Determination of Stress Intensity Factor and Interaction Behavior of Radial Cracks in an Un-notched Round Bar

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***Abstract*— Cracks emanating from the surface of a bar, power transmission shafts, and pressure vessels when undergoing cyclic loading. Multiple radial cracks also originating from the inner surface of cylinders when undergoing cyclic internal pressure. In general, surface cracks in structural components are often approximated by elliptical, semi elliptical cracks, circular or semi circular cracks. Due to complexities in analysis of multiple cracks, many researchers have studied the damage tolerance analysis of single crack in rectangular plate, notched and un-notched round bar and thus the effect of crack interaction has been neglected. In order to predict safe-life of such cracked components, it is important to know the individual stress intensity factors (SIF) under mixed mode loading. This paper presents the SIF and interaction behavior of multiple radial cracks in an un notched round bar subjected to far filed tension loading. Three surface cracks of same dimensions (crack depth and aspect ratios) were introduced around the circumference of the cylindrical rod specimen. The crack depth ratio (a/d) ranging between 0.1 and 0.3 was considered for two different crack aspect ratios (a/c = 0.6 and 1.0). The crack tip region was meshed with iso-parametric singular elements to incorporate the singularity of stress and strain fields. The SIF values of single and multiple cracks are compared to determine the interaction behaviour of radial cracks on SIF. Comparison SIF of radial cracks with published SIF of single crack showed a good agreement at short crack depths ((a/d) <0.3). Beyond the crack depth ratio of 0.3 higher SIF is observed. This is due to interaction effect of radial cracks which increases with crack depth ratio. SIF are higher at the surface (S/S_0 = ±1) of a semi-circular crack whereas SIF are higher at the crack middle region (S/S_0 =0) for semi elliptic cracks independent of crack depth ratios.**

Key words - Iso-parametric element, Interaction behavior, Crack aspect ratio, multiple cracks, Stress Intensity Factor

I. INTRODUCTION

Highly stressesd aircraft structures and automobile components may develop cracks during their service life. The existence of such cracks and their subsequent growth can cause a loss of strength and a reduction in service life of a structure. Fracture mechanics provides a means of quantitatively assessing the behavior of cracks. Knowledge of this behavior will help to improve the safety of the structure during its service life. The application of the principles of fracture mechanics to a practical problem requires knowledge of the crack size (a), the service stress (σ), the stress intensity factor (K) and the appropriate properties of the materials. Stress Intensity Factor (SIF) solutions are available for wide range of geometric configurations for both two and three dimensional geometries. Most of the two dimensional cracks propagate into three-dimensional cracks and it may be simplified into 3-D planar cracks for the purpose of design and analysis. Usually, for convenience, these 3D cracks are simplified into circular (a/c = 1.0), elliptical (a/c = 0.2) and straight cracks. Because of the complexity of elastic 3D cracked bodies, analytical solution is possible only for single embedded circular or elliptical crack subjected to simple far field tensile loading in the elastic range. For a single and surface crack no closed-form solution is available, though many semi-analytical solutions have emerged. Accurate estimation of stress intensity factor along the border of the crack helps to predict the reliable crack growth. Number of SIF solutions have been developed for both semi elliptical (a/c =0.2) crack front and semi-circular (a/c=1) crack fronts in notched and un-notched round bars subjected to both tension and bending loading. Generally, a surface crack has been approximated by a semi-circular, semi-elliptical or straight edge crack in a bar [1]. The geometry correction factor (Y) used in SIF calculations ($Y = K/\sigma\sqrt{\pi a}$) depends on the crack depth ratio (a/d), crack aspect ratio (a/c) and the location of the point considered along the crack front (P/P_0) as shown in fig 1(a). SIF for three dimensional cracked bodies have been evaluated by theoretical, numerical and experimental methods. While majority of the studies consider through-the-thickness cracks, the SIF problem becomes complex, when there is a surface crack. Shih and Chen [2] studied the SIF for a semi-elliptic surface crack in a round bar and observed that, the SIF increased with an increase in crack depth ratio (a/d), and the SIF values decreased with increase of aspect ratio (a/c). A three parameter (a, c and d and location of point along crack profile) relationship for SIF was obtained by numerical curve fitting method. It is noted that the relationship is limited to mode-I fracture. Cai and Shin [3] obtained negative SIF values when they used the above three parameter approach, possibly due to errors in selection of order of polynomials. Couroneau and Royer [4] and Carpinteri [5] determined the SIF for a surface crack in a round bar subjected to constant amplitude tension cyclic load to understand the transitions in crack profile during fatigue loading and observed a significant influence of crack aspect ratio on crack front evolution. Several numerical simulations have been carried out to estimate SIF values for a notched round bar. Toribio et al. [6] numerically estimated the mode-I SIF for a cracked bolt subjected to tension and bending loads. Noda and Takase [7] estimated the SIF values for a V-notched round bar for various shaped notches. The estimation is independent of geometric parameters such as, stress concentration factor. Guo et al. [8] determined the SIF values for a semi elliptical surface crack in a notched tensile round bar for different notch shapes and stress concentration factors. Their analysis showed that the SIF values are strongly dependent on the stress concentration factor, and not on the shape of the notch geometry (U, V etc).

However when the interacting multiple cracks are exists in a structural component, the analysis becomes too complex. Thus in literature there seems to be a lack of study for the interaction behavior of multiple cracks. In fact very few solutions [9, 10] for multiple semi-elliptical surface cracks in a semi infinite solid have been observed. Since analytical solutions to such problems are not available due to inherent complexity, many numerical techniques have been implemented. In this paper using finite element analysis, an extension is made to deal with the problem of interacting multiple cracks in an un notched round bar. The prime objectives of the present paper are,

1. Determination of SIF for a single and radial cracks in an un-notched round bar subjected to far field tensile loading
2. Analyze the effect of crack depth (a/d) and aspect ratios (a/c) on SIF of radial cracks in a round bar.
3. Determination of interaction behaviour of radial cracks subjected to far field tensile loading
4. Comparison of SIF of radial cracks with published data.

2.0 DESCRIPTION OF THE FINITE ELEMENT MODEL

The numerical estimation of SIF for radial cracks in a round bar subjected to tensile loading condition was carried out using ABAQUS Finite Element software. A 3-D un notched round bar with radial cracks (Fig.1a) of same size was considered for the analysis.

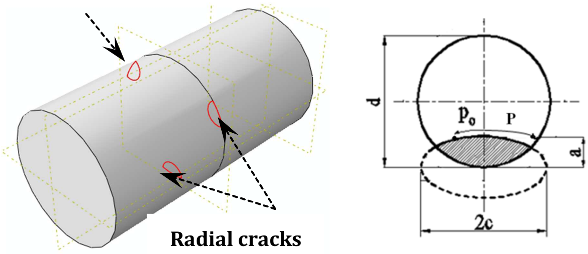


Fig.1a. Radial cracks in an un notched round bar

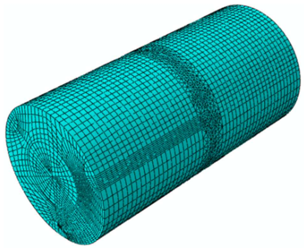


Fig.1b. Meshed un notched round bar with radial cracks

`The dimensions of the round bar are listed in Table.1. Two aspect ratios (a/c) and four crack depth ratios (a/d) were considered in the present work to understand the effect of these variables on SIF. The meshed model of the rectangular bar is shown in Fig.1b. Semi minor and major axis ‘a’ and ‘c’ correspond to the crack depth and transverse length of the crack.

Table.1 Dimensions of a un notched round bar with radial crack

Sl No	Parameters	Dimensions
1	Diameter of the bar	10 mm
2	Height of the bar	20 mm
4	Aspect ratio (a/c)	0.6, and 1.0
5	Crack depth ratio (a/d)	0.1, 0.2, and 0.3

2.1 Crack Modeling

3-D meshed model of the un notched round bar with radial cracks is shown in Fig1b. The region around the crack front was partitioned from the rectangular bar to apply fine mesh around it (Fig.2a). In the present work, the “Contour integral evaluation” approach was used to estimate the SIF values around the crack region. The onset of cracking in quasi-static problems can be studied using contour integral evaluation method; however, contour integral estimation does not predict the crack growth. Each contour is a ring of elements surrounding the crack front. Many contour integral evaluations are possible at each location along the crack front whereas in the present work three contours were created around the crack front as shown in Fig.2b.

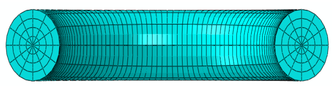


Figure 2(a) : Meshed crack region

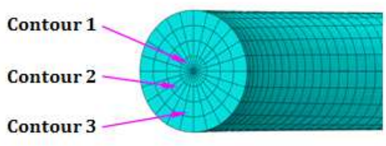


Fig. 2(b). Contours around crack region

2.2 Specification of crack front and crack extension direction

Figure 3a shows the un notched round bar with radial cracks. The crack front is the region which defines the first contour. The crack front in the round bar can be considered to be equivalent to the crack line in three dimensional problems. The direction of the virtual crack extension can be specified at each node along the crack front by specifying either the normal to the crack plane (n) or by specifying the crack extension direction. Since the present problem is 3-Dimensional in nature the crack propagation direction can not be predicted and thus, 'normal to the crack plane' approach was used to define the crack extension direction. In most of the fracture problems, the singularity at the crack tip should be considered for small strain analysis since it improves the accuracy of SIF values. To obtain a square root singularity, collapsed second order elements are used. To create a 3D crack tip singularity, 20-node brick and 27-node brick elements can be used with a collapsed face. If all the midface nodes are moved to their quarter points closest to the crack line, $\frac{1}{\sqrt{r}}$ singularity can be modeled. In the present work several contours have been considered around the crack front region and the top surface of the bar is constrained for all degrees of freedom and a tensile load is applied at the bottom surface as shown in Fig. 3b.

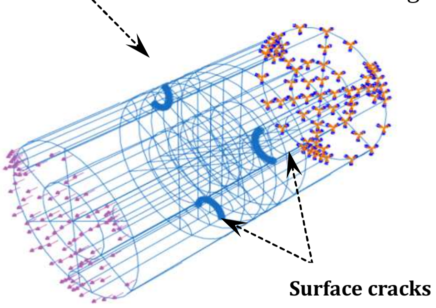


Fig.3a. FE model with radial cracks

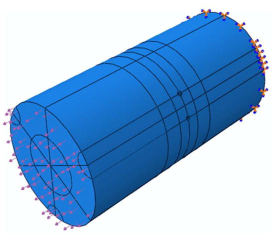
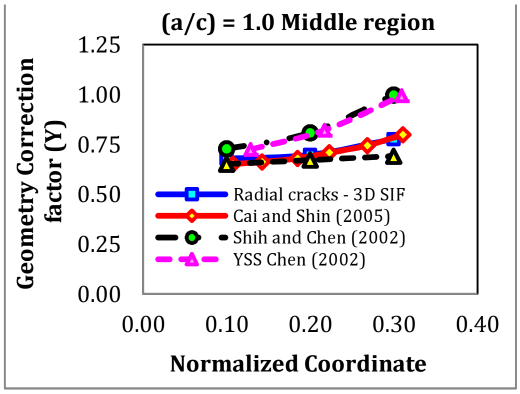


Fig.3b. FE model with boundary conditions

3.0 RESULTS AND DISCUSSION

SIF of un-notched round bar with radial cracks has been carried out using 3-D finite element analysis. At the middle region of the crack front ($P/P_0 = 0$) good agreement of SIF is observed between radial cracks and results of Cai and Shin (2005), Shih and Chen (2002), YSS Chen (2002) Courneau and Royer (1998) as shown in Fig. 4a. Multiple radial cracks SIF are higher at the crack surface region where the crack front interacts with free surface ($P/P_0 = \pm 1$) of the bar as shown in Fig. 4b. This is due to the interaction effect of radial cracks which accelerates the crack growth at higher crack depths.



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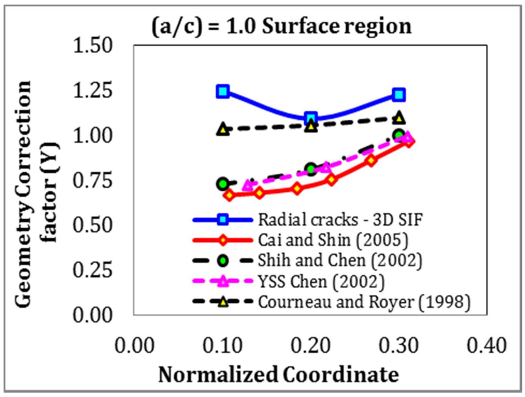

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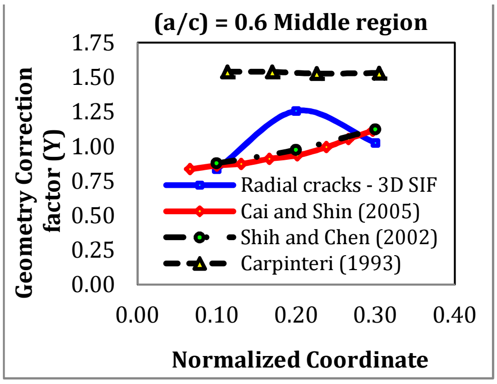
Fig.4. Comparison of present results with published data – semi elliptic crack (a/c) = 0.1

Figure 5a shows the SIF comparison of multiple radial cracks with published data for a semi elliptic crack. At the middle region ($P/P_0 = 0$), SIF of radially cracked bar is higher than published data. This is due to accelerated crack growth which might have caused by the multiple radial cracks whereas at the crack surface region ($P/P_0 = \pm 1$) marginal agreement of SIF between radial cracks and published data was observed (Fig. 5b).



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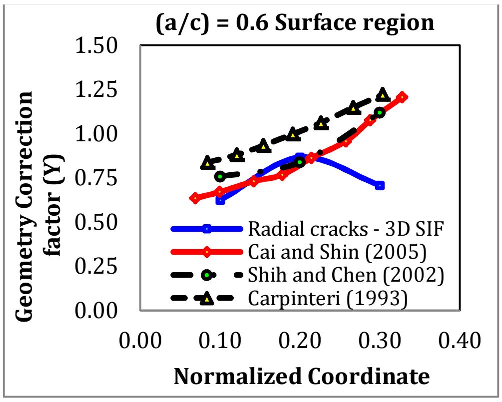


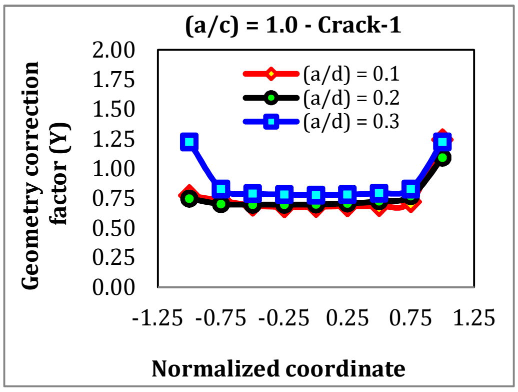
(b)

Fig.5. Comparison of present results with published data – semi elliptic crack (a/c) = 0.6

3.1 Effect of crack depth ratio (a/d) on SIF

Figure 6 shows the variation of SIF for a semi-circular crack (Fig. 6a) and semi-elliptic (Fig.6b) for different crack depth ratios (a/d) ranging between 0.1 and 0.3. The SIF values are measured along the entire crack front and normalized coordinate system was used to plot the SIF. P/P_0 values of ±1 represents the left and right surface of the crack front and $P/P_0 = 0$ represents middle region of the crack front. The SIF values of semicircular crack are higher at the crack surface region ($P/P_0 = \pm 1$) compared to middle region ($P/P_0 = 0$) at short crack depths ($a/d < 0.3$) as shown in Fig.6a.

Higher SIF values were observed at the crack middle region for an elliptic crack (Fig. 6b) irrespective of the crack depth ratios considered in the present work. Thus one can expect higher crack growth rate at the middle region of a semi-elliptic crack as the crack depth ratio increases. Non symmetric SIF distribution is observed for a semi- elliptic crack. The non-symmetric distribution is due to the interaction behaviour of the growing cracks as the crack depth ratio increases. The interaction behavior of the radial cracks increases the additional effect of mode-II and mode –III fracture.



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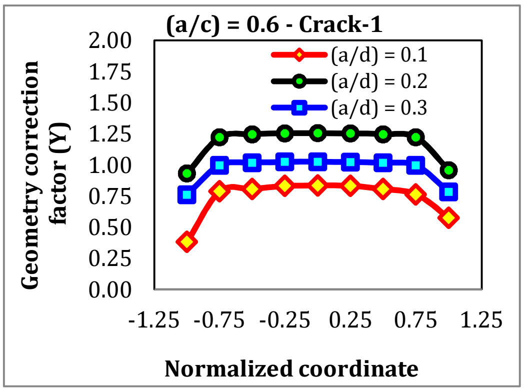

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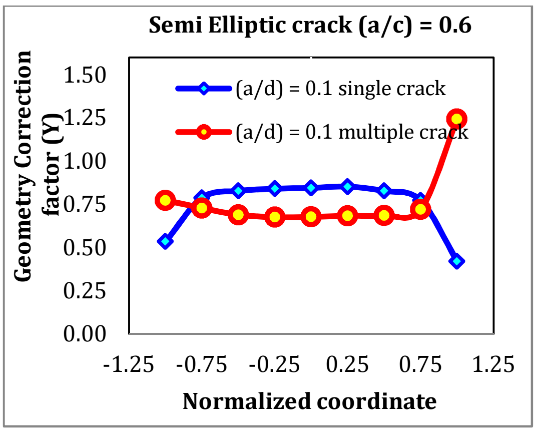
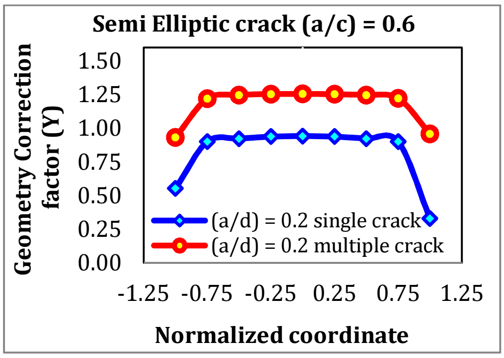
Fig.6. Effect of crack depth ratio on SIF

3.2 SIF comparison of single and multiple cracks

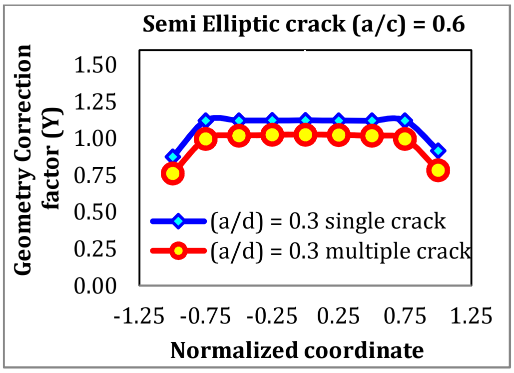
Figure 7 shows the variation of SIF for multiple semi-elliptic cracks located radially in a un notched round bar. Crack depth ratios ranging between 0.1 and 0.3 were considered. It is observed that the effect of multiple radial cracks is more pronounced at the surface region of the crack compared to middle region (Fig.7a). As the crack depth ratio increases, SIF values are higher at the middle region at higher crack depths (Fig.7c).



(a). (a/d) = 0.1



b. (a/d) = 0.2



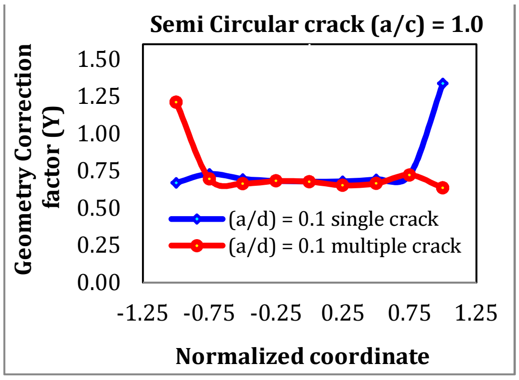
(c). (a/d) = 0.3

Fig.7. SIF comparison of single and multiple cracks – semi elliptic crack (a/c) = 0.6

Figure 8 shows the SIF variation for single and radial semi circular cracks in an un-notched round bar. Higher SIF and higher growth rate is observed for multiple radial cracks compared with single crack in a bar (Fig. 8a). It is noted that the middle region SIF is independent of number of cracks. As the crack depth ratio increases the effect of multiple radial cracks is also increases considerably.

3.3 Effect of crack aspect ratio (a/c) on SIF

Figure 9 shows the effect of crack aspect ratio (a/c) on SIF for radial cracks in an un notched round bar subjected to far-field tensile loading. Aspect ratios of 0.6 and 1.0 were considered in the present work and SIF values were measured along the crack front as shown in the Fig. 9. It is observed that SIF values of elliptic crack are higher at the crack middle region (P/P_0 = 0) and SIF values of semicircular crack are higher at the surface region of the crack (P/P_0 = ±1). As the crack depth ratio (a/d) increases, the geometry of the circular crack transforms into semi elliptic and straight crack. Thus one can expect a higher crack growth rate for semi elliptic crack at the middle region



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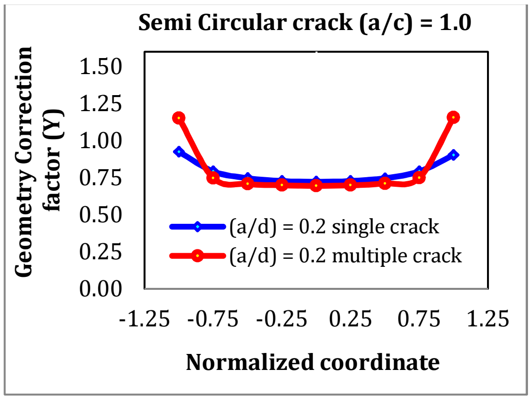

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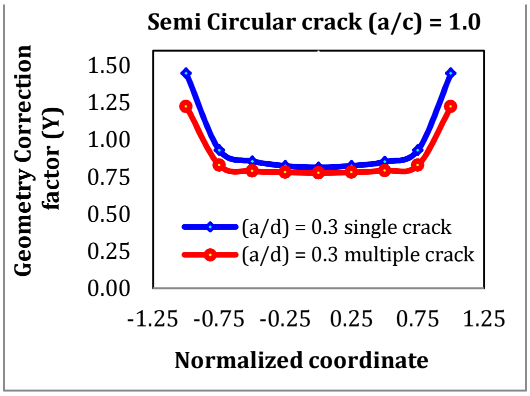

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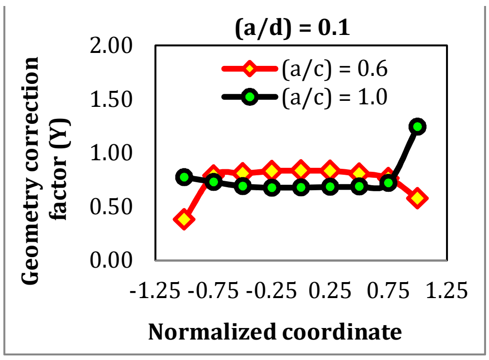
Fig.8. SIF comparison of single and multiple cracks – semi circular crack (a/c) = 1.0



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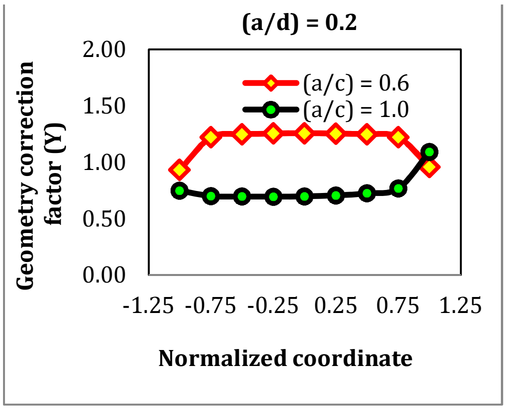

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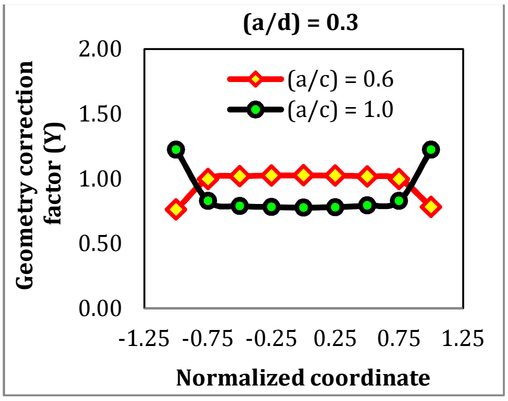


(c)

Fig.9. Effect of crack aspect ratio on SIF

3.4 Effect of interaction behavior among radial cracks on SIF

Figure 10 shows the interaction behavior of radial semi circular cracks on SIF. The interaction behavior of radial cracks 1&2 cracks 2 &3 and cracks 3&1 are calculated from the following relation.

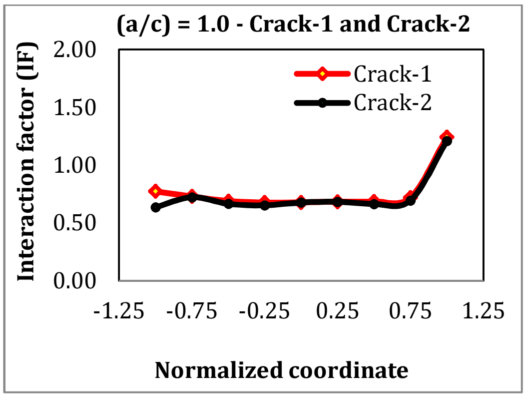
$$\text{Interaction factor} = K_{1,1}/K_{1,2} \qquad (1)$$

Where

$K_{1,1}$ – mode I SIF for the radial crack1

$K_{1,2}$ _ mode I SIF for the radial crack 2

It is observed that middle region of the crack is independent of number of crack geometry and thus the interaction behavior is marginal as shown in Fig 10a to 10c. The surface crack interaction increases with crack depth ratio (a/d) at the surface region. Non symmetric distribution of SIF and interaction factor is observed due to additional influence of mode II and mode III fracture due to radial cracks.



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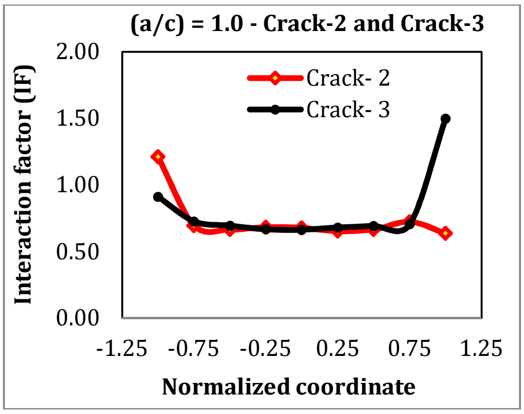

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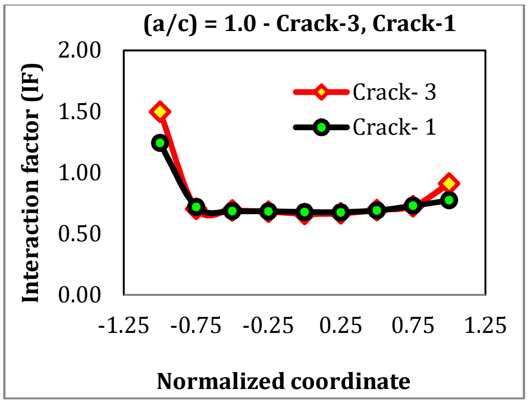


(c)

Figure 10: Variation of crack Interaction factor – semi circular crack (a/c) = 1.0

The interaction factors of semi- elliptic ((a/c) = 0.6) radial cracks for various crack depths are listed in Table.2. The tabulated values can be used to predict the SIF and life estimation for radial cracks in an un notched round bar.

Table 2. Interaction factor for radial semi- elliptic cracks (a/c = 0.6)

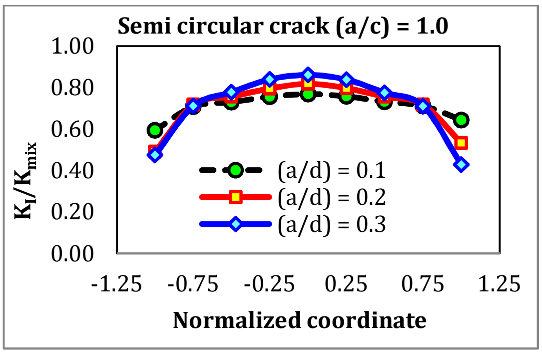
Geometric location (P/P_0)	(a/d)		
	0.1	0.2	0.3
-1.00	1.2178	0.6464	0.9123
-0.75	1.0091	0.9358	1.0008
-0.50	1.0368	0.9784	0.9957
-0.25	1.0362	0.9946	1.0010
0.00	1.0011	1.0043	1.0006
0.25	1.0021	1.0114	1.0004
0.50	1.0318	1.0240	1.0002
0.75	1.0363	1.0269	1.0012
1.00	1.0274	0.9463	1.0033

3.5 Effect of mixed mode fracture on SIF

The effect of torsion that causes a mixed mode fracture of un notched round bar was carried out numerically. The 'kinematic coupling' constraint was used to apply the torque at the bottom surface. In this approach all the surface nodes are attached to a common reference point through which the torque is applied to the surface. The combined torsion and tensile load causes mixed mode fracture and the mixed SIF was calculated from the following relation.

$$K_{mix} = \left[K_I^2 + K_{II}^2 + \frac{K_{III}^2}{(1-\nu)}\right]^{0.5} \qquad (2)$$

Where, K_I, K_{II}, and K_{III} are the Mode-I, Mode-II and Mode III SIF of un - notched round bar. The effect of individual fracture modes ($K_{(I,\ II\ and\ III)}/K_{mix}$) is plotted for various crack depth ratios and crack aspect ratios. Figure 10 shows the effect of individual fracture modes on SIF for a particular semi circular crack ((a/c) = 1.0). At the crack surface region ($P/P_0 = \pm 1$) dominant mode of fracture is mode II (Fig. 11b) whereas mode I and mode III fracture are higher at the crack middle region (P/P_0 = 0). Thus one can expect the in plane and out of plane shear failure modes at the surface region for a semi-circular crack. As the crack depth ratio (a/d) increases, mode I SIF at the crack surface region decreases due to the simultaneous increase of mode II fracture. At the crack middle region, mode I SIF increases with the increase of crack depth ratio. This is due to decrease of out plane shear fracture (mode-III) at the middle region with increase of crack depth ratio (Fig. 11c)



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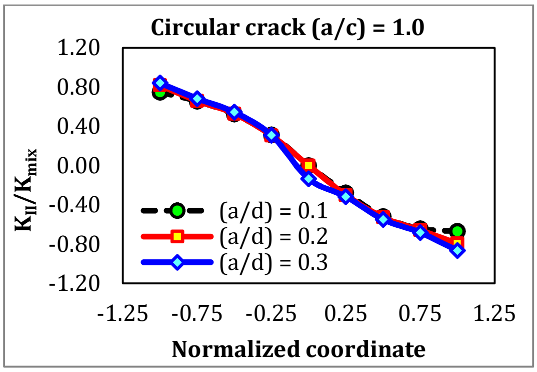

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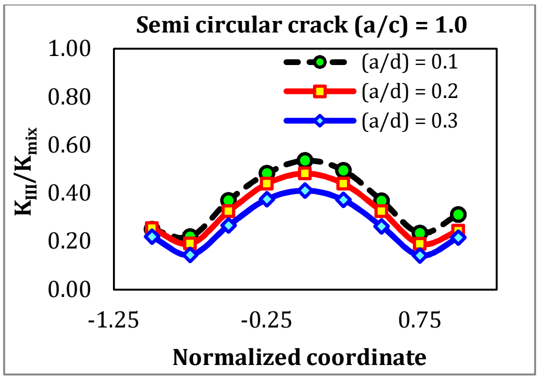
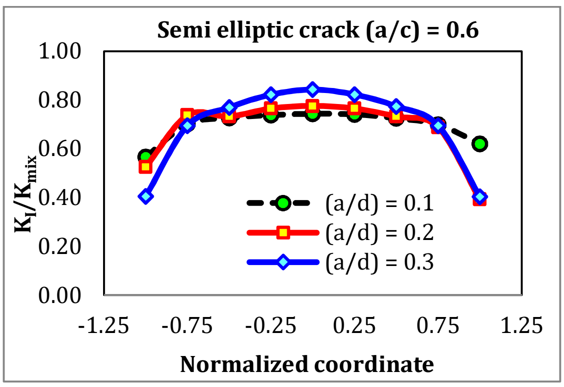


(c)

Fig.11. Effect of individual fracture modes – semi circular crack (a/c) = 1.0

Figure 12 shows the effect of individual fracture modes on failure of un notched round bar with multiple semi-elliptic cracks. Similar to the results of semi circular cracks, mode I SIF value increases with increase of crack depth ratio at the middle region of the crack ($P/P_0 = 0$). In contrast to the SIF results of semi circular cracks, a shift in mode II and mode III SIF is noted at a crack depth ratio of 0.2 (Fig 12(b) and 12(c)). The transition may be due to crack shape transition of semi elliptical crack into Semi circular crack.



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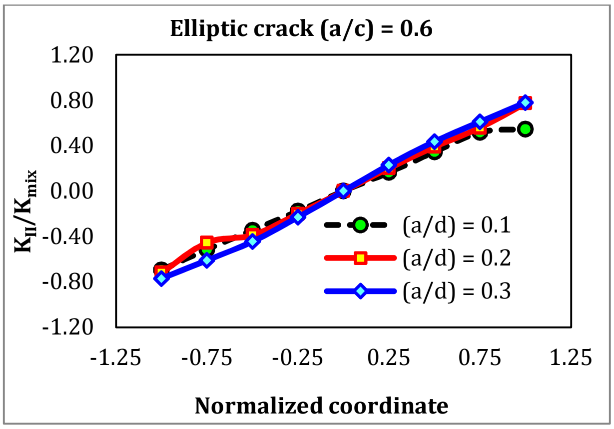

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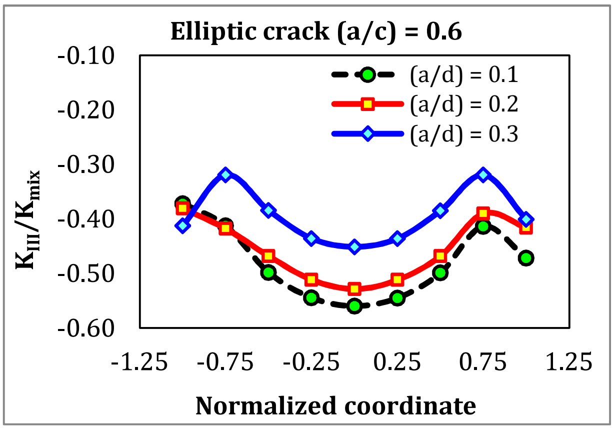

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Fig.12. Effect of individual fracture modes – semi circular crack (a/c) = 1.0

4. CONCLUSIONS

SIF of radial cracks in an un-notched round bar subjected to far field loading condition has been estimated numerically. Comparison of SIF of single and multiple cracks reveals the following conclusions.

1. At the middle region ($S/S_0 = 0$) of the crack front, SIF values are higher for semi-elliptic crack than semi-circular crack irrespective of the crack depth ratios considered. Thus one can expect higher crack growth rate at the middle region for a semi-elliptic crack compared to semi-circular surface crack.

2. As the aspect ratio (a/c) increases, SIF values decreases considerably along the crack front. This is due to curvature effect of the surface cracks with increasing aspect ratios.

3. The interaction effect of radial cracks is marginal at the middle region of the crack front whereas at the crack surface region ($S/S_0 = \pm 1$) the interaction effect is more significant.

4. The effect of interaction behaviour increases with crack depth ratio and it is higher for the deep cracks.

5. Comparison of present radial crack SIF values with published data shows a good agreement of SIF at the middle region of the crack front whereas at the crack surface region multiple cracks SIF values ar higher the single crack SIF.

6. The mixed mode fracture analysis suggests that the influence of mode II and mode III fracture can not be neglected at the crack surface region whereas at the middle region, mode I fracture governs the fracture.

5.0 REFERENCES

[1] Shivakumar, K.N., and J.C. Newman,"Stress intensity factors for large aspect ratio surface and corner cracks at a semi-circular notch in tension specimen". *International Journal of Engineering Fracture Mechanics*, 38, 467-473, 1991.

[2] Shih, Yan-Shin., and Jien-Jong Chen, "The stress intensity factor study of an elliptical cracked shaft", *International Journal of Nuclear Engineering and Design*, 214, 137-145, 2002.

[3] Cai, C.Q., and C.S. Shin,"A normalized area-compliance method for monitoring surface crack development in a cylindrical rod", International journal of fatigue, 27, 801-809. 2005.

[4] Couroneau, N. and J. Royer,"Simplified model for the fatigue growth analysis of surface cracks in round bars under mode I. *International Journal of Fatigue*, 20, 711-718, 1998.

[5] Carpinteri, A, "Shape change of surface cracks in round bars under cyclic axial loading", *International Journal of Fatigue*, 15, 21-26. 1993.

[6] Toribio, J., V. Sanchez-Galvez, M.A. Assize, and J.M Campos, "Stress intensity factor solutions for a cracked bolt under tension bending and residual stress loading", *International Journal of Engineering Fracture Mechanics*, 39, 359-371, 1991.

[7] Noda, N.A., and Y. Takase,"Generalized Stress Intensity Factors of V-shaped notch in a round bar under torsion, tension, and bending", *International Journal of Engineering Fracture Mechanics*, 70, 1447-1466, 2003.

[8] Guo, W.,H. Shen, and H. Li, "Stress Intensity Factors for elliptical surface cracks in round Bars with different Stress concentration coefficient", *International Journal of Fatigue*, 25, 733-741, 2003.

[9] Perl, M., and R. Arone," Stress Intensity Factors for Large Arrays of Radial Cracks in Thick Walled Steel Cylinders", *International Journal of Engineering Fracture Mechanics, 25, 341-348* 1986.

[10] Shu, H.M., J. Petit and G. Bezine, "Stress Intensity Factors for Radial Cracks in thick Walled Cylinders–II Combination of Auto frettage and Internal Pressure", *International Journal of Engineering Fracture Mechanics, 49, 625-629*,1994.