Profiled Shaft and Rotor Dynamics

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***Abstract*— Shaft-rotor systems consisting of multi disks and profiled shafts are taken into consideration. The determination of deflection, slope, shear force and bending moment at the extremities of the shaft are done using conventional mathematical procedures. Transfer matrix method (TMM) is used for the computation of the resonance, critical speed or whirling frequency conditions. For particular profiles and rotational speeds and lengths, the response of the system is determined. A built-in profiled shaft-rotor system with two disks and an impulse load of 1N on the disk at the free end of the system is investigated for illustration purpose. The step response of the multi-disk profiled shaft-rotor system is also found.**

***Keywords*— profiled; rotor; frequency; response; transfer matrix**

I. Introduction

Rotor dynamics and stability of shaft-rotor systems has been the concern of engineers and scientists for more than a century, and it will continue to persist as an active area of research and study in coming future. Rotating shafts are employed in industrial machines like steam and gas turbines, internal combustion engines, compressors and pumps for power transmissions. Increasing demand for power and high speed transportation makes the study of vibratory motion essential. Rotor-dynamics [1] deal with dynamics of rotating machines. It is different from structural vibration analysis because of the gyroscopic moments. Transfer matrix method is commonly employed for rotor dynamics analysis and is one of the accurate analytical methods.

Basic idea of Transfer Matrix Method (TMM) was first given by Holzer for finding natural frequencies of torsional systems. Myklestad [2, 3] adapted the method for computing natural frequencies of airplane wings. Prohl [4] applied it to rotor-bearing systems in which gyroscopic moments were included. Lund [5] used complex variables and showed how system damping could be accounted for including self-exciting influences, such as oil whip and/or internal frictions. The above developments led to the method came to be known as "The Transfer Matrix Method" [6]. Computation of critical speed is the main parameter for the design of shaft-rotor systems. Improved method for computing critical speeds and rotor stability is done by Murphy and Vance [7]. Whalley and Abdul Ameer [8] used frequency response analysis for profiled shafts to study the dynamic response of shaft-rotor system, but with single disk. Also, the effects of rotor length and rotating speed were not included. They studied the system behavior for the shafts-rotors with diameters which are functions of their lengths. They derived an analytical method using Euler-Bernoulli beam theory combined with the transfer matrix method. Here, profiled shaft with dual-disks with different profiles, lengths and rotational speeds have been considered with the method TMM for frequency response calculation and then is validated with Whalley and Ameer [8] for a single disk. Further the step response of the system is also found.

Since less work has been done on the profiled shaft-rotor systems, the dynamics of the system is to be considered. Further, most of the works on rotating systems have been done with particular cylindrical element type shaft-rotor with uniform cross-sectional area. In this paper, rotor-shaft with continuously varying cross-sectional area is presented. Also, effects of important factors like rotor-lengths and rotor-speeds are shown separately.

Nomenclature

A(x) = area of cross-section
C(x) = compliance per unit of length
L(x) = inertia per unit of length
L=length of shaft
E=modulus of elasticity
F(s) = system model matrix
$M_y(x, s)$ =bending moment in x–y plane
$q_y(x, s)$=shear force
Y(x, s) =vertical deflection of shaft
θ(x, s)=slope of the shaft
I(x) =mass moment of inertia
ρ=material density
J=shaft polar moment of inertia
Ω=whirling frequency
Γ(s) =wave propagation factor
Ω=shaft–rotor rotational speed
R(s) =rigid rotor model matrix
m=rigid disk mass
r(x)=shaft radius at distance x from bearing

II. Mathematical modelling

A. Shaft Model

The vibrating shaft model is illustrated in the figure below. Input and output relationship for deflection, slope, bending moment and shear force for the distributed parameter shaft model [8] is given by,

$$(y_2, \theta_2, M_{y2}, Q_{y2})^{\mathrm{T}} = F(s)\,(y_1, \theta_1, M_{y1}, Q_{y1})^{\mathrm{T}} \qquad (1)$$

Where y, θ, M_y and Q_y are the deflection, slope, bending moment and shear force respectively.

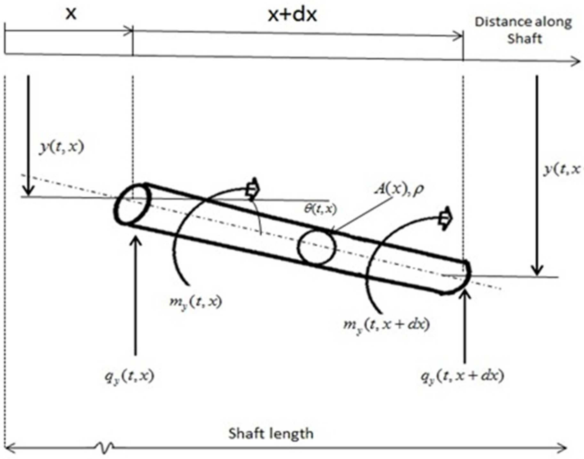

Fig. 1. Vibrating shaft model

Where F(s) =

$$\begin{bmatrix} \frac{\cos\sqrt{\Gamma(l)}l+\cosh\sqrt{\Gamma(l)}l}{2} & \frac{\sin\sqrt{\Gamma(l)}l+\sinh\sqrt{\Gamma(l)}l}{2\sqrt{\Gamma(l)}} & \frac{C(0)(\cos\sqrt{\Gamma(l)}l-\cosh\sqrt{\Gamma(l)}l)}{2\Gamma(l)} & \frac{C(0)(\sin\sqrt{\Gamma(l)}l-\sinh\sqrt{\Gamma(l)}l)}{2\Gamma(0)\sqrt{\Gamma(0)}} \\ \frac{-\sqrt{\Gamma(0)}(\sin\sqrt{\Gamma(l)}l-\sinh\sqrt{\Gamma(l)}l)}{2} & \frac{\cos\sqrt{\Gamma(l)}l+\cosh\sqrt{\Gamma(l)}l}{2} & \frac{-C(0)(\sin\sqrt{\Gamma(l)}l+\sinh\sqrt{\Gamma(l)}l)}{2\sqrt{\Gamma(0)}} & \frac{C(0)(\cos\sqrt{\Gamma(l)}l-\cosh\sqrt{\Gamma(l)}l)}{2\Gamma(0)} \\ \frac{\Gamma(0)(\cos\sqrt{\Gamma(l)}l-\cosh\sqrt{\Gamma(l)}l)}{2C(0)} & \frac{\sqrt{\Gamma(0)}(\sin\sqrt{\Gamma(l)}l-\sinh\sqrt{\Gamma(l)}l)}{2C(0)} & \frac{\cos\sqrt{\Gamma(l)}l+\cosh\sqrt{\Gamma(l)}l}{2} & \frac{\sin\sqrt{\Gamma(l)}l+\sinh\sqrt{\Gamma(l)}l}{2\sqrt{\Gamma(0)}} \\ \frac{-\Gamma(0)\sqrt{\Gamma(0)}(\sin\sqrt{\Gamma(l)}l+\sinh\sqrt{\Gamma(l)}l)}{2C(0)} & \frac{\Gamma(0)(\cos\sqrt{\Gamma(l)}l-\cosh\sqrt{\Gamma(l)}l)}{2C(0)} & \frac{-\sqrt{\Gamma(0)}(\sin\sqrt{\Gamma(l)}l-\sinh\sqrt{\Gamma(l)}l)}{2} & \frac{\cos\sqrt{\Gamma(l)}l+\cosh\sqrt{\Gamma(l)}l}{2} \end{bmatrix}$$

B. Rigid disk

The output vector from the shaft will become the input for the rigid disk model, as shown in Fig. 2. i.e., for single disk and shaft model, we have
$y_3(s) = y_2(s)$,$\Theta_3(s) = \Theta_2(s)$,$M_{Y3}(S) = -J\Omega s\Theta_2(s) + M_{y2}(s)$ and $Q_{y3}(s) = ms^2Y_2(s) + Q_{y2}(s)$

Hence $(Y_3(s), \theta3(s), My3(s), Qy3(s))^T = R(s)$ $(Y_2(s),\theta_2(s),My_2(s),Qy_2(s))$

Where, R(s)= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -J\Omega s & 1 & 0 \\ ms^2 & 0 & 0 & 1 \end{bmatrix}$ (2)

III. RESULTS AND DISCUSSIONS

A. Cantilever Shaft-rotor system with Multi Disks

A Cantilever profiled Rotor-Shaft System with two disks at different lengths is shown in Fig 2. By replacing l_1, m_1, J_1 and l_2, m_2, J_2 instead of l, m and J as in [6], for suffixes 1 and 2 respectively for the rotors-shafts and neglecting the bearing effects, as proceeded in the above sections, in the same way for the dual disk systems illustrated in Fig. 2 can be formulated as-

H(s) =$R_2(s).F_2(s)R_1(s).F_1(s)$, and (3)

$(Y_1(s), Y_5(s))$,$(\Theta_1(s), \Theta_5(s))$, $\left(M_{y1}(s), M_{y5}(s)\right)$, and $(Q_{y1}(s), Q_{y5}(s))$ are the deflections, slopes, Bending Moments and Shear Forces at the Fixed and Free end respectively. Default data for the system illustrated in Fig. 2 is given below:

$$m_1 = 0.6 \text{ kg}, m_2 = 0.7 \text{ kg},$$
$$d_1 = 0.08 \text{ m}, d_2 = 0.09 \text{ m},$$
$$\rho = 7800\ {kg}/{m^3},\ E = 209\text{x}10^9 Pa$$
$$\omega = 10000 \text{ rpm and}$$
$$l_1 = l_2 = 0.075 \text{ m}, r_0 = 0.005 \text{ m}$$

From eq. (A1.11), we get

$H(s) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$, and input-output vectors relationship is given by:

$$\begin{bmatrix} y_5 \\ \theta_5 \\ M_{y5} \\ Q_{y5} \end{bmatrix} = H(s) \begin{bmatrix} y_1 \\ \theta_1 \\ M_{y1} \\ Q_{y1} \end{bmatrix}$$

After applying the boundary conditions, we get deflection y_5 at the free end of the system, so we will ultimately get the transfer function, where the profile equation of the shaft-rotor is given by [8] $r(x) = r0(1 - NN(x^2))$

For example, transfer function for NN=40 and rotational speed of 10000 rpm with 1 N vertically downward force on the disk is:

$$\frac{1.667s^4 + 7.521\text{x}10^4 s^3 + 7.351\text{x}10^8 s^2 + 3.597\text{x}10^{11} s + 2.359\text{x}10^{15}}{s^6 + 4.513\text{x}10^4 s^5 + 4.444\text{x}10^8 s^4 + 2.856\text{x}10^{11} s^3 + 1.631\text{x}10^{15} s^2 + 1.065\text{x}10^{17} s + 1.467\text{x}10^{20}}$$

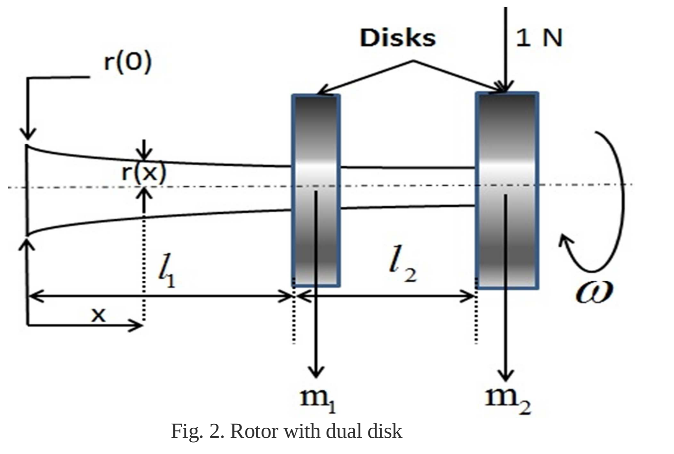

Fig. 2. Rotor with dual disk

The profile equation for the shaft-rotor [8] is given by $r(x) = r_0(1 - NN(x^2))$ For NN=40, r0=0.005, 10,000 rpm and unit force, for single disk the results are matching as in Whalley and Ameer [8], as shown in Fig. 3.

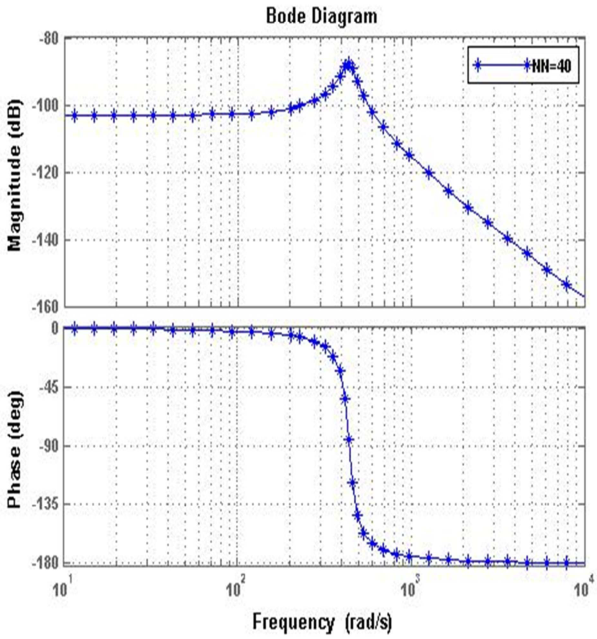

Fig. 3. Bode plot for single disk

B. Results for Dual Disks

Profile equation [8] is: $r(x) = r_0(1 - NN(x^2))$ Bode plots for different profile values, rotating speeds and shaft-rotor lengths have been obtained and are shown in Fig. 4, Fig. 5 and Fig. 6 respectively and the results obtained are tabulated for better understanding.

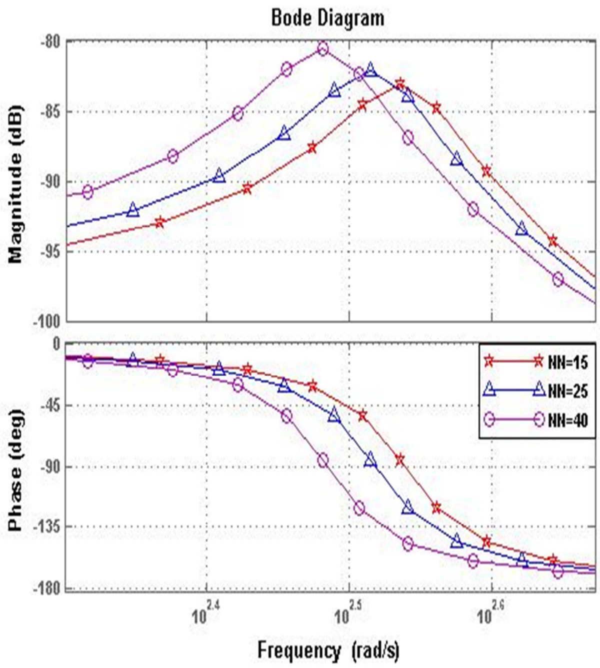

Fig. 4. Bode diagram for varying profiles

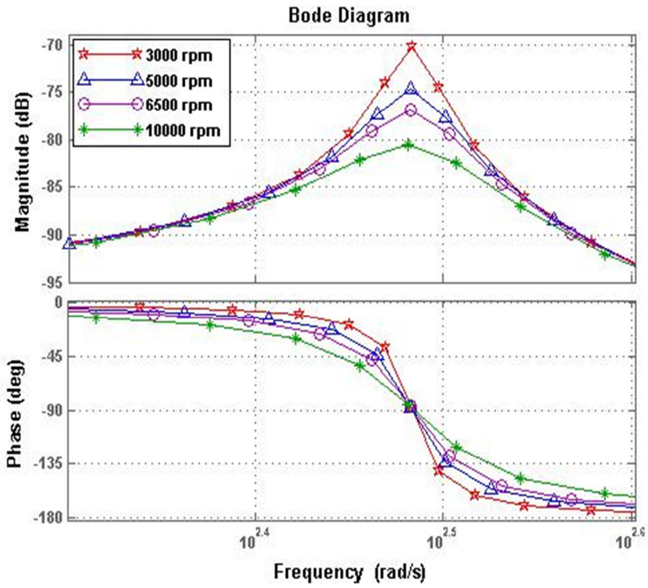

Fig. 5. Bode diagram for different rotor speeds

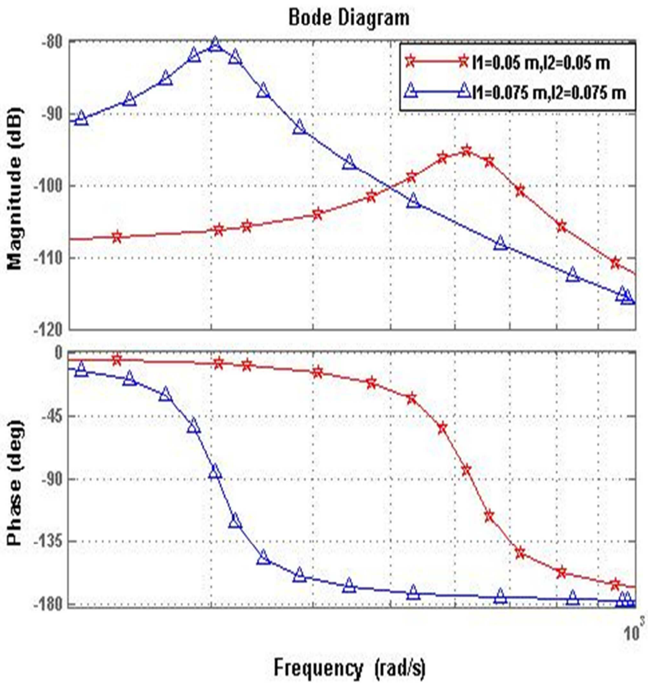

Fig. 6. Bode diagram for different rotor lengths

The x-axis of bode plot represents the frequency in rad/s, while y-axis represents the amplitude in dB. However they can be represented by other units. The whirling frequency is noted at the peaks in magnitude diagram of bode. All the plots are obtained using MATLAB® software.

Table I. Results obtained from Bode plots.

l_1 (m)	l_2 (m)	Speed (rpm)	Value of NN	Critical Frequency (rad/sec)	Amplitude (dB)
0.050	0.050	10000	40	620	-95.1
0.075	0.075	10000	40	303	-80.6
0.075	0.075	10000	15	343	-83.1
0.075	0.075	10000	25	327	-82.1
0.075	0.075	3000	40	304	-70.1
0.075	0.075	5000	40	304	-74.5
0.075	0.075	6500	40	304	-76.8

C. Step Response

For NN=40, at 11000 rpm, the step response, following an impulse of unit load (in Newton) at the free end, gives the characteristics shown in Fig. 7. This indicates that steady state conditions will be restored in approximately 0.134 s. At lesser rotational speeds the effects would be much greater because of the reduction in the gyroscopic couple. In Fig. 7, at 5500 rpm the maximum overshoot remains almost unchanged, but its settling time is almost double than for 11000 rpm, i.e., 0.276 s, after the same impulse disturbance. As shown herein, this reveals a part of the problem.

IV. CONCLUSION

The establishment of the vibrational characteristics of multi disk profiled shaft-rotor systems present challenging problems. The vibration analysis with the help of bode plot has been done for the multi-disks profiled rotor system.

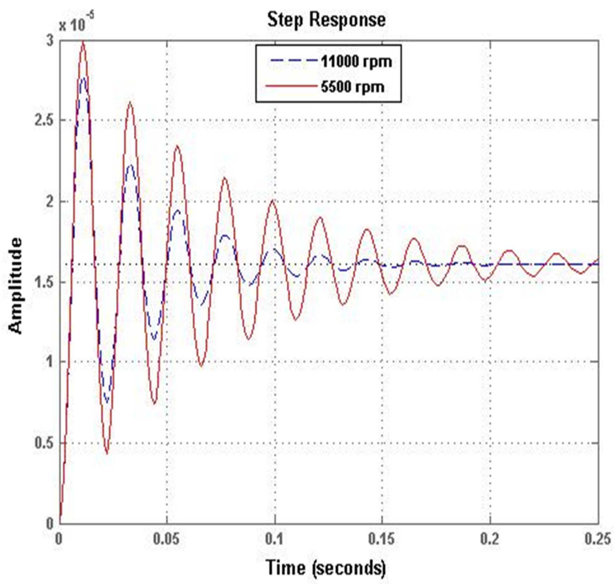

Fig. 7. Step Response

Here, multi-disk-rotor systems are shown in Fig. 2 for the illustration purposes. For a particular speed and varying NN, as shown in Fig. 4 and results shown in Table I, the resonant frequency decreases with the increase in the value of NN while amplitude increases for values of NN from 20 to 50. For varying speed, we get the bode plot as shown in Fig. 5. Keeping the lengths and value of NN unchanged, the critical frequency remains almost unaltered as shown in the Table I, while the amplitude goes on decreasing. For different shaft-rotor lengths, we get the bode plot as shown in Fig. 6, where the speed and value of NN kept unchanged, the critical frequency decreases tremendously with increasing length as shown in Table I, while the amplitude goes on increasing. Additionally the step response is also shown in Fig. 7 and hence the analysis of multi-disk profiled shaft-rotor system has been done clearly, including effects of rotor-lengths and rotor-speeds, on the dynamic magnification and whirling speeds of the system, which are not shown in [8].

Gears and other such rotating devices can be mounted instead of cylindrical discs and further calculations can be made.

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