, which is essential for user at time of machining in many cases for physical constraints avoidance. For proposed configuration, position coordinates are captured for limiting range of spherical joints movement to avoid interference between link and mobile platform as well as recirculating ball screws as translational actuators. Linear actuation of individual legs with magnitude T1, T2 and T3 for the range of 0.1 unit with step size of 0.01 unit is applied for determination of tool tip coordinates. The obtained position coordinates (x, y, z) of tool tip are exported to excel program. The coordinate's data are captured for the rotary base positions of (ϕ_0): 0˚, 30˚, 60˚, 90˚, 120˚. A 3-PRS configuration repeats the same coordinates for any angular increment after 120˚ due to its axi-symmetry nature. The workspace of parallel manipulator is developed using such tool tip point clouds process sequentially as shown in fig. 5. Using MATLAB by reading excel file and 3D-surface is generated using surf command as shown in fig. 6. Similarly, linear actuation of pair of legs with magnitude T1 and T2, T2 as well as T3 and T1 simultaneously for same range and step size is applied without time lag for determination of tip coordinates. It means both legs are acuated simultaneously and at same position with reference to fixed base at any time. The tool tip coordinates are captured in excel file. Using MATLAB program, the workspace is developed after sequential processing of tool tip coordinates data captured earlier. It is observed that inner surface is generated through actuation of two screws, while outer surface is generated due to actuation of single leg as shown in fig. 7.

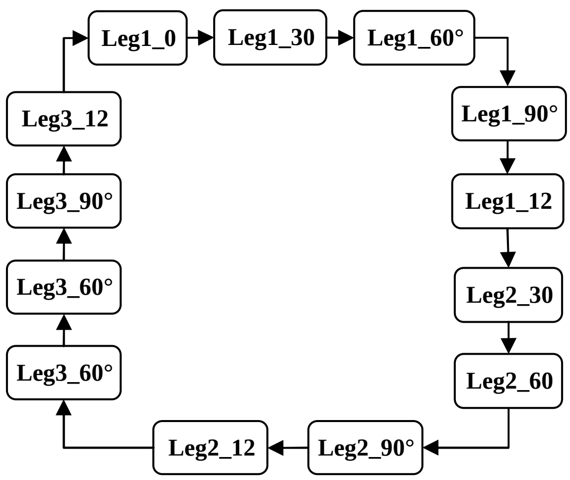

Fig. 5 Sequential processing of captured tip coordinates with single leg actuation for workspace generation

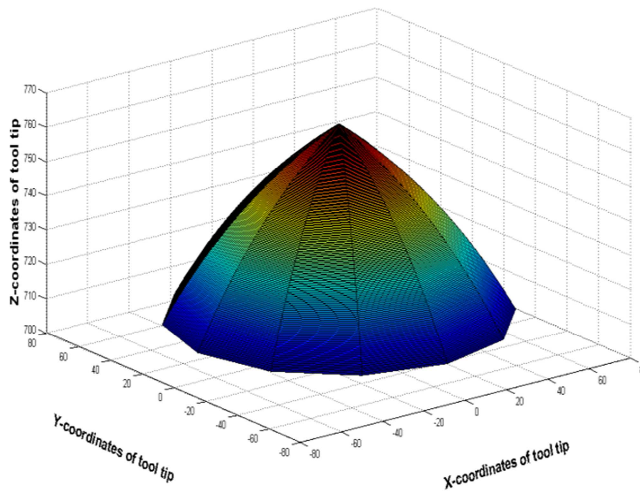

Fig. 6 Work space generation using individual link actuation with constant velocity for 3-PRS

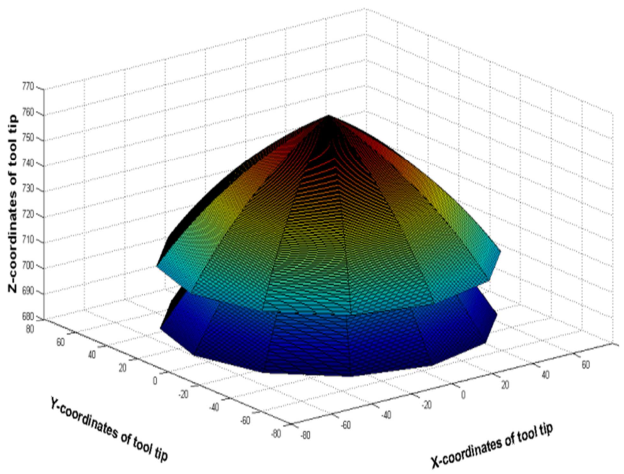

Fig. 7 Workspace for all possible combination of link actuation with constant velocity & without lag for 3-PRS

The generate workspace has almost similar shape as reported by L W Tsai and Sameer Joshi [20] for 3DOF spatial mechanisms.

VII. CONCLUSION

The kinematic analysis of 3-PRS parallel manipulator is carried out using vector algebra and three loop closure equations are formulated. The solutions of these three non linear higher order loop closure equations are worked out using BEZOUT'S resultant and SYLVESTER'S method. The obtained results are same but the Bezout's resultant less time consuming compared Sylvester method due to its higher size matrix problem. The tool-tip coordinates are captured and exported in excel format. The exported results were opened out using file operations in MATLAB software. The 3-dimensional surface of the exported result is plotted, which is the workspace of the 3DOF parallel manipulators end-effector. The singularity analysis is carried out and jacobian matrix is derived. Normally, near the singular configurations parallel manipulator's experiences poor performance. Singularity can be a kind of situation where the manipulator has additional uncontrollable DOF or loss of any existing DOF. Algebraically, singularities represent rank deficiency of Jacobian matrix if two elements of any row or column of $[J^*]$ matrix becomes zero. The parallel manipulator with 3-DOF with rotary base using 3-RPS configuration can be used for machining tasks on prismatic surfaces, inclined drilling, slotting operations with narrow range and in medical science that require high dexterity, high accuracy, high loading capacity and considerable stiffness.

APPEENDIX

Coefficients of (12)

$C_{11} = 3bU - \sqrt{3}\,Up$, $C_{12} = 3bU - \sqrt{3}Up$,

$C_{13} = 2U(T_1 - T_2)$, $C_{14} = -2U(T_1 - T_2)$,

$C_{15} = U^2$, $C_{16} = -2U^2$,

$C_{17} = 2U^2 + p^2 + (T_2 - T_1)^2 + 3b^2 - 2\sqrt{3}pb - q^2$

$C_{21} = 3bU - \sqrt{3}\,Up$, $C_{22} = 3bU - \sqrt{3}Up$,

$C_{23} = 2U(T_2 - T_3)$, $C_{24} = -2U(T_2 - T_3)$

$C_{25} = U^2$, $C_{26} = -2U^2$,

$C_{27} = 2U^2 + p^2 + (T_2 - T_3)^2 + 3b^2 - 2\sqrt{3}pb - q^2$

$C_{31} = 3bU - \sqrt{3}\,Up$, $C_{32} = 3bU - \sqrt{3}Up$,

$C_{33} = 2U(T_1 - T_3)$, $C_{34} = -2U(T_1 - T_3)$

$C_{35} = U^2$, $C_{36} = -2U^2$,

$C_{37} = 2U^2 + p^2 + (T_1 - T_3)^2 + 3b^2 - 2\sqrt{3}pb - q^2$

Coefficients of (14)

$L_1 = -C_{11} - C_{12} + C_{15} + C_{17}$, $L_2 = 2C_{14}$,

$L_3 = -C_{11} + C_{12} - C_{15} + C_{17}$, $L_4 = 2C_{13}$,

$L_5 = 4C_{16}$, $L_6 = L_4$, $L_7 = C_{11} - C_{12} - C_{15} + C_{17}$,

$L_8 = 2C_{14}$, $L_9 = C_{11} + C_{12} + C_{15} + C_{17}$

$M_1 = -C_{21} - C_{22} + C_{25} + C_{27}$, $M_2 = 2C_{24}$,

$M_3 = -C_{21} + C_{22} - C_{25} + C_{27}$, $M_4 = 2C_{23}$,

$M_5 = 4C_{26}$, $M_6 = M_4$, $M_7 = C_{21} - C_{22} - C_{25} + C_{27}$,

$M_8 = 2C_{24}$, $M_9 = C_{21} + C_{22} + C_{25} + C_{27}$

$N_1 = -C_{31} - C_{32} + C_{35} + C_{37}$, $N_2 = 2C_{34}$,

$N_3 = -C_{31} + C_{32} - C_{35} + C_{37}$, $N_4 = 2C_{33}$,

$N_5 = 4C_{36}$, $N_6 = N_4$, $N_7 = C_{31} - C_{32} - C_{35} + C_{37}$,

$N_8 = 2C_{34}$, $N_9 = C_{31} + C_{32} + C_{35} + C_{37}$

Coefficients of (18)

$A_1 = L_1N_4 - L_4N_1$, $A_2 = L_1N_5 - L_4N_2$, $A_3 = L_1N_6 - L_4N_3$

$A_4 = L_2N_4 - L_5N_1$, $A_5 = L_2N_5 - L_5N_2$, $A_6 = L_2N_6 - L_5N_3$

$A_7 = L_3N_4 - L_6N_1$, $A_8 = L_3N_5 - L_6N_2$, $A_9 = L_3N_6 - L_6N_3$

$B_1 = L_1N_7 - L_7N_1$, $B_2 = L_1N_8 - L_7N_2$, $B_3 = L_1N_9 - L_7N_3$

$B_4 = L_2N_7 - L_8N_1$, $B_5 = L_2N_8 - L_8N_2$, $B_6 = L_2N_9 - L_8N_3$

$B_7 = L_3N_7 - L_9N_1$, $B_8 = L_3N_8 - L_9N_2$, $B_9 = L_3N_9 - L_9N_3$

$C_1 = L_4N_7 - L_7N_4$, $C_2 = L_4N_8 - L_7N_5$, $C_3 = L_4N_9 - L_7N_6$

$C_4 = L_5N_7 - L_8N_4$, $C_5 = L_5N_8 - L_8N_5$, $C_6 = L_5N_9 - L_8N_6$

$C_7 = L_6N_7 - L_9N_4$, $C_8 = L_6N_8 - L_9N_5$, $C_9 = L_6N_9 - L_9N_6$

Coefficients of (19)

$I_5 = Z_1 {x_3}^4 + Z_2 {x_3}^3 + Z_3 {x_3}^2 + Z_4 x_3 + Z_5$

$I_4 = Z_6 {x_3}^4 + Z_7 {x_3}^3 + Z_8 {x_3}^2 + Z_9 x_3 + Z_{10}$

$I_3 = Z_{11} {x_3}^4 + Z_{12} {x_3}^3 + Z_{13} {x_3}^2 + Z_{14} x_3 + Z_{15}$

$I_2 = Z_{16} {x_3}^4 + Z_{17} {x_3}^3 + Z_{18} {x_3}^2 + Z_{19} x_3 + Z_{20}$

$I_1 = Z_{21} {x_3}^4 + Z_{22} {x_3}^3 + Z_{23} {x_3}^2 + Z_{24} x_3 + Z_{25}$

$Z_1 = A_1 C_1 - B_1 B_1$

$Z_2 = A_1 C_2 + A_2 C_1 - 2B_1 B_2$

$Z_3 = A_1 C_3 + A_2 C_2 + A_3 C_1 - 2B_1 B_3$

$Z_4 = A_2 C_3 + A_3 C_2 - 2B_2 B_3$

$Z_5 = A_3 C_3 - B_3 B_3$

$Z_6 = A_1 C_4 + A_4 C_1 - 2B_1 B_4$

$Z_7 = A_1 C_5 + A_2 C_4 + A_4 C_2 + A_5 C_1 - 2B_1 B_5 - 2B_2 B_4$

$Z_8 = A_1C_6 + A_2C_5 + A_3C_4 + A_4C_3 + A_5C_2 + A_6C_1 - 2B_1B_6 - 2B_2B_5$
$-2B_3B_4$

$Z_9 = A_2 C_6 + A_3 C_5 + A_5 C_3 + A_6 C_2 - 2B_2 B_6 - 2B_3 B_5$

$Z_{10} = A_3 C_6 + A_6 C_3 - 2B_3 B_6$

$Z_{11} = A_1 C_7 + A_4 C_4 + A_7 C_1 - 2B_1 B_7 - B_4 B_4$

$Z_{12} = A_1C_8 + A_2C_7 + A_4C_5 + A_5C_4 + A_7C_2 + A_8C_1 - 2B_1B_8 - 2B_2B_7$
$-2B_4B_5$

$Z_{13} = A_1C_9 + A_2C_8 + A_3C_7 + A_4C_6 + A_5C_5 + A_6C_4 + A_7C_3 + A_8C_2 + A_9C_1$
$-2B_1B_9 - 2B_2B_8 - 2B_3B_7 - 2B_4B_6 - B_5B_5$

$Z_{14} = A_2C_9 + A_3C_8 + A_5C_6 + A_6C_5 + A_8C_3 + A_9C_2 - 2B_2B_9 - 2B_3B_8$
$-2B_5B_6$

$Z_{15} = A_3 C_9 + A_6 C_6 + A_9 C_3 - 2B_3 B_9 - B_6 B_6$

$Z_{16} = A_4 C_7 + A_7 C_4 - 2B_4 B_7$

$Z_{17} = A_4 C_8 + A_5 C_7 + A_7 C_5 + A_8 C_4 - 2B_4 B_8 - 2B_5 B_7$

$Z_{18} = A_4C_9 + A_5C_8 + A_6C_7 + A_7C_6 + A_8C_5 + A_9C_4 - 2B_4B_9 - 2B_5B_8$
$-2B_6B_7$

$Z_{19} = A_5 C_9 + A_6 C_8 + A_8 C_6 + A_9 C_5 - 2B_5 B_9 - 2B_6 B_8$

$Z_{20} = A_6 C_9 + A_9 C_6 - 2B_6 B_9$

$Z_{21} = A_7 C_7 - B_7 B_7$

$Z_{22} = A_7 C_8 + A_8 C_7 - 2B_7 B_8$

$Z_{23} = A_7 C_9 + A_8 C_8 + A_9 C_7 - 2B_7 B_9 - B_8 B_8$

$Z_{24} = A_8 C_9 + A_9 C_8 - 2B_8 B_9$

$Z_{25} = A_9 C_9 - B_9 B_9$

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