Dynamic Analysis Including Stability of Flexibly Supported Narrow Hydrodynamic Journal Bearings with Micropolar Lubricant

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***Abstract*— The present paper deals with the static and dynamic characteristics including the stability of flexibly supported narrow hydrodynamic journal bearings lubricated under the micropolar lubricants. From the basic equations *i.e.* the equation of continuity of mass, the equations for the balance of linear momentum and the balance of angular momentum, which are coupled together due to the existence of two kinematic vector fields of the microstructures and characterizing the micropolar fluid, the derivation of Reynolds equation has been modified. The theoretical prediction of hydrodynamic pressures in the bearing is obtained by the solution of modified Reynolds equation satisfying the appropriate boundary conditions. The steady state pressure profile is obtained easily by an analytical method. The steady state parameters like load carrying capacity and attitude angle can be easily obtained once the pressure profile over the entire bearing surface has been found. First order perturbations of eccentricity ratio and attitude angle are used to calculate the dynamic pressure and the resulting equations are solved by analytical method. Dynamic characteristics are obtained in terms of stiffness and damping coefficients by the help of perturbed pressures. Stiffness and damping coefficients aside, the dynamic characteristics also include the threshold stability and the whirl ratio.**

Keywords*— *Hydrodynamic journal bearing; Micropolar fluid; Linear analysis; Perturbation technique; Stability parameter

I. Introduction

The rapid development of science and technology has reflected on the innovation design and development of modern machine equipments with non Newtonian fluid at high precisions and optimum performance ever in adverse working environment. Since the development of the concept of the hydrodynamic lubrication about 120 years ago with the experiments **[1-2]** and later on, lot of theoretical, experimental and analytical studies were presented by a number of authors. To compliment the deficiencies of the classical continuum theory a number of contemporary studies **[3-5]** on the micro-continuum approach were started developing through a number of researches during the mid and later years of sixties of the late century.

A few more experiments **[6-7]** showed that the mixing of some extremely small amount of polymeric additives in the lubricating fluid reduced the skin friction near a rigid body than in such a fluid to an extent of 30-35%. The development of the theory of micropolar fluid was initiated with the development of the theory on micro-continuum approach. For the application of the classical continuum theory in the field of film lubrication, the most practical importance is whether the deep molecular orientation results in any rheological abnormalities in the vicinity of the solid surfaces and the significant differences in the behavior of the molecules of the liquid in intimate contact with and adhering to the surfaces.

The application of the theory of micropolar fluid in the study of the flow problems in hydrostatic step seal and externally pressurized conical step bearing**[14]** showed an increase in the non-dimensional flow flux and load carrying capacity with increase in the micropolar effect between two infinitely long parallel plates. Similar results **[15]** obtained in analysis of short bearing problem with micropolar lubrication. The fluid pressure was found to increase in comparison to that in the Newtonian flow and was represented by a surface depending on two groups of micropolar parameters.

A good amount of work has been reported on the steady state characteristics of rotor/bearing systems lubricated with micropolar fluids. The steady state characteristics of an infinitely long journal bearing **[8]** are studied by considering a two dimensional flow field. There are several studies carried by many authors on non-Newtonian fluid. In spite of all those studies the gradual development of a full consistent theory along with the acceptable simplification for achieving the practical applicability may be attributed to Eringen and his co-investigators **[9-12]**. The study was later extended **[13]** for the micropolar fluid flow between two concentric cylinders for Couette flow and Poiseuille flow. The velocity, microrotational velocity, shear stress difference and the couple stress profiles for both cases were presented and discussed. An investigation **[15]** on flexibly supported finite turbulent flow oil journal bearings was reported which clearly indicates that higher threshold of stability is achieved in micropolar lubrication compared with Newtonian lubrication. Stability analysis of finite journal bearing under micropolar lubricants was presented **[14]** for both linear and non linear which shows the non linear stability provides better stability than the linear.

However, it has been observed that no investigation was available for the dynamic analysis including stability of flexibly supported narrow hydrodynamic journal bearings. The dynamic analysis including stability of flexibly supported narrow hydrodynamic journal bearing has been carried out in the present paper with first order perturbation of eccentricity ratio and attitude angle to obtain the dynamic pressures.

NOTATION

C	Radial clearance (m)
D	Journal diameter (m)
D_{ij}	Damping co-efficients of micropolar fliud film, for i=R,φ and j=R,φ (Ns/m)
$\bar{D}_{ij}$	Dimensionless damping co-efficient of micropolar fluid film
F_i	Force component along the R and φ direction
H	Local film thickness (m)
$\bar{h}$	Non-dimensional film thickness =h/C
l_m	Non-dimensional characteristic length of the micropolar fluid=Λ/C
M	Mass parameter (kg)
N	Coupling number= $[\chi/(2\mu+\chi)]^{1/2}$
T	Time (s)
Φ_0	Steady state attitude angle (rad)
Φ	Attitude angle (rad)
U	Velocity of journal=ωR (m/s)
$\bar{K}$	Non dimensional stiffness co-efficient of the support
$\bar{m}$	Non-dimensional parameter =$MC\omega^2/W_0$
$\bar{M}_C$	Critical value of non-dimensional parameter
D	Damping co-efficient of the support
$\bar{d}$	Non-Dimensional damping co-efficient of the support
P	Micropolar film pressure in the film region in the non-linear analysis (Pa)
$\bar{p}$	Non-dimensional micropolar film pressure in the film region in the non-linear analysis.
S_{ij}	Stiffness of the micropolar fluid film for i=R,φ and j=R, φ (N/m)
$\bar{S}_{ij}$	Dimensionless damping co-efficient of the micropolar fluid film = $2C^3S_{ij}/(\mu\omega R^3 L)$ For i=R , φ and j=R , φ
Λ	Characteristic length of the micropolar fluid= $[\gamma/(4\mu)]^{1/2}$ (m)
α,β,γ	Viscosity co-efficient of the micropolar fluid (kg m/s)
λ_R	Whirl ratio
ρ	Mass density (kg/m^3)
τ	Non-dimensional time = ωt
K	Stiffness co-efficient of the support
θ	Circumferential coordinate (rad)
ε_0	Steady state eccentricity ratio
X	Cartesian coordinate axis along the circumferential direction = $R\theta$ (m)
$\bar{z}$	Non-dimensional Cartesian coordinate axis along the bearing axis = $2z/L$
$\bar{w}_0$	Non dimensional steady state load in bearing $\bar{w}_0 = 2C^2 w_0/\mu\omega R^3 L$
ε_i	Perturbed eccentricity for i=R and φ
z	Cartesian coordinate axis along the bearing axis (m)
w_0	Steady state load in bearing

II. ANALYSIS

A. Modified Reynolds Equations

The governing modified Reynolds equation for two-dimensional flow of micropolar lubricant under steady state condition is written as follows **[14]**.

$$\frac{\partial}{\partial x}\left[\frac{h_0^3}{\mu}\Phi(\Lambda,N,h_0)\frac{\partial p_0}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{h_0^3}{\mu}\Phi(\Lambda,N,h_0)\frac{\partial p_0}{\partial z}\right]=6U\frac{\partial h_0}{\partial x} \tag{1}$$

With the following substitution

$$\theta=\frac{x}{R},\qquad \bar{z}=\frac{2z}{L},\qquad \bar{h}_0=\frac{h_0}{C},\qquad \bar{p}_0=\frac{p_0C^2}{\mu\omega R^2},\qquad l_m=\frac{C}{\Lambda}$$

Equation (1) will reduce to its non-dimensional form as:

$$\frac{\partial}{\partial\theta}\left[\bar{g}(l_m,N,\bar{h}_0)\frac{\partial\bar{p}_0}{\partial\theta}\right]+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left[\bar{g}(l_m,N,\bar{h}_0)\frac{\partial\bar{p}_0}{\partial\bar{z}}\right]=\frac{1}{2}\frac{\partial\bar{h}_0}{\partial\theta} \tag{2}$$

Where,

$$\overline{\Phi}(l_m,N,\bar{h}_0)=1+\frac{12}{\bar{h}_0^2l_m^2}-6\frac{N}{\bar{h}_0l_m}\coth\left(\frac{Nl_m\bar{h}_0}{2}\right)$$

and

$$\bar{g}(l_m,N,\bar{h}_0)=\frac{\bar{h}_0^3}{12}\overline{\Phi}(l_m,N,\bar{h}_0)=\frac{\bar{h}_0^3}{12}+\frac{\bar{h}_0}{l_m^2}-\frac{N\bar{h}_0^2}{2l_m}\coth\left(\frac{Nl_m\bar{h}_0}{2}\right)$$

Equation (2) is further simplified to the following non-dimensional form under steady state condition

$$C_A\frac{\partial\bar{h}_0}{\partial\theta}\frac{\partial\bar{p}_0}{\partial\theta}+C_B\left\{\frac{\partial^2\bar{p}_0}{\partial\theta^2}+\left(\frac{D}{L}\right)^2\frac{\partial^2\bar{p}_0}{\partial\bar{z}^2}\right\}=\frac{1}{2}\frac{\partial\bar{h}_0}{\partial\theta} \tag{3}$$

Where,

$$C_A = \frac{\bar{h}_0^2}{4} + \frac{1}{l_m^2} - \frac{N\bar{h}_0}{l_m}\coth\left(\frac{Nl_m\bar{h}_0}{2}\right) + \frac{N^2\bar{h}_0^2}{4}\cos ech^2\left(\frac{Nl_m\bar{h}_0}{2}\right)$$

and

$$C_B = \frac{\bar{h}_0^3}{12} + \frac{\bar{h}_0}{l_m^2} - \frac{N\bar{h}_0^2}{2l_m}\coth\left(\frac{Nl_m\bar{h}_0}{2}\right)$$

B. Perturbation method

The concept of the perturbation technique is to provide small order perturbation to the eccentricity ratio and attitude angle assuming the journal to execute small harmonic oscillation about its static equilibrium position, to eliminate the time dependent terms without serious error. The non-dimensional pressure and local film thickness **[14]** are as follows:

$$\bar{p} = \bar{p}_0 + \bar{p}_1\varepsilon_1 e^{i\lambda R\tau} + \bar{p}_2\varepsilon_0\phi_1 e^{i\lambda R\tau} \tag{4}$$

$$\bar{h} = \bar{h}_0 + \varepsilon_1 e^{i\lambda R\tau}\cos\theta + \varepsilon_0\phi_1 e^{i\lambda R\tau}\sin\theta$$

Where;

$\bar{h}_0 = 1 + \varepsilon_0\cos\theta$; $\varepsilon = \varepsilon_0 + \varepsilon_1 e^{i\lambda R\tau}$;

$\phi = \phi_0 + \phi_1 e^{i\lambda R\tau}$ and $\lambda_R = \frac{\omega_p}{\omega}$

C. Equation and pressure profile for steady- state and dynamic conditions

Considering the Ocvirk solution (1955) for narrow bearing and solved (3) using the analytical method to obtain the steady state pressure distribution $\bar{p}_0$, satisfying the boundary conditions as given in (6) results in:

$$\bar{p}_0 = \left(\frac{L}{D}\right)^2 \frac{\varepsilon_0\sin\theta}{4C_B}\left(1-\bar{z}^2\right) \tag{5}$$

Boundary conditions for (3) are as follows:

$$\bar{p}_0(\theta, \pm 1) = 0$$

$$\frac{\partial\bar{p}_0(\theta, 0)}{\partial\bar{z}} = 0$$

$$\frac{\partial\bar{p}_0(\theta_2, \bar{z})}{\partial\theta} = 0 \qquad \bar{p}_0(\theta, \bar{z}) = 0 \qquad \text{for} \quad \theta \geq \theta_2 \tag{6}$$

Modified Reynolds equation in rotating coordinate system in micropolar lubrication **[14]** is given as:

$$\frac{\partial}{\partial x}\left[\frac{h^3}{\mu}\Phi(\Lambda, N, h)\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{h^3}{\mu}\Phi(\Lambda, N, h)\frac{\partial p}{\partial z}\right] = 6\frac{\partial(Uh)}{\partial x} + 12\frac{\partial h}{\partial t} - 12\omega_p\frac{\partial h}{\partial\theta} \tag{7}$$

Equation (7) for two-dimensional flow of micropolar lubricant with the following substitutions:

$\theta = \frac{x}{R}$, $\bar{z} = \frac{2z}{L}$, $\bar{h} = \frac{h}{C}$, $\bar{p} = \frac{pC^2}{\mu\omega R^2}$,

$l_m = \frac{C}{\Lambda}$, $\tau = \omega t$

will reduce to its non-dimensional form as:

$$\frac{\partial}{\partial\theta}\left[\bar{g}\left(l_m, N, \bar{h}\right)\frac{\partial\bar{p}}{\partial\theta}\right] + \left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left[\bar{g}\left(l_m, N, \bar{h}\right)\frac{\partial\bar{p}}{\partial\bar{z}}\right] = \frac{1}{2}\left(1-2\phi'\right)\frac{\partial\bar{h}}{\partial\theta} + \frac{\partial\bar{h}}{\partial\tau} \tag{8}$$

Where,

$$\bar{g}\left(l_m, N, \bar{h}\right) = \frac{\bar{h}^3}{12}\bar{\Phi}\left(l_m, N, \bar{h}\right) = \frac{\bar{h}^3}{12} + \frac{\bar{h}}{l_m^2} - \frac{N\bar{h}^2}{2l_m}\coth\left(\frac{Nl_m\bar{h}}{2}\right),$$

$$\bar{\Phi}\left(l_m, N, \bar{h}\right) = \left\{1 + \frac{12}{\bar{h}^2 l_m^2} - 6\frac{N}{\bar{h}\, l_m}\coth\left(\frac{Nl_m\bar{h}}{2}\right)\right\}$$

and

$$\phi' = \frac{\partial\phi}{\partial\tau}$$

Equation (8) is expanded with all the substitution made and neglecting the higher order terms of $\left(\varepsilon_1 e^{i\lambda_R t}\right)$ and $\left(\varepsilon_0\phi_1 e^{i\lambda_R t}\right)$. Collecting the zeroth and first order terms of ε_1 and $\varepsilon_0\phi_1$ the following two equations are obtained.

$$C_B\left(\frac{D}{L}\right)^2\frac{\partial^2\bar{p}_1}{\partial\bar{z}^2} + (C_A\cos\theta)\left(\frac{D}{L}\right)^2\frac{\partial^2\bar{p}_0}{\partial\bar{z}^2} = -\frac{1}{2}\sin\theta + i\lambda_r\cos\theta \tag{9}$$

$$C_B\left(\frac{D}{L}\right)^2\frac{\partial^2\bar{p}_2}{\partial\bar{z}^2} + (C_A\sin\theta)\left(\frac{D}{L}\right)^2\frac{\partial^2\bar{p}_0}{\partial\bar{z}^2} = \frac{1}{2}\cos\theta + i\lambda_r\left(\sin\theta - \frac{1}{\varepsilon_0}\frac{\partial\bar{h}_0}{\partial\theta}\right) \tag{10}$$

Where, C_A and C_B are obtained from (3).

The boundary conditions for the perturbed pressures are as follows:

$$\bar{p}_1(\theta, \pm 1) = \bar{p}_2(\theta, \pm 1) = 0$$

$$\frac{\partial\bar{p}_1(\theta, 0)}{\partial\bar{z}} = \frac{\partial\bar{p}_2(\theta, 0)}{\partial\bar{z}} = 0 \tag{11}$$

$$\bar{p}_1(\theta_1, \bar{z}) = \bar{p}_2(\theta_1, \bar{z}) = 0,$$

$$\bar{p}_1(\theta, \bar{z}) = \bar{p}_2(\theta, \bar{z}) = 0, \qquad \theta \geq \theta_2$$

Where, θ_2 represents the angular coordinate where the film start and cavitates. Equation (9) and (10) are solved using analytical method to obtain the perturbed pressure distribution $\bar{p}_1$ & $\bar{p}_2$, satisfying the boundary conditions as mentioned in (11):

$$\bar{p}_1 = C_B\left(\frac{L}{D}\right)^2\left[\frac{1}{4}\left(\frac{C_A\varepsilon_0\sin\theta\cos\theta}{C_B} - \sin\theta\right) + i\lambda_r\cos\theta\right]\left[\bar{z}^2 - 1\right]$$

$$\bar{p}_2 = C_B\left(\frac{L}{D}\right)^2\left[\frac{1}{4}\left(\frac{C_A\varepsilon_0\sin^2\theta}{C_B} + \cos\theta\right) + i\lambda_r\sin\theta\right]\left[\bar{z}^2 - 1\right]$$

D. Stiffness and damping coefficients

The components of the dynamic load along the line of centre's and perpendicular to the line of centre's corresponding to perturbed pressure $\left(p_1 \varepsilon_1 e^{i\lambda_R t}\right)$ in R-direction **[15]** can be written as under

$$(W_d)_R e^{i\lambda R\tau} = -2\int_0^{L/2}\int_{\theta_1}^{\theta_2} p_1 \cos\theta \varepsilon_1 e^{i\lambda R\tau} R d\theta dz \tag{12}$$

$$(W_d)_\phi e^{i\lambda R\tau} = -2\int_0^{L/2}\int_{\theta_1}^{\theta_2} p_1 \sin\theta \varepsilon_1 e^{i\lambda R\tau} R d\theta dz \tag{13}$$

As the journal executes small harmonic oscillation about its steady state position in an elliptic orbit, the components of dynamic load can be expressed as a spring load and viscous damping load as

$$(W_d)_R e^{i\lambda R\tau} = S_{RR} C\varepsilon_R + D_{RR}\frac{d}{dt}(C\varepsilon_R) \tag{14}$$

$$(W_d)_\phi e^{i\lambda R\tau} = S_{\phi R} C\varepsilon_R + D_{\phi R}\frac{d}{dt}(C\varepsilon_R) \tag{15}$$

Combining (12) through (15), non-dimensionalising and noting that $\varepsilon_R = \varepsilon_1 e^{i\lambda_R t}$, the following non-dimensional components of stiffness and damping coefficient result in R-direction

$$\bar{S}_{RR} = -\operatorname{Re}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_1 \cos\theta d\theta d\bar{z}\right]$$

$$\bar{S}_{\phi R} = -\operatorname{Re}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_1 \sin\theta d\theta d\bar{z}\right]$$

$$\bar{D}_{RR} = -\operatorname{Im}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_1 \cos\theta d\theta d\bar{z}\right]\Big/\lambda_R$$

$$\bar{D}_{\phi R} = -\operatorname{Im}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_1 \sin\theta d\theta d\bar{z}\right]\Big/\lambda_R \tag{16}$$

Similarly, considering dynamic displacement of the journal along ϕ direction, it can be shown that:

$$\bar{S}_{R\phi} = -\operatorname{Re}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_2 \cos\theta d\theta d\bar{z}\right]$$

$$\bar{S}_{\phi\phi} = -\operatorname{Re}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_2 \sin\theta d\theta d\bar{z}\right]$$

$$\bar{D}_{R\phi} = -\operatorname{Im}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_2 \cos\theta d\theta d\bar{z}\right]\Big/\lambda_R$$

$$\bar{D}_{\phi\phi} = -\operatorname{Im}\left[2\int_0^1\int_{\theta_1}^{\theta_2} \bar{p}_2 \sin\theta d\theta d\bar{z}\right]\Big/\lambda_R \tag{17}$$

Where,

$$\bar{S}_{ij} = \frac{2C^3 S_{ij}}{\mu\omega R^3 L}, \bar{D}_{ij} = \frac{2C^3 D_{ij}}{\mu R^3 L}, \ i = R,\phi \text{ and } j = R,\phi$$

E. Equation of motion

Referring to Fig.1, the equations of motion of the journal, assuming the journal to be rigid can be written as under

Equation of motion of rotor in 'r' direction is

$$m_R\left[A_r + \frac{d^2 e_r}{dt^2} - e_r\left(\frac{d\phi}{dt}\right)^2\right] + S_{rr}e_r + D_{rr}\frac{de_r}{dt} + S_{r\phi}e_\phi + D_{r\phi}\frac{de_\phi}{dt} = w_0\cos\phi \tag{18}$$

Equation of motion of rotor in 'ϕ' direction is

$$m_R\left[A_\phi + \frac{d^2 e_\phi}{dt^2} + 2e_r e_\phi\right] + S_{\phi\phi}e_\phi + D_{\phi\phi}\frac{de_\phi}{dt} + S_{\phi r}e_r + D_{\phi r}\frac{de_r}{dt} = w_0\sin\phi \tag{19}$$

Equation of motion of the bearing (neglecting the weight of bearing) in 'r' direction

$$m_b[A_r] - S_{rr}e_r - D_{rr}\frac{de_r}{dt} - S_{r\phi}e_\phi - D_{r\phi}\frac{de_\phi}{dt} = -\left(Kx_b + d\frac{dx_b}{dt}\right)\sin\phi \tag{20}$$
$$-\left(Ky_b + d\frac{dy_b}{dt}\right)\cos\phi$$

And equation of motion of bearing in 'ϕ' direction is

$$m_b[A_\phi] - S_{\phi\phi}e_\phi - D_{\phi\phi}\frac{de_\phi}{dt} - S_{\phi r}e_r - D_{\phi r}\frac{de_r}{dt} = -\left(Kx_b + d\frac{dy_b}{dt}\right)\cos\phi \tag{21}$$
$$+\left(Ky_b + d\frac{dy_b}{dt}\right)\sin\phi$$

Where, K and d are the external stiffness and damping co-efficient respectively.

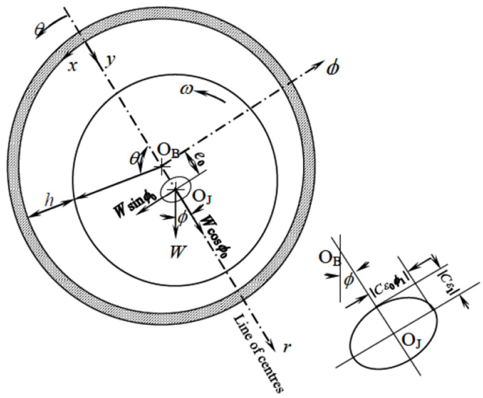

Fig.1 Configuration of bearing geometry

F. Stability characteristics

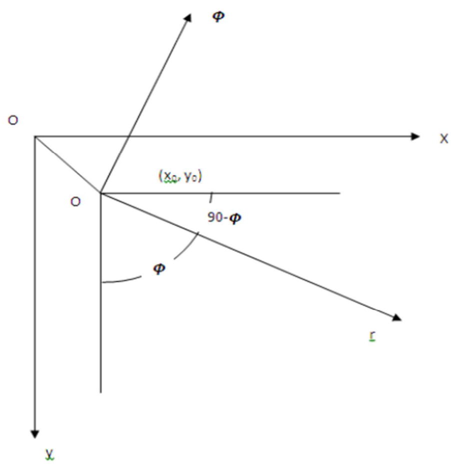

Fig. 2 Schematic diagram of co-ordinate system

Here for the system as shown in Fig.2

$A_r = \ddot{x}_b \sin\phi + \ddot{y}_b \cos\phi$

And

$A_\phi = \ddot{x}_b \cos\phi - \ddot{y}_b \sin\phi$

Now the following substitution are made

$x_b = c.\bar{x}_b = c\left(\bar{x}_0 + \bar{x}_1 e^{i\lambda_R t}\right)$,

$y_b = c.\bar{y}_b = c\left(\bar{y}_0 + \bar{y}_1 e^{i\lambda_R t}\right)$,

$\phi = \phi_0 + \phi_1 e^{i\lambda_R t}$,

$e_r = \bar{e}_r .c = c\left(\varepsilon_0 + \varepsilon_1 e^{i\lambda_R t}\right)$,

$e_\phi = \bar{e}_\phi .c = c\left(\varepsilon_0\phi_0 + \varepsilon_0\phi_1 e^{i\lambda_R t}\right)$,

$\cos\phi = \cos\phi_0 - \phi_1 e^{i\lambda_R t}\sin\phi_0$,

$\sin\phi = \sin\phi_0 - \phi_1 e^{i\lambda_R t}\cos\phi_0$,

$S_{ij} = \dfrac{\eta U R^2 L}{2C^3}.\bar{S}_{ij}$, $\quad D_{ij} = \dfrac{\eta R^3 L}{2C^3}.\bar{D}_{ij}$, $\quad w_0 = \dfrac{\eta U R^2 L}{2C^3}.\bar{w}_0$

$T = \omega_p .t \qquad \omega_p/\omega = \lambda \qquad m_b/m_R = Z \qquad \bar{m} = \dfrac{m_R C w^2}{w_0}$

Now using the above substitution and equation co-efficient on both sides, we have the equation of motion in the following non-dimensional form

$$-\lambda^2\bar{m}\bar{w}_0\bar{P}_1 + \left(-\lambda^2\bar{m}\bar{w}_0 + \bar{S}_{rr} + i\lambda\bar{D}_{rr}\right)\varepsilon_1 + \left(\bar{S}_{r\phi}\varepsilon_0 - i\lambda\bar{D}_{r\phi}\varepsilon_0 + \bar{w}_0\sin\phi_0\right)\phi_1 = 0 \tag{22}$$

$$\lambda^2\bar{m}\bar{w}_0\bar{q}_1 + \left(\bar{S}_{\phi r} + i\lambda\bar{D}_{\phi r}\right)\varepsilon_1 + \left(-\lambda^2\bar{m}\bar{w}_0\varepsilon_0 + \bar{S}_{\phi\phi}\varepsilon_0 + i\lambda\bar{D}_{\phi\phi}\varepsilon_0 - \bar{w}_0\cos\phi_0\right)\phi_1 = 0 \tag{23}$$

$$\left(-z\lambda^2\bar{m}\bar{w}_0 - \bar{k} - i\lambda\bar{d}\right)\bar{P}_1 + \left(-\bar{S}_{rr} - i\lambda\bar{D}_{rr}\right)\varepsilon_1 + \left(-\bar{S}_{r\phi}\varepsilon_0 - i\lambda\bar{D}_{r\phi}\varepsilon_0 - \bar{k}\bar{b}_2\right)\phi_1 = 0 \tag{24}$$

$$\left(z\lambda^2\bar{m}\bar{w}_0 - \bar{k} - i\lambda\bar{d}\right)\bar{q}_1 + \left(-\bar{S}_{\phi r} - i\lambda\bar{D}_{\phi r}\right)\varepsilon_1 + \left(-\bar{S}_{\phi\phi}\varepsilon_0 - i\lambda\bar{D}_{\phi\phi}\varepsilon_0 - \bar{k}\bar{b}_1\right)\phi_1 = 0 \tag{25}$$

Where,

$\bar{P}_1 = \left(\bar{x}_1\sin\phi_0 + \bar{y}_1\cos\phi_0\right)$,

$\bar{q}_1 = \left(\bar{y}_1\sin\phi_0 - \bar{x}_1\cos\phi_0\right)$,

$\bar{b}_1 = \left(\bar{x}_0\sin\phi_0 + \bar{y}_0\cos\phi_0\right)$,

$\bar{b}_2 = \left(\bar{y}_0\sin\phi_0 - \bar{x}_0\cos\phi_0\right)$

Now for non-trivial solution, the determinant of (22), (23), (24) and (25) must vanish. Equating real and imaginary parts of equation (obtained by expanding the determinant of the equations for non-trivial solution) separately to zero, the following two equations are obtained

$$A_{11}\bar{m}^4 + A_{12}\bar{m}^3 + A_{13}\bar{m}^2 + A_{14}\bar{m} + A_{15} = 0 \tag{26}$$

$$A_{21}\bar{m}^3 + A_{22}\bar{m}^2 + A_{23}\bar{m} + A_{24} = 0 \tag{27}$$

G. Methods of solution

Equation (3), (9) and (10) are solved analytically using the boundary condition as stated in (6) and (11) to obtain the steady state and the perturbed pressure distribution $\bar{p}_0$, $\bar{p}_1$ and $\bar{p}_2$ from which the steady state load and the component of stiffness and damping coefficients are evaluated. Finally the critical mass parameter $\left(\bar{M}_C\right)$ & whirl ratio $\left(\lambda_R\right)$ can be determined by satisfying (26) & (27) employing Newton Raphson method.

III. RESULTS AND DISCUSSION

A. Steady state characteristics

Since the steady state characteristics *viz.*, load carrying capacity and attitude angle of the journal are dependent on the steady state film pressure $\bar{p}_0$, which, in turn, depends upon the micropolar parameters l_m, N, and eccentricity ratio ε_0, detailed parametric studies are done to show their effects on the respective non-dimensional form of steady state load and attitude angle and the results have been compared with the respective values for Newtonian fluid.

B. Effect of coupling number and eccentricity ratio on Load carrying capacity

When coupling number N is taken as a parameter then the variation of the dimensionless load capacity of the journal bearings as a function of l_m for L/D = 0.1 and ε_0 = 0.5 is shown in Fig. 3. In Fig.3 we can see that for any coupling number N, the load capacity reduces with l_m and approaches asymptotically to the Newtonian value as $l_m \to \infty$ at a finite value of l_m. The load parameter increases as coupling number is increased.

This is usually happened due to significant effect of micropolarity in both cases either the characteristic material length is large or clearance is small. The second case requires more attention here as clearance is usually very small in hydrodynamic journal bearing. As $l_m \to \infty$ the velocity and the other flow characteristics will reduce to their equivalents in the Newtonian theory with μ everywhere replaced by $(\mu + 1/2\chi)$, as gradient of microrotational velocity across film thickness is very small. Hence effectively the viscosity has been enhanced. So, when the non-dimensional load has been referred to the Newtonian viscosity, it is increased by a factor $(\mu + 1/2\chi)/\mu$ at $l_m \to \infty$.

In Fig.4 it can be seen that for a particular value of l_m, the non-dimensional load carrying capacity though increases in both types of lubrication as ε_0 is increased, the rate of increase in $\bar{W}_0$ in micropolar lubrication is found to be more rapid than that in the Newtonian. It is also found that the load parameter at any eccentricity ratio is considerably higher than that for Newtonian value at lower values of l_{m} and converges to that for Newtonian fluid as $l_m \to \infty$.

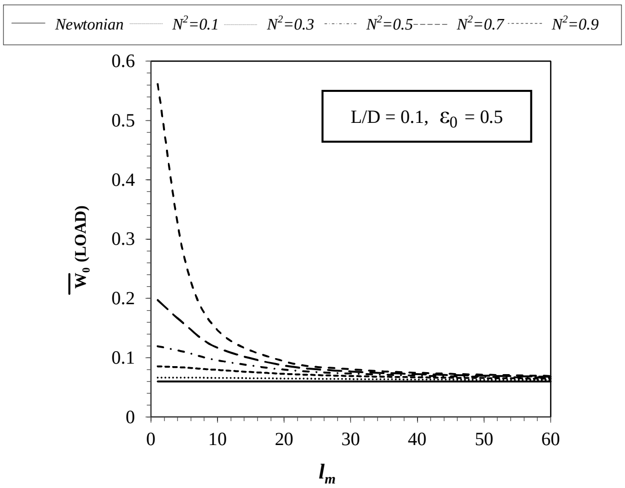

Fig.3. Variation of $\overline{W}_0$ with l_m for various values of N^2

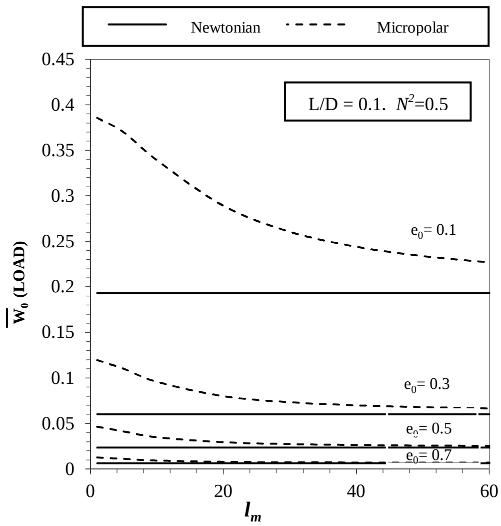

Fig.4. Variation of $\overline{W}_0$ with l_m for various values of ε_0.

that for the Newtonian fluid. The reason is that components of non-dimensional forces along the line of centres and perpendicular to it will be increased by the same factor $(\mu + 1/2\chi)/\mu$ and the attitude angle being the function of the ratio of the two will remain unaffected.

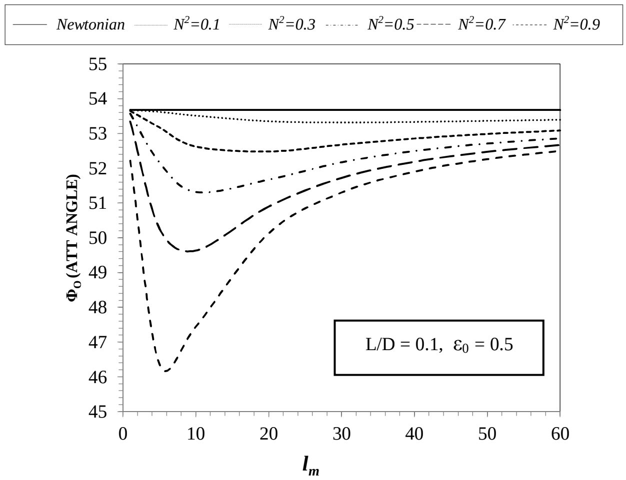

Fig.5. Variation of Φ_0 with l_m for various values of N^2

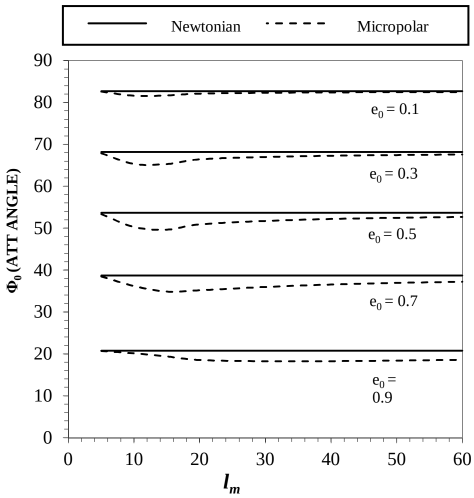

Fig.6. Variation of Φ_0 with l_m for various values of ε_0.

C. Effect of coupling number and eccentricity ratio on Attitude angle

Fig.5 shows the variation of the attitude angle with micropolar parameter l_{m} for L/D = 0.1 and ε_0 = 0.5 when coupling number N as treated a parameter. It can be seen from the figure that for a particular value of l_{m} attitude angle decreases as N is increased. Furthermore, as l_{m} increases the values of the attitude angle converge asymptotically to that for the Newtonian fluid. For any coupling number, attitude angle initially decreases with increase in l_{m} reaching a minimum and then reversing the trend as l_{m} is further increased.

It is found that the optimum value of l_{m} at which Φ_0 becomes a minimum increases with a decrease in N. Further at l_{m} = 0, the attitude angle remains the same as

Effect of the eccentricity ratio on attitude angle has been shown in Fig.6, for L/D = 0.1 and N^2 = 0.5. It is found from the figure that at any l_{m}, attitude angle decreases with increase in ε_0. Furthermore, the effect of micropolar parameter l_{m} becomes more pronounced at higher ε_0. The optimum micropolarity is found to be almost independent of ε_0 and occurs at l_{m} around 10.

D. Non-dimensional components of stiffness and damping coefficients

The dynamic characteristics are expressed in terms of non-dimensional components of stiffness and damping coefficients and the critical mass parameter along with the corresponding whirl ratio. These characteristics representing the stability of the journal which is dependent on the steady state and perturbed film pressures $\overline{p}_0$, $\overline{p}_1$ and $\overline{p}_2$ and thus, in turn, depend upon the micropolar parameters (l_m, N) and eccentricity ratio (ε_0). Detailed parametric studies are done to show their effects on the non-dimensional components of stiffness and damping coefficient and finally on critical mass parameter and whirl ratio. Fig.7 shows the combined effect of l_m and N on critical mass parameter at various values of N keeping ε_0 = 0.1, $\overline{K} = 0.001$, $\overline{d} = 0.001$ and L/D = 0.1.

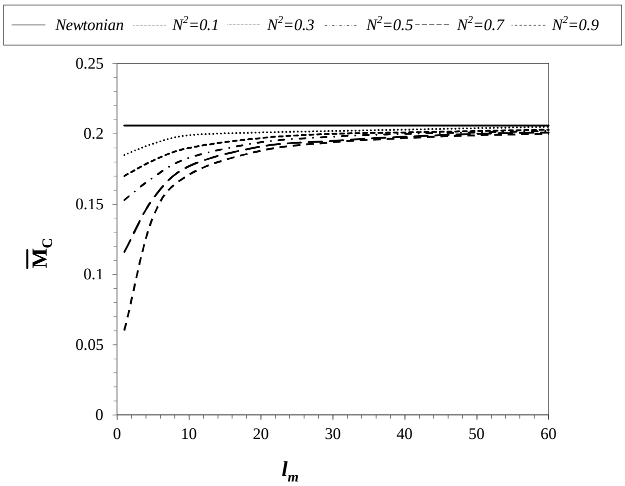

Fig.7. Variation of $\left(\overline{M}_C\right)$ with l_m for various values of N^2

As l_m grows indefinitely the threshold of stability decreases with decrease in N and the micropolar fluid approaches to Newtonian fluid. Stability improves initially as l_m is increased, reaches a maximum to that for Newtonian fluid at large value of l_m. The increase in the stability parameter in micropolar lubricant regime can be explained from the steady state analysis in which it has been shown in steady state analysis that load parameter of a journal bearing increases in the micropolar fluid than in the Newtonian fluid. Therefore, with same load, journal will run at higher eccentricity ratio in a Newtonian fluid as compared to micropolar lubricant and consequently the stability parameter in micropolar lubricant regime will increase.

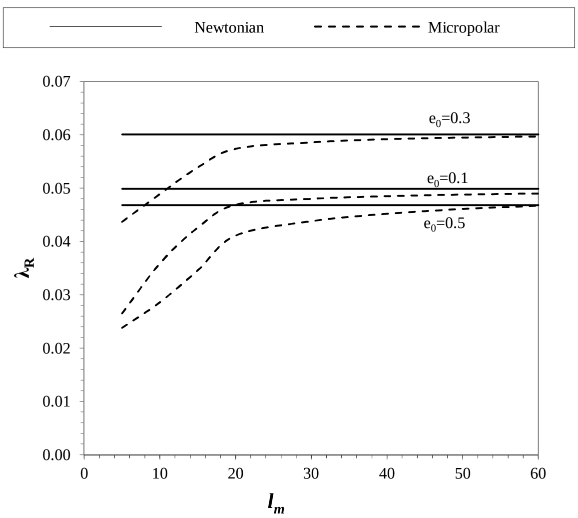

Fig.8. Variation of λ_R with l_m for various values of ε_0.

The effect of ε_0 on whirl ratio can be found from Fig.8 as a function of l_m for N^2 = 0.3 and L/D = 0.1. It can be seen from the figure that as eccentricity ratio increases, whirl ration increases but in the range of 0.1 to 0.3. After that whirl ratio decreases. Whirl ratio increases with increase in l_m, and ultimately converges to that for Newtonian fluid when l_m is indefinitely large.

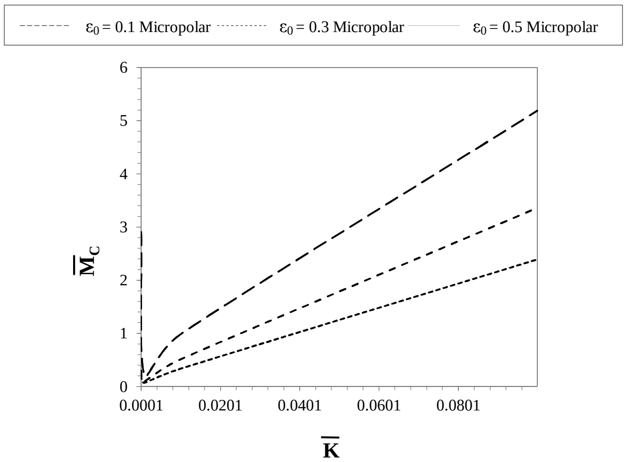

Fig.9. Variation of $\left(\overline{M}_C\right)$ with $\left(\overline{K}\right)$ for various values of ε_0.

Fig.9 shows the variation of critical mass parameter $\left(\overline{M}_C\right)$ with external stiffness $\left(\overline{K}\right)$ at various values of ε_0 at N^2 = 0.1, l_m =5, $\overline{d} = 0.001$, $\overline{z} = 0.1$ and L/D= 0.1.It is found from the figure that critical mass parameter $\left(\overline{M}_C\right)$ initially decrease at very low value of $\left(\overline{K}\right)$ and then increase with increasing the value of $\left(\overline{K}\right)$ at different eccentricity ratio. As shown in the figure that the critical mass parameter increases with decrease in eccentricity ratio and the threshold stability decreases with eccentricity ratio.

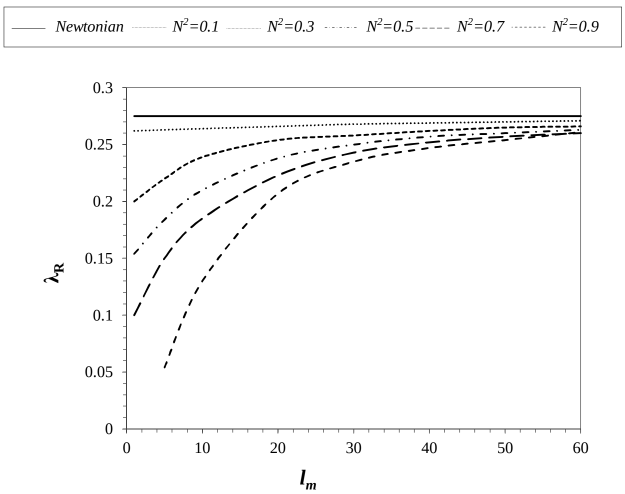

Fig.10. Variation of λ_R with l_m for various values of N^2.

Fig.10 shows the variation of whirl ratio with l_m, when N is taken as a parameter for $\varepsilon_0 = 0.5$ and L/D = 0.1. It can be seen from the figure that as coupling number increases, whirl ratio decreases. As a parametric variation of N, the whirl ratio increase with increase in l_m at particular eccentricity ratio and converges to that for Newtonian fluid when l_m is indefinitely large.

IV. CONCLUSION

From the studies and the results reported in the paper, the following conclusions may be drawn.

(i) The non-dimensional load carrying capacity increases as the coupling number increases and non-dimensional characteristic length decreases. The load parameter converges to the Newtonian value as $N \rightarrow 0$ and $l_m \rightarrow \infty$.
(ii) Other parameters remaining the same, the load parameter increases in both types of lubrication with the increase in eccentricity ratio. The load parameter remains always higher in the micropolar fluid as compared with the Newtonian fluid.
(iii) At a certain value of l_m, all dimensionless response coefficients increase as the eccentricity ratio increases when other parameters remain the same.
(iv) More micropolarity represents more stability in the bearing. However, for a specific micropolar fluid an optimum micropolar effect can be found by the suitable choice of the lubricant's grain size.
(v) Higher eccentricity ratio favors the bearing towards higher stability.
(vi) Threshold stability increase with damping.
(vii) Higher threshold of stability is achieved in micropolar lubrication compared with Newtonian lubrication.

ACKNOWLEDGMENT

'Acknowledgement'-although a light surrounding term contains the deepest sense of gratitude, that not only services within the boundary line of a few calculations oriented papers, but also it denotes an extensive range of emotional aspects in the mind of person concerned. Especially, who is grateful for a praiseworthy remuneration, of course, remuneration does not exist on the logic of financial benefit, but it brings forth only personal satisfaction. It sincerest implication goes into the farthest core of my heart, being fully aware of the fact, that this paper would never attain successful completion, but for the kind and direct guidance of *Dr. A. k. Chattopadhyay***,** which is exclusively fathomless indeed, existing vividly in the procedure as well as the relevant steps formulated by the author.

I am highly grateful to the authorities of Thapar University, Patiala for providing this opportunity to carry out the research work.

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