# Mathematical Modelling and Simulation Of Semi-Active Suspension System For An 8×8 Armoured Wheeled Vehicle With 11 DOF

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Abstract—Stability and ride comfort in multi-wheeled military vehicles depend on a combination of vertical motion and angular motion of pitch and roll. Since multi-wheeled military vehicles are intended to deliver accurate firing attack, stability and ride comfort for the military vehicles is a must. This study focuses on modelling and predicting vehicle response under the source of road disturbance. A mathematical model has been developed and control algorithm has been verified with the purpose/objective of reducing the unwanted sprung mass motions such as heave, pitch and roll. Semi active suspension gives better ride comfort with consumption of fraction of power required for active suspension. In this paper, the comparative performance of passive suspension along with two semi-active suspension control logics namely, on-off skyhook and continuous skyhook has been demonstrated through simulations. The ride comfort is evaluated in terms of rms acceleration at CG in vertical direction (Z), which is the major contributor for ORV Measurement.

Keywords—AFV Suspension; Semi Active Suspension; Sky hook damping; MagetoRheological Damper; Ride comfort Analysis; Mathematical modelling;

#### I. INTRODUCTION

Here in this study a mathematical model of the suspension system of an 8x8 armored fighting vehicle is developed in Simulink using first principles modeling technique for passive suspension system. Subsequently modeling is done for on-off skyhook and continuous skyhook by replacing the damping force gain in each suspension centers. Two types of damping control strategy are considered for the vibration control, viz. on-off sky hook damping control and continuous skyhook damping control, in both control strategies, the damping force can be controlled using electric current in an appropriate magneto rheological damper in actual case. The damping force is varied by controlling the current flow to the damper. The viscosity of the fluid inside the damper is proportional to the magnetic field produced due to the current flowing through the damper. The damping force is decided by the control system from the velocity of unsprung mass vertical velocity and relative velocity across the damper.

The suspension model considered is having 11 DOF Viz. 8 vertical wheel displacements and Roll, Pitch and bounce.

A first order lag with 0.001 second time lag is given to the damping force in semi-active system to compensate for the loop execution time & inertia of damper parts in actual process.

Three kinds of disturbances are simulated, 3 Hz sinusoidal pitch mode disturbance of 5 cm amplitude with 90 degree phase difference between front and rear axles, 3 Hz sinusoidal roll mode disturbance of 5 cm amplitude with 90 degree phase difference between left and right wheels and Step input of 10 cm between 4 axles with a time delay of 2.5 sec between the axles which depicts a step climbing exercise. The spectrum of frequency covered under primary ride is up to 5 Hz. Hence an exciting frequency of 3 Hz is taken for the comparison.

#### II. CONTROL STRATEGIES CONSIDERED

In semi active suspension system, the damping force is controlled to improve ride comfort. Force from the road is transmitted to the vehicle body through the suspension spring and damper. For instance if damper is inactive the force that is transmitted to the body would be less by a factor damping constant times damper velocity. This would surely reduce the acceleration of the vehicle body but due to the trailing oscillation, the r.m.s acceleration level would increase, creating discomfort to the passengers due to high ORV. On the other hand for cases like bump and step climbing it is advisable to avoid jerks to some extent. For achieving semi active damper control two types of logics are being used.

## A. On-off Skyhook damping control [2]

In on-off skyhook control the damping value is selected as high, if the product of absolute velocity of sprung mass and relative velocity of sprung mass with respect to un-sprung mass is greater than zero and else low damping. The damping force is given by the equation(1)

$$f_d = \begin{cases} c_{sky} \dot{z}_{abs} & \text{if } \dot{z}_{abs} \, \dot{z} \ge 0\\ c_{min} \dot{z}_{abs} & \text{if } \dot{z}_{abs} \, \dot{z} < 0 \end{cases}$$
(1)

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#### B. Continuous Skyhook damping control [2]

In continuous skyhook, if the product of absolute velocity of sprung mass and relative velocity of sprung mass with respect to un-sprung mass is greater than zero, the force is proportional to ratio of absolute velocity to relative velocity of sprung mass with respect to unsprung mass. The damping force is given by the equation(2)

$$f_{d} = \begin{cases} Sat_{C \in [c_{min}:c_{max}]} \left| c_{sky} \frac{\dot{z}_{abs}}{\dot{z}} \right| \dot{Z}_{abs} & if \quad \dot{z}_{abs} \dot{z} \ge 0 \\ c_{min} \dot{z}_{abs} & if \quad \dot{z}_{abs} \dot{z} < 0 \end{cases}$$
(2)

## III. ASSUMPTIONS IN MODELLING

The mathematical equations governing the suspension system are derived by simplifying the system by considering the following assumptions. The components are considered as linear, No bump stop contact, All displacements are measured with respect to equilibrium conditions and hence gravity is ignored, Small displacements, Ground contact is maintained continuously, Rigid vehicle body and No damping in tyre.

The parameters like spring stiffness, damping constant, moment of inertia (Ixx & Iyy), unsprung mass, vehicle weight, track width, position of axles with respect to CG etc. are given to the Simulink model. The effective Damper and spring force occurring at the wheel is considered for the simulation, the component stiffness and damping required is calculated by considering the mechanical advantage and line of action of tyre force by virtue of component location.

## IV. 11 DOF VEHICLE MODEL& EQUATIONS



FIGURE.1

Figure.1 shows the vehicle suspension system of the considered armoured fighting vehicle which consists of the sprung mass connected to eight unsprung masses total with a 11 DOF. The sprung mass is represented as a plane and is allowed to heave, pitch and roll, while the unsprung

masses are allowed to bounce vertically with respect to the sprung mass.

The equation governing the suspension system is derived from figure.1 as follows.

Sprung mass displacements at suspension centers for  $1^{st}$  and  $4^{th}$  axles are given by the following equations similarly equations with same signs are used for  $2^{nd}$  and  $3^{rd}$  axles.

$$z_{l1} = z - \theta x_{l1} - \varphi \times \frac{t}{2}$$
(3)

$$z_{r1} = z - \theta x_{r1} + \varphi \times \frac{t}{2} \tag{4}$$

$$z_{l4} = z + \theta x_{l4} - \varphi \times \frac{t}{2} \tag{5}$$

$$z_{r4} = z + \theta x_{r4} + \varphi \times \frac{t}{2} \tag{6}$$

Equation for unsprung mass from figure.1 is

$$m_{u}\ddot{z}_{uln} = k_{tln}(z_{gln} - z_{uln}) - k_{ln}(z_{uln} - z_{ln}) - c_{ln}(\dot{z}_{uln} - \dot{z}_{ln})$$
(7)  
$$m_{u}\ddot{z}_{urn} = k_{trn}(z_{grn} - z_{urn}) - k_{ln}(z_{urn} - z_{rn}) - c_{rn}(\dot{z}_{urn} - \dot{z}_{rn})$$
(8)  
Where n=1-4

This gives 8 equations which represent the 8 wheel displacement motions.

Equation for the sprung mass is given by

(a) Equation for bouncing

$$m\ddot{z} = \sum_{\substack{n=1 \\ n=1}}^{n=14} k_n (z_{un} - z_n) + c_n (\dot{z}_{un} - \dot{z}_n) + \sum_{\substack{n=r+1 \\ n=r+1}}^{n=r+1} k_n (z_{un} - z_n) + c_n (\dot{z}_{un} - \dot{z}_n)$$
(9)

(b) Equation for pitching

$$l_{yy}\ddot{\theta} = \sum_{n=l1}^{n=l4} x_n \times \{k_n(z_{un} - z_n) + c_n(\dot{z}_{un} - \dot{z}_n)\} + \sum_{n=r1}^{n=r4} x_n \times \{k_n(z_{un} - z_n) + c_n(\dot{z}_{un} - \dot{z}_n)\}$$
(10)

(c) Equation for Rolling

$$I_{xx}\ddot{\varphi} = \sum_{n=1}^{n=14} \frac{t}{2} \times \{k_n(z_{un} - z_n) + c_n(\dot{z}_{un} - \dot{z}_n)\} - \sum_{n=1}^{n=14} \frac{t}{2} \times \{k_n(z_{un} - z_n) + c_n(\dot{z}_{un} - \dot{z}_n)\}$$
(11)

Where ' $z_g$ ' is the deflection of the tyre due to road irregularity, ' $z_u$ ' is the unsprung mass displacement from equilibrium position, 'z' is the vertical displacement of the sprung mass, ' $\theta$ ' and ' $\phi$ ' pitch angle and roll angle displacement from equilibrium condition, 't' track width and 'x' the distance of particular axle from CG.

Moment due to the force at reference points 1 and 2 are taken as positive and reference points 3 and 4 are negative about y-axis. (This is taken care by giving negative values in Simulink model)

The suspension parameters are tuned as per Olleys Criteria by fixing the natural frequency of pitch, roll and bounce near to 1 Hz [1].

From equations derived for passive suspension model, semi active suspension model is made by replacing damping force part of equation (7) & (8) by the equation (1) & (2) for on-off and Continuous control logics respectively.

# V. MODELLING IN SIMULINK

The second order linear differential equations derived in section IV is modeled in Simulink, Initially model for a passive suspension system was made in Simulink and then the damper gain block whose output giving the damper force is replaced by a block for on-off and continuous which is inside by the on-off and continuous block of the Simulink model shown in figure.2

The model is having 8 inputs and 4 outputs. Independent signals can be given to each wheel station. The amplitude, phase lag and frequency can be varied by clicking the input signal block. The outputs are pitch, roll angle, vertical displacement and acceleration at CG.

The type of disturbance is selected by the switch block for sinusoidal or step and the signal is given to blocks of passive, on-off and continuous blocks. the output is directly going to the Matlab Workspace from where it is plotted.

First order lag of 0.001 time delay is used for the damping force to smoothen the sudden change in damping force as in actual case

The parameters used in the model are given in table. I.

SI No	MODEL PARAMETERS				
	Parameter	notation	Value		
1	Mass of the vehicle	т	23600 kg		
2	Each unsprung mass	$m_u$	200 kg		
3	Pitch Inertia	I <sub>yy</sub>	$90000 \ kgm^2$		
4	Roll inertia	I <sub>xx</sub>	32000 kgm <sup>2</sup>		
5	Spring Stiffness	k	156kN/m		
6	Passive damping constant	С	14.5 kNs/m		
7	On-off skyhook max damping constant	C <sub>sky on-off</sub>	29 kNs/m		
8	Continuous Sky hook max damping constant	C <sub>sky conti</sub>	44 kNs/m		
9	Min damping constant for Skyhook damping	C <sub>min</sub>	1.5 kNs/m		
10	1 <sup>st</sup> axle location from CG	<i>x</i> <sub>1</sub>	2.388 m		
11	2 <sup>st</sup> axle location from CG	<i>x</i> <sub>2</sub>	0.788 m		
12	3 <sup>rd</sup> axle location from CG	<i>x</i> <sub>3</sub>	-1.312 m		
13	4 <sup>th</sup> axle location from CG	$x_4$	-2.812 m		
14	Track width	t	2.4 m		

TABLE I.



FIGURE.3

The simulink model made for the suspension systems is shown in figure.2. The first subsystem block is containing the model of passive suspension system, second on-off skyhook and third block having continuous skyhook. Internal details of passive suspension block are shown in figure.3 which is again having subsystem blocks having heave, pitch, roll calculation block and suspension units block. Heave Pitch and Roll blocks is as detailed in figure.4. Suspension units block internal details are shown in figure.5 in which equations of left and right suspension units are grouped separately whose internal details are shown in figure.6

The internal details of blocks of unsprung mass shown in figure.6 are detailed in figure.7, when the same unsprung mass equation is modeled for sky hook damping control it is as shown in figure.7 where the damping force calculated by the product of passive damping constant and damper velocity and the same is replaced by a separate block which will produce the damping force as per the semi active requirement as per the logic given by the equation (1) & (2) respectively for on-off and continuous skyhook damping control. Figure.9 shows the Simulink model for damping force calculation for on-off sky hook damping and figure.10 shows Simulink model for damping force calculation for continuous skyhook damping control. The damping force is calculated by taking the sprung mass velocity at suspension mounts and unsprung mass velocity which gives the damping force as output.





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# VI. SIMULATIONS

Three set of simulations are done a) Sinusoidal Pitch model disturbance with 3 Hz and 5 cm amplitude to front and rear wheel with a phase difference of 90 degrees b) Sinusoidal Roll model disturbance with 3 Hz and 5 cm amplitude in which the left and right wheels are having 90 degree phase difference. c) Step input to different axles with time delay of 2.5 seconds which replicates a step climbing event. The simulation is done for 10 sec for each case. The solver used is variable step ode45, The results are plotted after the system got settled in case of sinusoidal disturbance. Normalised tyre force which is the ratio tyre force to corner weight which represents road holding capability of the vehicle is also plotted for the study.



-0.

-0.2

-0.3

1.4

Roll Angle [Degrees] 0.2 0 -0 2 -0.4 -0 3.6 3.7



Vertical Displacement of CG for Roll Mode Disturbance







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Comparison of r.m.s Vertical acceleration at CG						
SI	Disturbance	Passive	On-off	Continuous		
No	kind		Skyhook	Skyhook		
1	Pitch mode	3.06m/s <sup>2</sup>	$2.42m/s^{2}$	$2.05m/s^{2}$		
2	Roll mode	3.17m/s <sup>2</sup>	$2.65 m/s^2$	$2.11m/s^2$		
3	Step 10cm	$0.58m/s^{2}$	$0.51m/s^2$	$0.47m/s^2$		

# VII. RESULT & DISCUSSION

11 DOF model of 8x8 Armoured Fighting Vehicle has been developed and derived mathematically in this study. The comparative performance of passive suspension along with two semi-active suspension control logics namely, on-off skyhook and continuous skyhook has been demonstrated through simulations. Simulation studies have been performed for three conditions namely, (a) 3 Hz sinusoidal pitch mode disturbance of 5 cm amplitude with 90 degree phase difference between front and rear axles, (b) 3 Hz sinusoidal roll mode disturbance of 5 cm amplitude with 90 degree phase difference between left and right wheels and (c)Step input of 10 cm between 4 axles with a time delay of 2.5 sec between the axles.

From the simulation studies, it is clear that the unwanted body motions in form of body displacement, body rms acceleration, roll angle and pitch angle are significantly reduced in case of continuous skyhook control as compared to on-off skyhook and passive suspension system. Normalized tyre force (NTF) which is a direct representation of stability is also studied. Variations in the NTF are also less with continuous skyhook control. Hence the semi-active control logic finds promising application in the design of multi-axle multipurpose wheeled Armoured vehicles. The reduction in rms acceleration levels at CG indicates reduced fatigue to the combatants and military payloads. Also the reduction in acceleration has a promising chance of improving the firing accuracy of the platform.

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