

# Analysis and Control of a Six Link Serial Manipulator with Flexible Joints

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**Abstract** - In this paper, a classical PID controller has been developed for a six-link, six-joint flexible robotic manipulator model. The flexibility has been given to the joint of the system model constructed in sim-mechanics environment. In the first stage, the simmechanics simulation results have been presented to ensure greater acceptability of the flexible model over the rigid joint-link manipulator. Finally, with the help of SISO design toolbox, a classical PID controller has been incorporated. Gains of the PID controller have been tuned using three methods and compared. Results corresponding to the optimized gains have been presented and discussed.

**Keywords**- Serial Manipulator; Flexible Joints; Simmechanics; PID control; SISO design toolbox.

## I. INTRODUCTION

Flexible link manipulators are preferred over the rigid manipulators due to its high pay-load carrying capability and precise high speed operation. Research on the dynamic analysis of flexible manipulators has received much attention in the last couple of decades. Mechanical flexibilities on the manipulator can be added in both the links and joints. Link flexibility reduces the weight of the manipulator arms that are designed to operate at high speed with low inertia. Joint flexibility arises because of the elastic behavior of the joint transmission elements such as gears and shafts. Thus, flexible manipulators undergo two types of motion, i.e. rigid and flexible motion. Because of the interaction of these motions, the resulting dynamic equations of flexible manipulators are highly complex and, in turn, the control task becomes more challenging compared to that for rigid robots. The comparison results between the developed sim-mechanics model using joint flexibilities and that of rigid joint-link manipulators have been shown here, which clearly indicates the higher acceptable stability of the flexible one. Therefore, a first step towards designing an efficient control strategy for these manipulators must be aimed at developing accurate dynamic models that can characterize the above flexibilities along with the rigid body dynamics. Main focus of our research is to develop a simulink model using joint flexibilities and compare the results with that of rigid link manipulators.

The said problem can be considered as the development of a multi arm manipulation control for a single link flexible manipulator. Ata et al. [1] modeled a two link flexible manipulator using Bernoulli-Euler theory. A dynamic modeling technique has been addressed by Subudhi and Morris [2] for multiple flexible link-joint systems using Euler-Lagrange formulation. Swern & Tricamo [3] developed a control algorithm that eliminates the constraint forces supplying a position correction to each arm. A control model has also been proposed by Khemaissia [4] to generate goal directed multi joint-arm movement using hybrid neuro-genetic algorithm. Colbaugh et al [5] introduced a strategy aims at adaptively generating the position set points for the standard robot controllers, without requiring knowledge of both manipulator and payload model parameters and dynamics. Some more researchers also have studied the dynamic formulation of flexible manipulators using either Lagrangian formulation [6-9] or Euler formulations [10]. However, going through the literature it has been observed that validation of the analytic models for the flexible manipulators is difficult and often becomes impossible. Moreover, control of such manipulators is a complex problem. Quite a few researchers have tried to control flexible manipulators using different control algorithms such as Proportional Derivative Control Law [11], Robust Control Law [12], Neuro-Fuzzy-based control law [13] etc. However, there are some inherent problems existing with those control laws. Therefore, in the present work an attempt has been made to model a six link flexible manipulator with joint flexibility in simmechanics environment and control issues have been explored. Here, the model includes six links connected serially with six revolute joints. Where, the first link has been attached to the base and the final link has been used for the object manipulation task with some pay-load.

Rest of the paper is structured as follows: In Section 2, modeling of a 6-DOF flexible manipulator in Sim Mechanics environment is explained and PID-based control of the manipulator has been stated. Results are presented and discussed in Section 3. Finally, some concluding remarks are made in Section 4.

## II. MODELING AND CONTROL OF A 6-DOF FLEXIBLE MANIPULATOR

In the present work an attempt has been made to model a six link flexible planar manipulator with joint flexibility in simmechanics environment. The basic structure of the manipulator system has been shown in Figure 1.

Here, the model includes six links serially connected rigid links. The first link has been attached to the base and the final link has been used for the object manipulation task with some pay-load. Lengths of all the links have been taken as 1 meter each for simplicity. Mass of the first link has been taken as 1 Kg followed by gradual increase of mass of 1 Kg each for the successive links. As a result, mass of the end link has been considered as 6Kg. Simmechanics model of the said manipulator is shown in Figure 2. It is to be noted that two joint sensors has been used for taking position and speed feedback from first joint and sixth joint, respectively. In the simmechanics model, flexibility to the manipulator is incorporated attaching one spring-damper block to each revolutory joint. Both damping and stiffness coefficients have been taken as 0.5 N-m-s/rad and 0.5 N-m/rad, respectively for all the joints. Each joint spring-damper acts as an actuator, which applies the calculated damped torque to the link body. Moreover, CG positions of the links have also been varied slightly from their geometric center positions to explore more complexity in the problem.

### A. Control and Stability Analysis of 6-DOF Flexible Manipulator

Flexible Manipulators are found to be advantageous over the rigid ones. Dynamic equations of 6-dof rigid manipulators are readily available in the text books. However, a derivation of the dynamic equations corresponding to flexible manipulators is highly complex and cumbersome. Therefore, it has been modeled in the simmechanics environment. Thereafter, simulink model of the flexible manipulator system is converted to state-space system model and the system transfer function has been extracted from the state-space system. In this way, the system transfer function in Laplacian form has been obtained as,

$$\frac{3.41e^{-13}s^9 - 0.58s^8 + 1089s^7 - 3.4e^5s^6 + 1.24e^7s^5 - 1.003e^7s^4 - 7.34e^5s^3 - 1.94e^4s^2 - 227.2s - 0.99}{s^{10} + 601.6s^9 + 1.05e^5s^8 + 5.76e^6s^7 + 7.3e^7s^6 + 9.12e^7s^5 + 7.65e^6s^4 + 2.64e^5s^3 + 4588s^2 + 39.9s + 0.14}$$

Here, e is equal to 10. The root-locus graph and open loop bode plots are presented in Figure 3. It has been observed that the system has unstable open-loop bode. Therefore, it is required to stabilize the system. For the purpose of which, a classical PID controller technique has been applied.

On the other hand, it has been observed that the performance of PID controller depends on the selection of three gains values. The process of selection is very complex and demands a systematic approach. In the present study, tuning of PID gains has been made with the help of SISO design toolbox.

## III. RESULTS AND DISCUSSION

Main aim of this research is to develop a single link manipulator behaving like a utopian rod. Two models have been developed. Firstly one rigid link has been split into six rigid links connected serially through rotary joints. In the second model, one each spring and damper has been attached to each joints. Both the models are simulated in simulink environment. Table 1 shows the angular position and speed of joints 1 and 6 (connected to end effectors) for both the manipulators at different instant of time. It has been observed that the movement of the flexible manipulator is smoother than the rigid ones. Moreover, flexible one takes less time to settle down. Therefore, further analysis has been carried out with the flexible manipulator only.

Both flexible and rigid ones are unstable and needs a controller to stabilize. PID controller is one of the mostly used controller in this aspect and readily available in the Simulink environment. In the present study, the controlling action has been incorporated to the system using **SISO design tool box** in MATLAB environment. Classical PID control technique has been adopted in this case. The SISO tool provides the means for automated PID tuning and it automatically generates the compensator value. Table 2 shows the stability margin values for different controllers tuned using three methods. Tuning using Ziegler Nichols (ZN) method is mostly common and has been applied here. Both frequency response as well as step response based ZN tuning has been applied in the present study. Frequency response based ZN tuner provides larger stability margins compared to the other. Tuning of controller gains using Robust Response Time (RRT) method provides the best stability margin out of all the other methods (refer to Table 2). Compensator with which the best stability margins have been achieved is  $-\frac{0.0046(1+20s)}{s}$  (corresponding to RRT method). Gain

values corresponding to this compensator ( $K_P = -0.092$ ,  $K_D = 0$ ,  $K_I = -0.0046$ ) are noted for further processing. It is important to note that robust response time based tuning method has been made in automatic design mode.

It is also to mention that the negative value of the compensator (C) gain indicates the use of direct acting controller. It indicates an increase in the controlled variable (measurement) requires an increase in the control action. Otherwise, the feedback loop should be made positive (+ve).

Once the tuning is over, step and impulse response of the system is plotted corresponding to the best compensator (refer to Figure 5). It is observed that the system settles down almost at 200 seconds.

Finally,  $K_P$ ,  $K_D$  and  $K_I$  values are extracted corresponding to the best compensator, which are as follows.

$$K_P = -0.092, K_D = 0, K_I = -0.0046$$

With the above mentioned gain values the manipulator model is simulated. Figures 6 and 7 show the angular position of the first joint corresponding to two set of gain values as mentioned below.

**Set 1:**  $K_p = 10$ ,  $K_D = 0$ ,  $K_I = 1$  and **Set 2:**  $K_p = -0.092$ ,  
 $K_D = 0$ ,  $K_I = -0.0046$

It is to be noted that the above PID controller has been incorporated correspondingly to filter coefficient equal to 0.1997. It is seen that the manipulator settles down safely within 200 seconds with gains given in Set 2. On the other hand manipulator does not stop its motion with Set1 gain values.

The graphical results shown above clearly indicate the scopes for using such classical control techniques, like PID control, that can be adopted for stabilizing multi-DOF flexible system presented here, in spite of higher system instability.

#### IV. CONCLUDING REMARKS

In this paper, a comparative analysis has been made between a six DOF flexible manipulator system with joint flexibility and its rigid counterpart. Simulation result clearly shows greater stable performance of the flexible manipulator system, yet it is unstable system. The transfer function obtained from the flexible system, has been used to plot the root locus and bode diagram graphically, which indicates that the system is unstable. This initiates the necessity of proper controlling action to the modeled flexible manipulator system. For the purpose of which, a classical PID control technique has been chosen. However, performance of PID controller depends on its three gain values, which needs to be tuned properly. Three different tuning methods have been applied in the present work. Out of the three PID tuning approaches, RRT method has given the best performance. It is necessary to mention that the tuning is done with the help of MATLAB SISO design toolbox, which automatically updates the compensator. Step and impulse responses of the system have been observed with the best compensator. Best result has been observed with the gain values  $K_p = -0.092$ ,  $K_D = 0$ ,  $K_I = -0.0046$ . Finally, the PID controlled flexible manipulator is simulated with the above mentioned gain values. It has been observed that the performance of the manipulator is highly stable with the optimized gains. Otherwise, it continues its motion for a longer period of time.

The focus of future research work has been concentrated on dynamic modeling of such multi DOF

flexible manipulator system considering both joint as well as link flexibility. More emphasis will be paid on the development of a proper mechanism for tuning the PID controller gains. Authors are working towards those issues presently.

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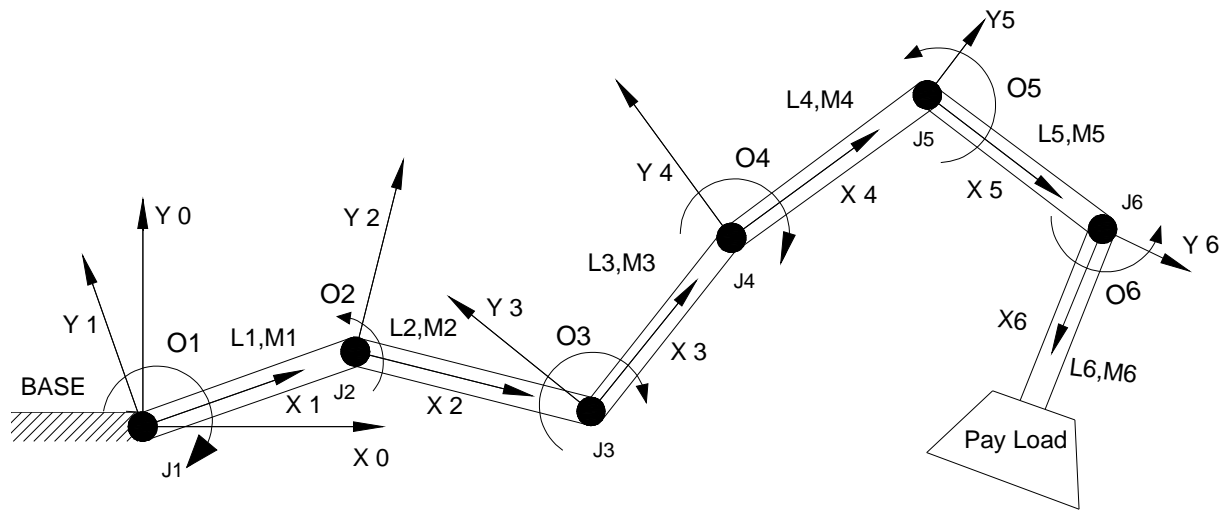


Figure 1: A six DOF flexible manipulator system.

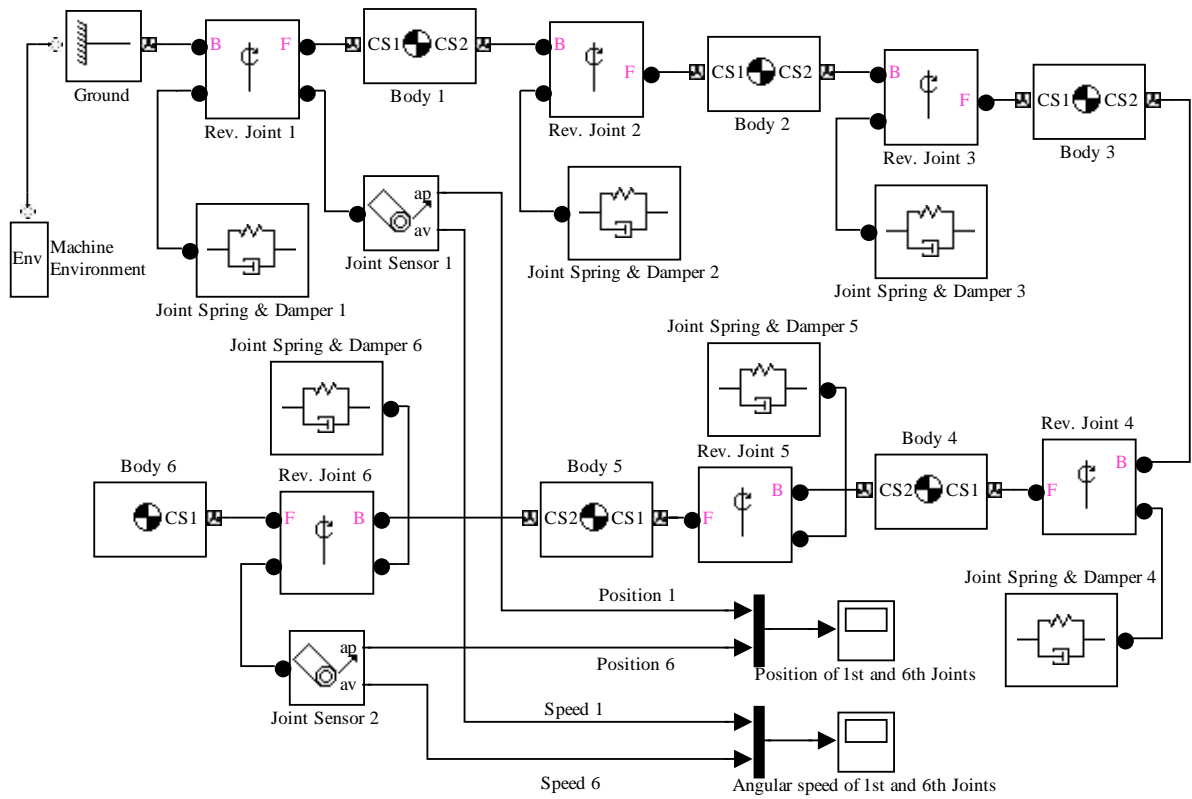


Figure 2: The sim-mechanics model of the flexible manipulator system.

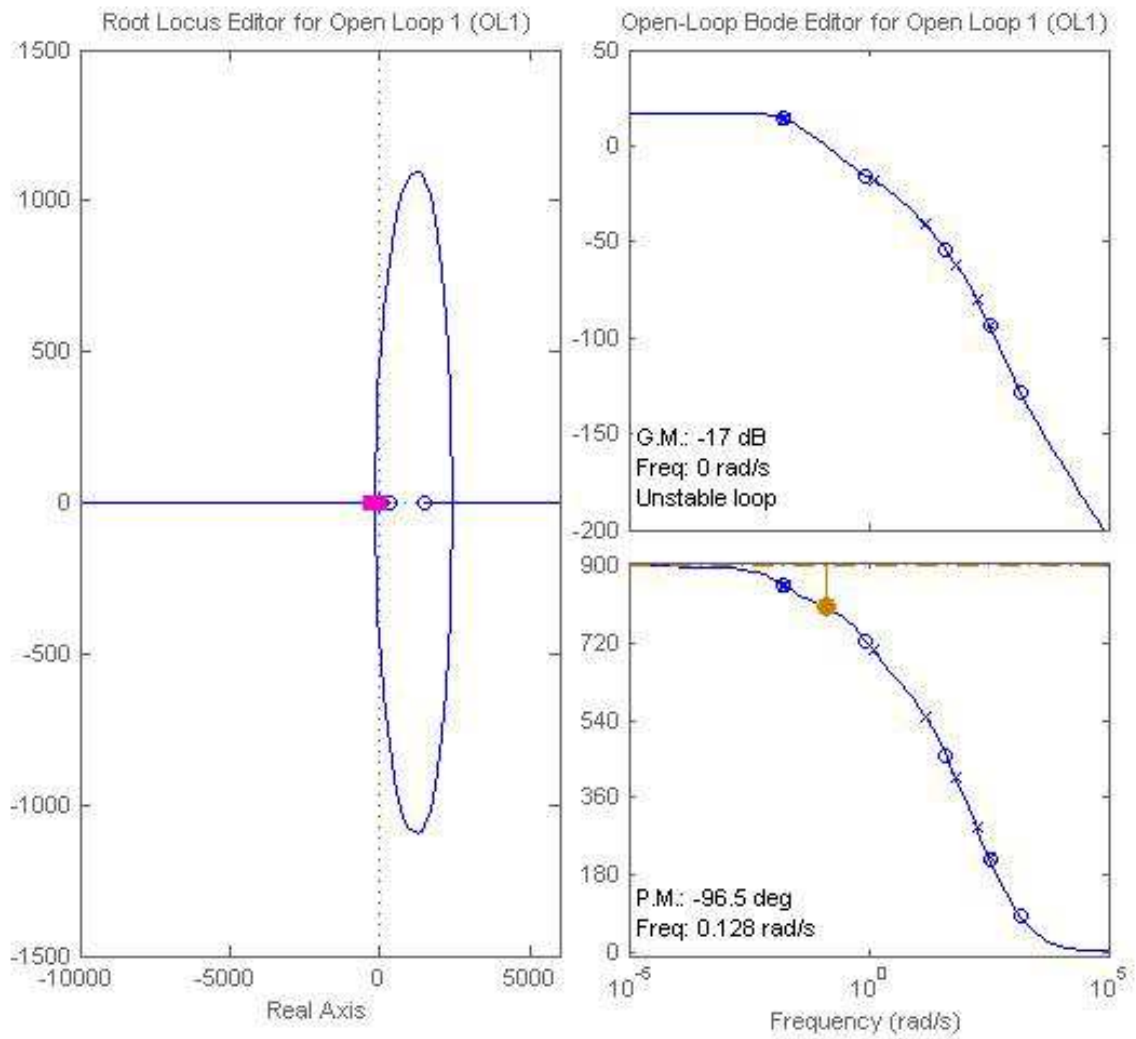


Figure 3: Open loop root locus & bode plot with simple system transfer function.

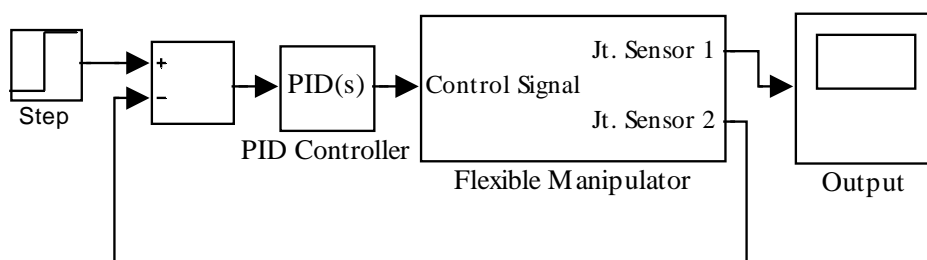


Figure 4: Simulink model of a PID controlled flexible manipulator.

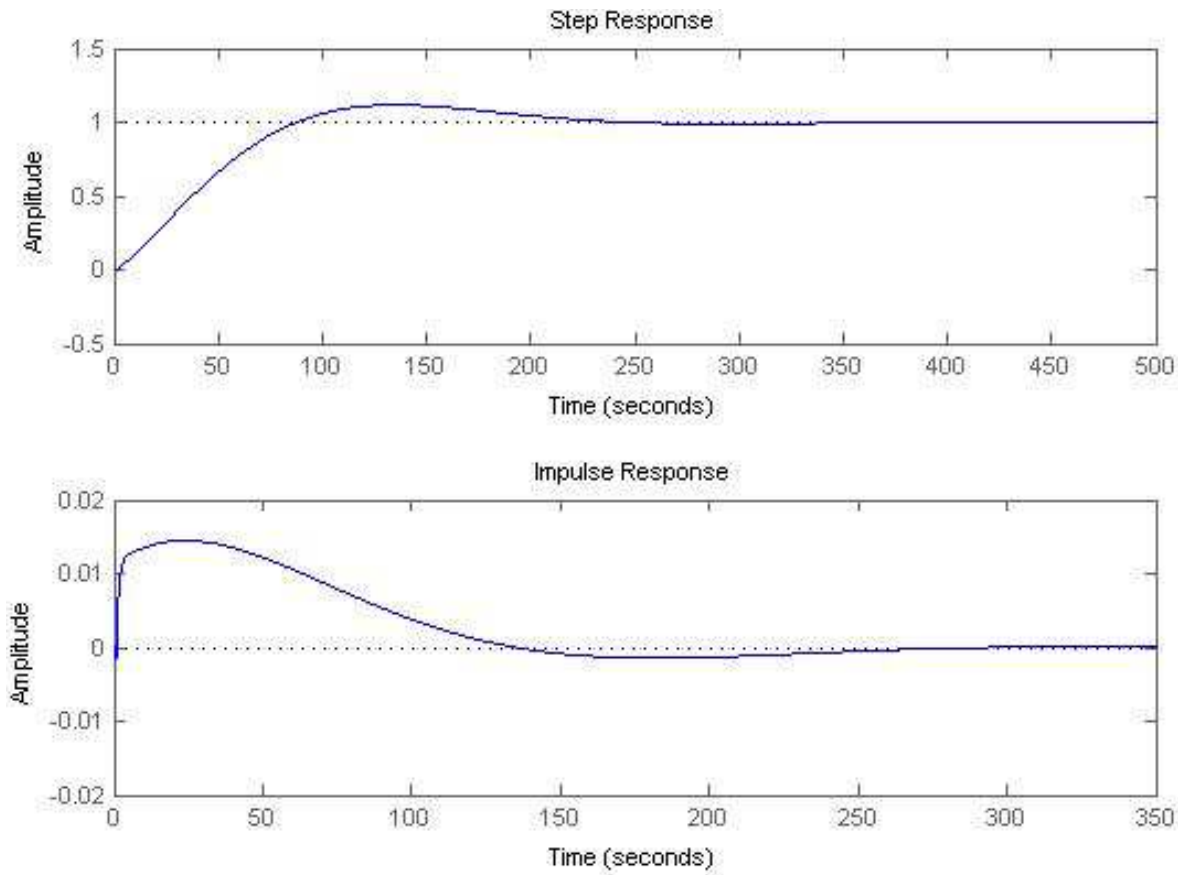


Figure 5: Step and Impulse response of the system transfer function output vs. control action.

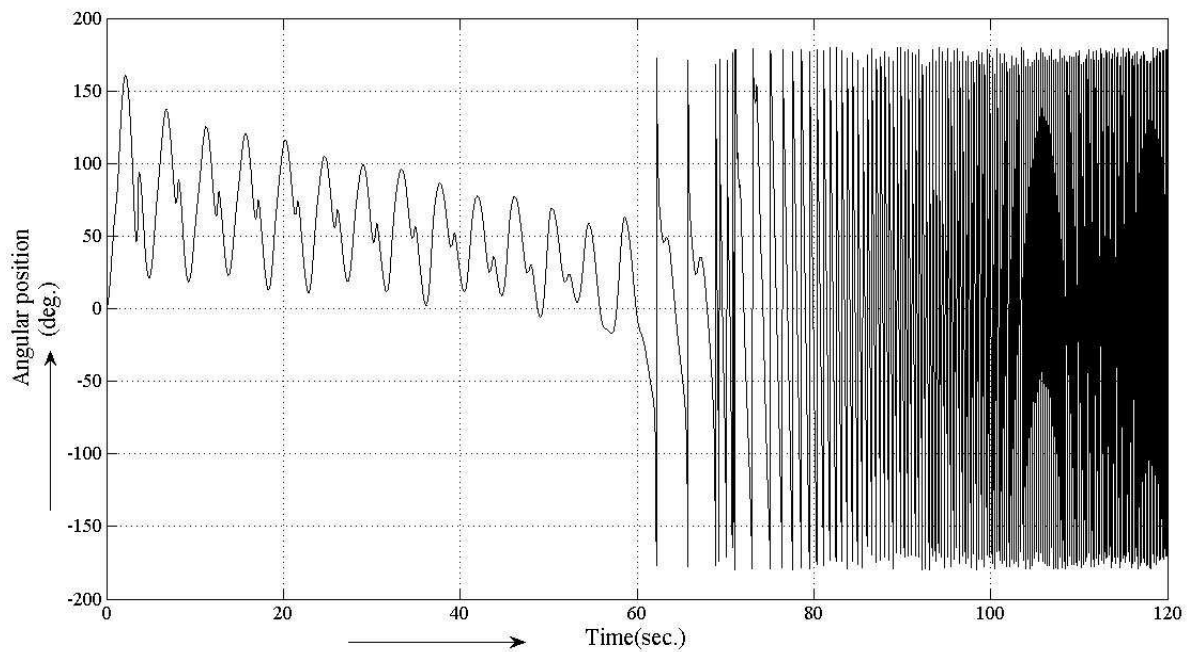


Figure 6: Angular position of the first joint vs. simulation time corresponding to Set 1 values.

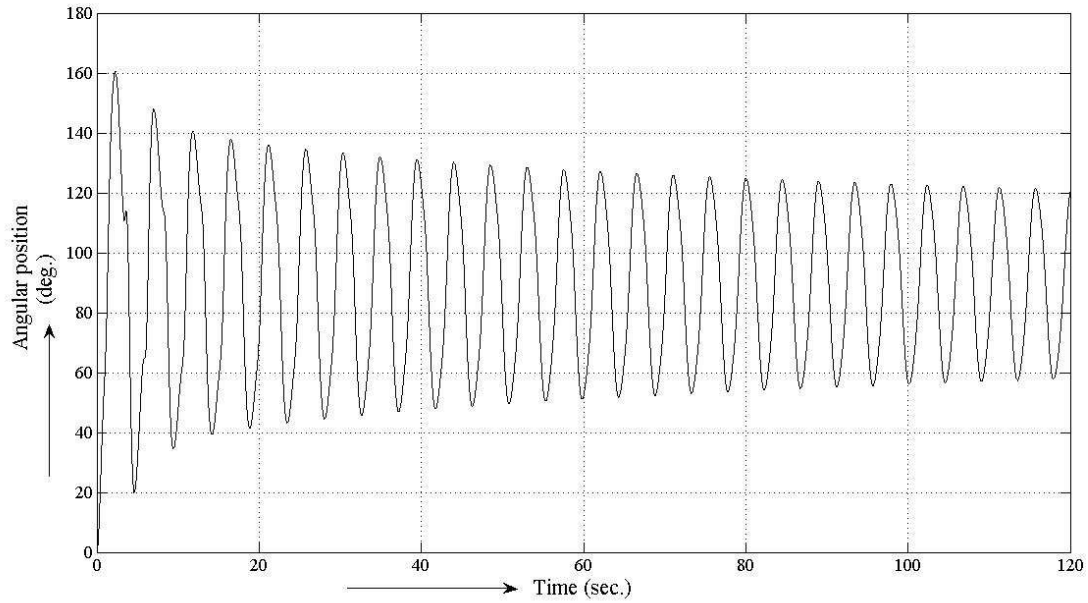


Figure 7: Angular position of the first joint vs. simulation time corresponding to Set 2 values.

Table 1: Comparisons among the position and speed of first and last joint of the manipulators.

Time (sec.)	Flexible Manipulator				Rigid Manipulator			
	Position (deg.)		Speed (deg./s)		Position (deg.)		Speed (deg./s)	
	Joint 1	Joint 6	Joint 1	Joint 6	Joint 1	Joint 6	Joint 1	Joint 6
0	0	0	0	0	0	0	0	0
5	-4.5	37.5	-11	19.5	100	43.5	-90	80
10	-6	51	-6.4	21	58	137.5	-810	-740
15	-4	66.5	11	37	-90	110	-62	-118
20	0	94	2.3	55	145	90	560	-310
25	1	114.5	2.6	23	-116	74	-470	-790
30	2.8	115	4	-8.8	-40	135	1350	-500

Table 2: Gain Margin (GM) and Phase Margins (PM) of the system for different controllers.

Controller Type	Ziegler Nichols-based PID tuning				Robust Response Time Method	
	Frequency Response-based		Step Response-based			
	GM (dB)	PM (deg)	GM (dB)	PM (deg)	GM (dB)	PM (deg)
P	6.02	45	3.75	30.5	9.05	60
PI	6.08	23.4	3.07	16.3	35.3	60
PID	4.85	26.4	1.18	36.2	36.7	60
PID with derivative filter	4.17	26.6	0.608	32.1	36.1	60