Second Order Sliding Mode Control for Single Link Flexible Manipulator

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***Abstract*— This paper presents a second order sliding mode control (SOSMC) for a single link flexible manipulator, which represents a class of under actuated systems. A second order super twisting algorithm (STA) is used to provide better positional accuracy and robustness against parametric variation and external disturbances with alleviation in chattering. A smooth control is synthesized using STA to exploit the robustness properties of sliding-mode controllers to ensure finite time convergence of the states. The flexible manipulator is actuated by a DC motor and the flexibility of the link is modeled as a linear torsional spring with stiffness. Comprehensive comparison between sliding mode control (SMC) and SOSMC is done in this study to show the effectiveness of the proposed strategy.**

Keywords – Flexible Link Manipulator; Higher Order Sliding Mode Control; Super Twisting Algorithm

I. Introduction

Flexible manipulators are robotic manipulators made of light-weight materials due to which, reduction in weight, lower power consumption and faster movement can be realized. Light-weight manipulators can be applied in general industrial processes; e.g. pick and place, in dangerous, monotonous and tedious jobs, in the space shuttle, in biomedical instrumentations etc. However, due to its light-weight, vibrations are inherent in the flexible link manipulator (FLM) that limits its wide applicability. The problem of link flexibility, not only makes the dynamic modeling of FLM very challenging but turns its uncertain behavior at the free end into a complicated control problem. The control strategies applied so far to flexible robots include proportional derivative control, computed torque control, active damping control, adaptive control, neural network based control, lead-lag control, sliding mode control, stable inversion in the frequency domain, stable inversion in the time domain, algebraic control, optimal and robust control, input shaping control and boundary control. On the problem of flexibility, many researchers have tried improving the dynamic models and incorporating different control strategies. However, in spite of all the research devoted to modeling and controlling these kind of robots, [1]- [4], there is no universal solution for the control. A survey paper [5], gives a complete, detailed overview of the work done in this field.

Difference between the actual plant dynamics and its mathematical model used for the design of the controller mostly come from external disturbances, unknown plant parameters, and unmodeled dynamics. Designing control that provides the desired closed-loop system performance in the presence of these disturbances/uncertainties is a challenging task. Hence the development of robust control methods attracted the control community in the modern era. But sliding mode control has been proved to be the most successful approach against the other robust control techniques as adaptive control, optimal control, $H\infty$ control and backstepping technique, etc in handling bounded uncertainties and external disturbances.

Sliding mode control (SMC) systems provide an effective and robust means of controlling nonlinear plants. The main advantage of this control methodology is insensitivity to disturbance and parameter uncertainties. This is achieved by steering the system states to a simple predefined function, called as sliding surface and designing a proper control input to maintain the states on this manifold thereafter. Significant research is been done in this field. The already matured classical SMC theory received a significant boost when new "second order" ideas appeared with "higher order" concepts [6]- [9]. The limitations of the conventional sliding modes are:
1. The classical sliding mode design approach requires sliding variable degree to be equal to one with respect to the control input which constrains the choice of the sliding variable.(Relative degree is the order of the derivative of the constrained variable, in which the control appears explicitly.)
2. SMC yields high frequency switching control action leading "chattering effect", which is difficult to avoid or attenuate.
These limitations of classical SMC are overcome by a generalized (r^{th}) order higher sliding mode controllers (HOSMC). Main features of HOSM are

- This can force the sliding variable and its(r-1)successive derivatives to zero.
- For this approach, there is no restriction on relative degrees.
- As the high frequency control switching is pushed

in the higher derivative of the sliding variable, chattering is significantly reduced.

- No detailed mathematical model of the plant is required.
- As the integration of the signum function is utilized in synthesizing the control, it becomes continuous.

. [6] [7] [9]
Several second order sliding modes control algorithms are introduced such as twisting and super-twisting controllers, the suboptimal control algorithm, the control algorithm with prescribed convergence law and the quasi-continuous control algorithm, etc.

A. Motivation

Conventional control techniques, though easy to design and implement, cannot give robustness against external and internal disturbances and parametric uncertainties. Classical SMC to control single link flexible manipulator (SLFM) has been implemented by many people [10]- [13], but the issue of chattering remains.
The troublesome chatter of SMC can be reduced by HOSMC approach removing the relative-degree restrictions, preserving the advantages of SMC and improving its accuracy. Out of second order sliding modes(SOSM), super-twisting algorithm is conceptually different from the others, as it depends only on the actual value of the sliding constraint, while the others need the first derivative of the sliding variable. Secondly, it is effective only for anti-chattering purposes as far as relative-degree one constraint variables are dealt with. Applications in HOSM are very recent though the theory is old. Hence, in this study, a second order super-twisting controller is used to control the position of the SLFM to reduce the chatter in the control input.

B. Outline of the paper

For a better understanding of this work, it is organized as follows. Starting with the introduction in Section I, physical model of the single-link flexible manipulator is presented in brief in Section II. The HOSM scheme is explained in Section III along with the super twisting algorithm. Section IV elaborates the design of a stable sliding surface and design of a control law. Section V shows the simulation results and Section VI concludes the work.

II. Dynamics of Single link flexible manipulator (SLFM)

In this work a single link manipulator actuated by a DC motor is considered as a plant. Fig.1 shows a schematic diagram of SLFM. Following are the parameters and constants of the plant:
α : Tip deflection (deg.),
θ : Motor shaft position (deg.),
T_l : Load Torque (N.m),
B_{eq} : Viscous damping coefficient,
K_{stiff} : Total stiffness of model (N.m/deg.).
J_{link} and J_{eq} : Moment of inertia of link and equivalent moment of inertia of the model resp in ($Kg\ m^2$).
Due to the flexible nature of the link, when it is actuated by an angle θ at the motor end, the tip gets displaced by an angle α, as shown in the diagram. Assuming, tip deflection angle α to be small, it is approximated as
$\alpha = \frac{D}{L}$, where D: Displacement of the tip of SLFM and L: Length of the link.
The flexibility of the link is modeled as a linear torsional spring with stiffness K.

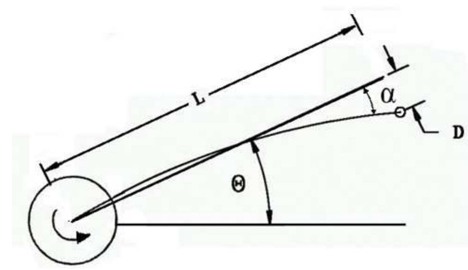

Fig. 1: Schematic diagram of flexible link manipulator

Courtesy:Quanser Manual

The equations of motion for this system, are taken from [14]. The system dynamics is obtained using the Euler-Lagrange formulation. The state space representation of SFLM is written as

$$\ddot{\theta} = \frac{T_l}{J_{eq}} - \frac{B_{eq}\dot{\theta}}{J_{eq}} + \frac{K_{stiff}\alpha}{J_{eq}} \quad (1)$$

$$\ddot{\alpha} = -K_{stiff}(\frac{1}{J_{eq}} + \frac{1}{J_{link}})\alpha + \frac{B_{eq}}{J_{eq}}\dot{\theta} - \frac{T_l}{J_{eq}}. \quad (2)$$

(1) and (2) represent the dynamics of FLM.
DC motor is used to generate the necessary torque. The schematic is shown in Fig. 2.

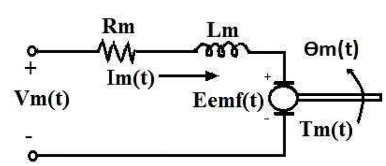

Fig. 2: DC Motor Model

Courtesy:Quanser Manual

Substituting the value of load torque from actuator model,

$$\ddot{\theta} = -p_1\dot{\theta} + p_2\alpha + p_3V_m \quad (3)$$

$$\ddot{\alpha} = p_1\dot{\theta} - p_4\alpha - p_3V_m \quad (4)$$

where V_m is the motor input voltage.
The nominal values are taken from [14] as $B_{eq} = 4 * 10^{-3}$

, J_{link} = 1.3978 , J_{eq}= 0.002 and K_{stiff}= 0.0057.
p_1, p_2, p_3, p_4 are constants obtained from the data of the model.
Defining $x_1 = \theta$, $x_2 = \alpha$, $x_3 = \dot{\theta}$, $x_4 = \dot{\alpha}$ and $V_m = u$ and using (3) and (4), the system is represented as

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= p_2 x_2 - p_1 x_3 + p_3 u \\ \dot{x}_4 &= -p_4 x_2 + p_1 x_3 - p_3 u. \end{aligned} \tag{5}$$

The state space representation is written in the compact form as

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + \mathbf{b}u \\ y &= C\mathbf{x} \end{aligned} \tag{6}$$

where, $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 212.7091 & -19.6573 & 0 \\ 0 & -616.9655 & 19.6573 & 0 \end{bmatrix}$

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 34.6024 \\ -34.6024 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The system dynamics with the disturbance can be written as

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}(u + \mathbf{d}) \tag{7a}$$
$$y = C\mathbf{x}. \tag{7b}$$

where 'd' represents lumped disturbance which includes parametric uncertainties and matched external disturbance. Some facts are

1) pair (A,b) is controllable,
2) A, b, C are known matrices ,
3) bounds on the uncertainties are known.

III. Higher Order Sliding Mode

A. Preliminaries

Various methods of chatter reduction have been reported in the literature such as use of sigmoid or saturation function, use of fractional order calculus, use of disturbance observer and many more [15]- [18], but they suffer from the main drawbacks of deterioration of accuracy and system robustness. The higher-order sliding mode approach has been developed over the last two decades not only for chattering attenuation but also for the robust control of uncertain systems with relative degree two and higher.
HOSM actually is a motion on the discontinuity set of a dynamic system understood in Filippov's sense [19]. The sliding order characterizes the dynamics smoothness degree in the vicinity of the mode. A sliding mode $\sigma \equiv 0$ may be classified by the number r, obtained by taking successive derivatives of $\sigma^{(r)}$, till u appears explicitly. That number is called the sliding order [6], [7]. The main problem in implementation of HOSMs is increasing information demand, i.e. sliding variable and its derivatives; which are generated through differentiators. Differentiators introduce the measurement noise, whose negative effects on the overall closed-loop performance dramatically increase with the number of differentiation stages [20], [21]. Examples of r-sliding modes attracting in finite time are known for r = 1 (which is trivial), for r = 2 [8], [19], [22]- [24], and for r = 3 [9]. Arbitrary order sliding controllers with finite-time convergence are recently presented [25], [26]. Generally, any r-sliding controller keeping, sliding surface σ = 0 needs σ, $\dot{\sigma}$,..., σ^{r-1} to be available. The only known exclusion is a "super-twisting" 2-sliding controller, which needs only measurements of σ. 2-sliding mode with respect to the constraint function σ is shown in Fig. 3.

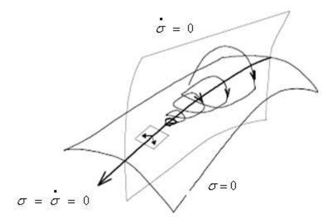

Fig. 3: Second order sliding mode trajectory

Courtesy: Proceeding on VSS10

B. Simplified block diagrams for SMC and HOSM

Consider a nominal system as

$$\dot{x} = A\mathbf{x} \; + B(u + \rho(t, x)), \tag{8}$$

where $\mathbf{x} \in \mathbb{R}^n, u \in \mathbb{R}$ and $\rho(t, x) \leq D_M$ the bounded uncertainty with known bounds. The sliding surface is designed such that $\sigma = C^T(\mathbf{x})$ and during sliding, $\sigma(t, x(t)) \equiv 0$. Differentiating this equation and from (8)

$$\dot{\sigma} = C^T A\mathbf{x} + C^T B u + C^T B \rho(t, x) \tag{9}$$
$$u = (C^T B)^{-1}(-C^T A\mathbf{x}) + \phi(\sigma) \tag{10}$$

Some choices of

$$\begin{aligned} \phi(\sigma) &= -k sgn(\sigma) \\ &= -\lambda\sigma - k sgn(\sigma) \\ &= -k|\sigma|^{1/2} sgn(\sigma). \end{aligned}$$

These are the well known reaching laws. The block diagram for this scheme is depicted in Fig.4.

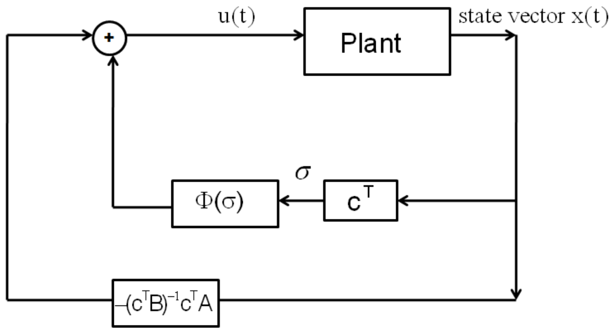

Fig. 4: Simplified representation of SMC scheme

To make this discontinuous control input, a continuous one, an integrator is added to mitigate the chattering. For super twisting algorithm, equation (9) is re-written

$$\dot{\sigma} = \phi(\sigma) + C^T Bu + C^T B\rho(t,x)$$

and this equation has to be finite time stable. Design $\phi(\sigma)$ such that with $\rho(t,x)$, σ goes to zero. With a choice of $\phi(\sigma) = -k|\sigma|^{1/2}sgn(\sigma)$, non-vanishing perturbations can not be rejected, hence it is modified as,

$$\left.\begin{array}{l} \phi(\sigma) = -k_1|\sigma|^{1/2}sgn(\sigma) + z \\ \dot{z} = -k_2 sgn(\sigma) \end{array}\right\} \tag{11}$$

This is a super-twisting algorithm (STA). The block diagram for the same is as shown in Fig.5.

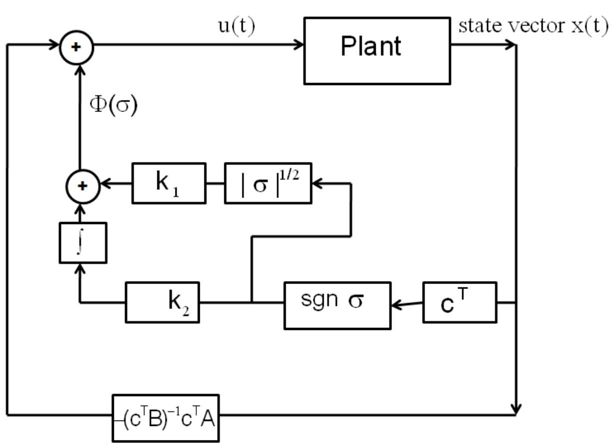

Fig. 5: Simplified representation of STA scheme

C. Super-twisting algorithm

This algorithm has been developed to control systems with relative degree one in order to avoid chattering in variable structure control. Also in this case the trajectories on the 2-sliding plane are characterized by twisting around the origin (Fig.6). The continuous control law u(t) is constituted by two terms; the first is defined by means of its discontinuous time derivative, while the other is a continuous function of the available sliding variable.

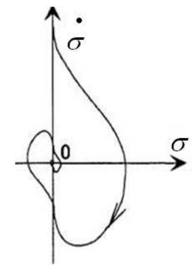

Fig. 6: Super-twisting algorithm phase trajectory

Courtesy: Proceeding on VSS10

The super twisting algorithm (STA) as given by(11) ensures exact finite time convergence, for any bounded uncertainty; for some constants k_1 and k_2 to the second sliding mode set $\sigma = \dot{\sigma}(t) = 0$; without usage of $\dot{\sigma}$ [25], [26]. If we consider system (11) as having σ as the measured output, the STA is an output-feedback controller.

IV. Design of a control

A. Design of a sliding surface

In this study, a sliding surface is chosen as

$$\sigma = \mathbf{c}^T(\mathbf{x}) \tag{12}$$

where c^T is a design parameter which is designed using QR decomposition method and the system state vector is used in designing the conventional sliding surface.

B. Design of Control Law

To design the control law, differentiating (12),

$$\dot{\sigma} = c^T(Ax + Bu) \tag{13}$$

Also as per super twisting algorithm,

$$\dot{\sigma} = -k_1|\sigma|^{1/2}sgn(\sigma) + z \tag{14}$$
$$\dot{z} = -k_2 sgn(\sigma) \tag{15}$$

From (13) to(15),

$$u = (c^T B)^{-1}(-c^T Ax - k_1|\sigma|^{1/2}sgn(\sigma) - \int k_2 sgn(\sigma)) \tag{16}$$

When this control input is substituted in the system equation, σ and $\dot{\sigma}$ approach to zero in finite time duration.

V. Simulation Results

Some simulation studies are carried out on a single link flexible manipulator plant to demonstrate the effectiveness of the proposed HOSM controller. The system is analyzed with $\pm 10\%$ parametric variations of their respective nominal values alongwith a matched disturbance of 0.01sin(t) in the input channel. The simulation parameters are:
For HOSMC, $k_1 = 4$ and $k_2 = 3$
and the sliding surface matrix is $c^T = [1.7321 \ -19.0164 \ 1.0784 \ -0.3358]$

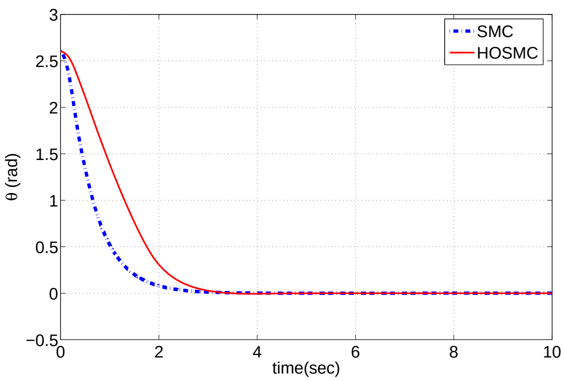

Fig. 7: Output Angular Displacement

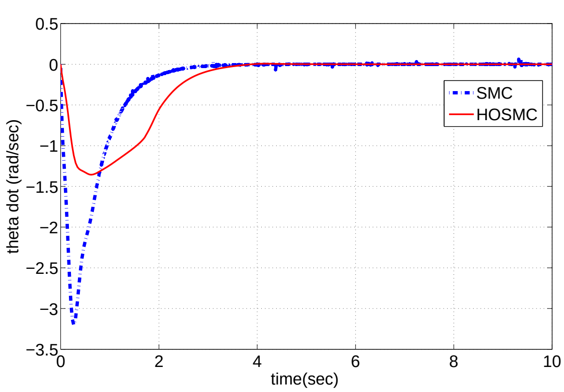

Fig. 8: Angular Velocity

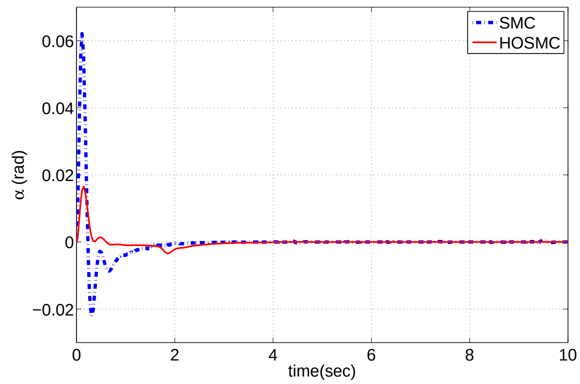

Fig. 9: Tip Displacement

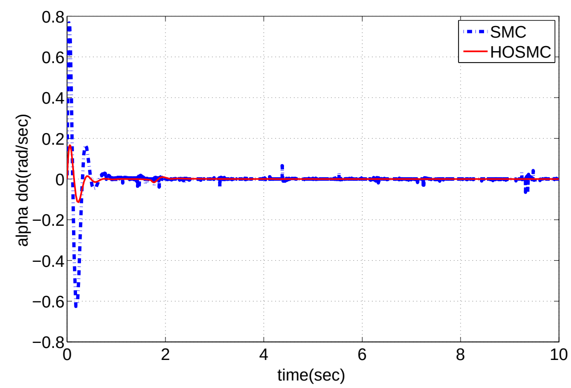

Fig. 10: Rate of change of Tip Displacement

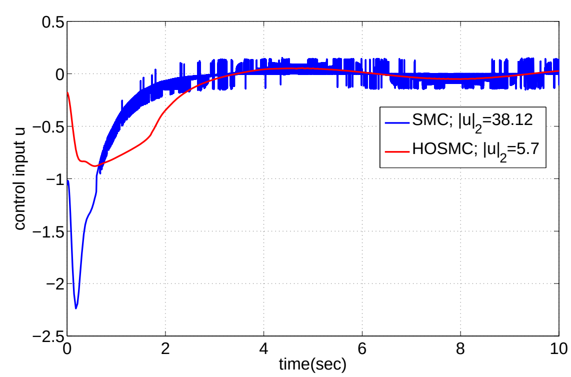

Fig. 11: Control Input

Fig. 7 shows output angular displacement while Fig. 8 shows angular velocity. Fig. 9 and Fig. 10 shows tip displacement and rate of change of tip displacement respectively. The angular displacement and angular velocity settle in finite time using higher order SMC and it is observed that HOSMC shows prominently less vibrations of the tip position. Thus the vibrations of a flexible link are reduced to a great extent using the proposed HOSMC. The maximal amplitudes during transients are reduced significantly using HOSMC as seen in all of the above graphs. The control efforts required by HOSMC are reduced by approximately 6 times than that of traditional SMC, as seen from Fig. 11. Also noticeable chattering attenuation in the control input for the proposed HOSMC can be observed in Fig. 11.

VI. Conclusion

A higher order sliding mode controller is proposed and successfully applied to control position of a single link flexible manipulator. Simulation results are presented to demonstrate the effectiveness of the HOSMC maintaining the advantages of SMC. The contributions of the work are
1) substantial reduction in the control effort,
2) alleviation of chattering in the control input to a great extent with respect to conventional SMC.
3) reduction in the vibrations of a tip of the flexible link.
Thus the proposed super twisting SOSMC outperforms the SMC in attenuating the chattering with less control efforts.

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