Semi-active vibration isolation of a quarter car model under random road excitations using Magnetorheological damper

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***Abstract*— Semi active control systems are becoming increasingly popular because they offer reliability of passive systems combined with high performance and versatility of active control systems but with low power consumptions. As Magnetorheological (MR) fluids can produce good controllable damping force under application of magnetic field, MR damper can be used as effective element in semi-active vibration control. A phenomenological model of MR damper is considered in the present study. Out of various semi-active control strategies, the on-off sky-hook control strategy is used in this work. To check the performance of the proposed on-off controller with MR damper, a single degree of freedom system with MR damper under sinusoidal excitation is first studied. This control scheme is then applied to a two degree-of freedom quarter car model. The parameters of MR damper in the model are varied by changing the input voltage according to the control scheme. The vehicle is assumed to travel with a constant forward velocity and excitation from road irregularities is simulated considering suitable profile spectral density of road. Performances of this controller for two types of road profiles, namely, sinusoidal road profile, random road profile are studied. The control strategy is found to be effective for a quarter car model. To examine optimal damper force vehicle required to isolate the vibration several performance indices have been chosen which are functions of vehicle performance measures such as sprung mass acceleration, vehicle handling and working space.**

Keywords— ***Magnetorheological damper; Quarter car model; Semi-active control.***

I. INTRODUCTION

The problems of road induced vibration of vehicle systems are generally solved by placing suspension systems between wheel and chassis. The suspension system consists of spring elements in parallel to damper elements. Passive suspension systems do not perform well over a wide range of operating conditions. Active and semi-active suspension systems eliminate this problem and improve the performance of the suspension systems. Several researchers studied different control systems for car vibration reduction [1].

Both linear and non-linear suspensions have been considered. For example, S. Narayanan and S. Senthil [2] considered quarter car model with passive nonlinear damping and hysteresis stiffness models. Equivalent linearization technique was used there to linearize the system along with optimal control strategy. Gopala Rao and Narayanan [2] and Narayanan and Senthil [3] used similar suspensions for optimal sky-hook control. Yi and Song [4] proposed adaptive semi active control strategy for quarter car model. Ahmadian and Pare [5] studied different semi-active control strategies for nonlinear quarter car model. They considered sky-hook, ground-hook and hybrid control strategies. Yoshimura, Kume, Kurimoto and Hino [6] studied the performance of a quarter car model using active sliding mode control.

Magnetorheological (MR) dampers are widely used in vehicle suspension system. Butz and Von Stryk [7] studied the response of quarter car model equipped with MR damper under step disturbance and sinusoidal bump. They considered different parametric models for MR damper and compared their results. Yao, Yap, Chen, Li and Yeo [8] studied sky-hook control with MR damper for quarter car model. Du, Sze and Lam [9] studied the semi-active H_∞ control of vehicle suspension system for a quarter car model equipped MR damper. Prabakar, Sujatha and Narayanan [10] studied the optimal preview control of a half car model under random road excitation, and the MR damper has been modeled as the modified Bouc-Wen model. Havelka and Musil [11] studied the performance of a quarter car model under random road conditions with non-linear adaptive model of MR damper. Yu, Dong, Choi and Liao [12] studied the vibration control response of a suspension system with magnetorheological damper of a passenger car. After developing a half-car model, a human simulated intelligent control (HSIC) scheme has been proposed to control the unwanted vibration. Prabhakar, Sujatha and Narayanan [13] studied performance of quarter car model under random road conditions. In this study the MR damper was modeled in two ways, namely Bingham model and modified Bouc-Wen model. For both the models, the parametric values were found using multi-objective optimization techniques.

A sky-hook based semi-active controller is considered in this work. A magnetorheological fluid (MR fluid) damper is used in on-off mode to isolate vehicle body from ground induced motion. The damper is modeled as modified Bouc-Wen model. Ground disturbance has been simulated as a random signal having spectral density found in typical road profile. The effects of voltage used in semi-active control of MR damper have been thoroughly investigated. An attempt has been made to obtain optimal value of the voltage to be used so that the suspension system performs satisfactorily. The problem is formulated as a multi-objective optimization problem involving maximum ride comfort, best road handling and least work-space requirement. The problem of chatter which appears in semi-active control system has been addressed. An anti-chatter solution is provided that reduces the jerk in the response of the vehicle body.

II. MANETORHEOLOGICAL FLUIDS AND DAMPERS

MR fluids consist of micron-sized, magnetically polarized particles dispersed in a non-magnetic medium. When a magnetic field is applied to the fluid, the particles get polarized. The interaction between these induced polarized particles causes the particles to form chain like structures, parallel to the applied magnetic field. These chain-like structures restrict the flow of fluid. Thus the fluid becomes semi-solid and exhibits visco-plastic behavior. The transition can be achieved within a few milliseconds after application of magnetic field. When the magnetic field is removed, the fluid particles return to their original positions and the fluid behaves like a Newtonian fluid.

The major advantage of these fluids over other mechanical interfaces is their ability to achieve a wide range of viscosity within very small time (within a millisecond). This provides an efficient way to control vibration. The advantages of using magnetorheological fluids over electrorheological fluids are that the magnetorheological fluids are more stable over a wide range of temperature and easy to operate. Based on its advantages magnetorheological fluids are used in various fields of applications. The main fields of applications are in mechanical structures, automobiles, optics, medical applications etc.

Various researchers modeled MR dampers in different ways like Bingham plastic model, Gamota-Filisko model, Bouc-Wen model etc. Out of these models, Bouc-Wen model is found out to be effective for MR damper. The schematic model is shown in the *Fig.1.a* and the force generated in Bouc-Wen model is given by

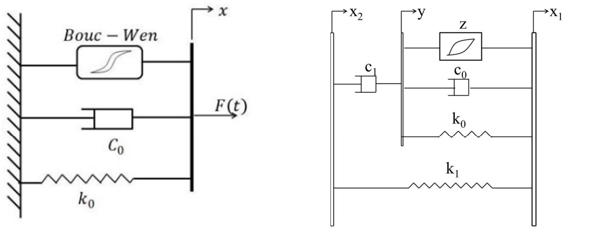
$$F = c_0\dot{x} + k_0(x - x_0) + \alpha z\,, \tag{1}$$

where the evolutionary variable z is governed by,

$$\dot{z} = A\dot{x} - \gamma|\dot{x}|z|z|^{(n-1)} - \beta\dot{x}|z|^{n} \quad , \tag{2}$$

with γ, β, A as the parameters of the model.

Force-velocity curves for Bouc-Wen model do not match with the experimental results near the zero velocity region. Spencer, Dyke, Sain and Carlson [14] modified the Bouc-Wen model (*Fig. 1.b*) considering the leakage flow



(a) (b)

Fig. 1. Schematic diagram of a) Bouc-Wen model, b) modified Bouc-Wen model.

and the accumulator present in the MR damper. This modified Bouc-Wen model is considered in the present study.

III. MATHEMATICAL MODILING

A. Quarter car model

The schematic diagram for quarter car model is shown in the *Fig.2*

The equations of motion of the quarter car model along with the MR damper which is modeled as modified Bouc-Wen model or Spencer model can be expressed as

$$m_1\ddot{x}_1 + k_{11}(x_1 - x_2) + f_{MR} = 0\,, \tag{3}$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_0) + c_2(\dot{x}_2 - \dot{x}_0) - k_{11}(x_1 - x_2) - f_{MR} = 0 \tag{4}$$

$$f_{MR} = k_1(x_1 - x_2) + k_0(x_1 - y) + c_0(\dot{x}_1 - \dot{y}) + \alpha z\,, \tag{5}$$

$$\dot{y} = \frac{1}{(c_0 + c_1)}\{\alpha z + k_0(x_1 - y) + c_0\,\dot{x}_1 + c_1\dot{x}_2\} \tag{6}$$

where x_1 and x_2 are the sprung mass displacement and the unsprung mass displacement, respectively. The evolutionary variable z is governed by the Bouc-Wen equation as follows,

$$\dot{z} = A(\dot{x}_1 - \dot{y}) - \gamma|\dot{x}_1 - \dot{y}|z|z| - \beta(\dot{x}_1 - \dot{y})z^2\,. \tag{7}$$

where α, β, γ and A are parameters of Bouc-Wen model. The evolutionary variable α and damping constants c_0 and c_1 are linearly dependent on the applied voltage, u, as

$$\alpha = \alpha_a + \alpha_b\,u \tag{8}$$

$$c_0 = c_{0a} + c_{0b}\,u \tag{9}$$

$$c_1 = c_{1a} + c_{1b}\,u \tag{10}$$

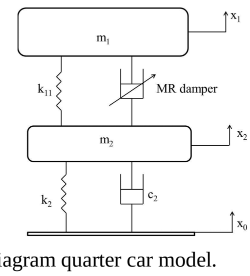


Fig. 2. Schematic diagram quarter car model.

B. Generation of random road profile

In the present study, the quarter car model shown in *Fig. 2* is considered to travel on a random road profile. The geometric profile of typical roads fits with the following simple analytical form

$$S_g(\Omega) = A_g\, \Omega^{-n} \qquad (11)$$

where, S_g is single sided power spectral density,

A_g is a coefficient depending upon road roughness.

n is rational number,

Ω is wave number, $(\Omega=2\pi/\lambda)$,

and λ is the wavelength.

Qualitatively, a large value of the exponent n accentuates the roughness at the longer wavelengths, while it suppresses the roughness at the shorter wavelengths. Hence, it is commonly accepted that the spectra corresponding to the geometrical profile of typical roads can be approximated by the following function,

$$S_g(\Omega) = S_g(\Omega_0)\,(\Omega/\Omega)^{-n_1},\ if\ \Omega \le \Omega_0 \qquad (12)$$

$$S_g(\Omega) = S_g(\Omega_0)\,(\Omega/\Omega)^{-n_2},\ if\ \Omega > \Omega_0 \qquad (13)$$

where, Ω_0 is a reference spatial frequency which is usually taken as $1/2\pi$. Here, $S_g(\Omega_0)$ provides a measure of road roughness. The exponents n_1 and n_2 are chosen such that $n_1 \neq n_2$.

Verros, Natsiavas and Papadimitriou [15] studied the generation of road profile as a function of time. If the vehicle is assumed to travel with a constant horizontal velocity v_0 then the force resulting from the road irregularities is,

$$x_g(\mathrm{t}) = \sum_{n=1}^{N} s_n \sin\{\mathrm{n}\,\omega_0\,\mathrm{t}+\phi_n\} \qquad (14)$$

where, amplitudes of excitation harmonics are evaluated as $s_n = \sqrt{2S_g(n\Delta\Omega)\Delta\Omega}$.

In (14) and expression for s_n,

- $\Delta\Omega = 2\pi/L$, where L is the length of the road,
- $\omega_0 = 2\pi v_0/L$, is the fundamental temporal frequency,
- ϕ_n is random variable, having a uniform distribution in the interval $(0,2\pi)$.

A good road profile is generated in *Fig. 3* assuming the values $n_1=2$, $n_2=1.5$ and $S_g(\Omega_0) = 16\times10^{-6}$ m2/ cycle m.

C. Control Strategy

Sky-hook control strategy is applied to the quarter car model. The sky-hook control strategy was first developed by Karnopp, Crosby and Harwood [16]. They considered the control force generated by the controller to be equal to the force of a passive damper which is connected to an inertial reference frame (sky). The approximation to an ideal skyhook is given by the following control logic,

$$F_{sky-hook} = \begin{cases} c_{sky}(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2), & \text{if } \dot{\mathrm{x}}_1(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2)>0 \\ 0, & \text{if } \dot{\mathrm{x}}_1(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2)\le 0 \end{cases} \qquad (15)$$

However, the strategy has been modified here as [17].

$$F_{sky-hook} = \begin{cases} F_{\max}, & \text{if } \dot{\mathrm{x}}_1(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2)>0 \\ F_{\min}, & \text{if } \dot{\mathrm{x}}_1(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2)\le 0 \end{cases} \qquad (16)$$

The damper force switches between off and on states rapidly. This may lead to chattering of the system. Chatter occurs near the switching point, i.e., either $\dot{x}_1$ or $(\dot{x}_1-\dot{x}_2)$ is zero. The chattering can be reduced by introducing a dead band around the switching point [1].

This method allows for relative displacement and velocity dead bands within which the condition function is given only the value *0*, irrespective of sign of $\dot{x}_1$ or $(\dot{x}_1-\dot{x}_2)$. Then the new condition function is,

$$F_{sky-hook} = \begin{cases} F_{\max}, \text{if } \dot{\mathrm{x}}_1(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2)>0 \\ \quad and\ sw\{\dot{\mathrm{x}}_1,(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2),\varepsilon_1,\varepsilon_2\}=1 \\ F_{\min},\ otherwise \end{cases} \qquad (17)$$

where, the switching function $sw\{\dot{\mathrm{x}}_1,(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2),\varepsilon_1,\varepsilon_2\}$ is governed by

$$sw\{\dot{\mathrm{x}}_1,(\dot{\mathrm{x}}_1-\dot{\mathrm{x}}_2),\varepsilon_1,\varepsilon_2\} = \begin{cases} 1, & if\ |\dot{x}_1|\ge\varepsilon_1\ and\ |\dot{x}_1-\dot{x}_2|\ge\varepsilon_2 \\ 0, & otherwise \end{cases} \qquad (18)$$

D. Performance Index

Several measures of performance of vehicle suspension system have been proposed in literature. Three of them are found to be useful. They are given by,

$$J_1 = \frac{1}{m_1 g}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\{\mathrm{m}_1\ddot{\mathrm{x}}_1(\mathrm{i})\}^2} \qquad (19)$$

$$J_2 = \frac{1}{m_1 g}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\{k_{11}[x_1(\mathrm{i})-\mathrm{x}_2(\mathrm{i})]\}^2} \qquad (20)$$

$$J_3 = \frac{1}{m_1 g}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\{F_w(\mathrm{i})\}^2} \qquad (21)$$

These performance measures represent acceleration of the vehicle body, relative displacement (workspace) between body and chassis and handling which is proportional to F_W the force developed between wheel and ground, i.e.,

$$F_w = k_2(\mathrm{x}_2-\mathrm{x}_0)+\mathrm{c}_2(\dot{\mathrm{x}}_2-\dot{\mathrm{x}}_0) \qquad (22)$$

The performance measures are often antagonistic. Hence, Suitable multi objective optimization process is required to get overall optimal performance of suspension system.

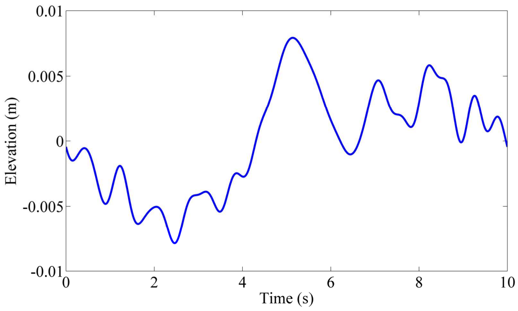


Fig. 3. Random road profile.

IV. Results and Discussion

In this work we consider a MR damper which has been modelled as modified Bouc-Wen model [14]. The model parameters are taken from [18] and are listed in Table I.

TABLE I. PARAMETER VALUES OF MODIFIED BOUC-WEN MODEL

c_{1a}	c_{1b}	c_{0a}	c_{0b}	α_a	α_b
14649 Ns/ m	34622 Ns/m V	784 Ns/ m	1803 Ns/ m V	14221 N/ m	384360 N /m V
k_0	**k_1**	**γ**	**B**	**A**	**N**
37810 N / m	617.21 N/ m V	136320 m-2	2059020 m-2	2679 m-1	2

The parameter values of quarter car model (*Fig.2*), used in simulation, are shown in Table II.

TABLE II. PARAMETER VALUES OF QUARTER CAR MODEL

m_1	m_2	k_{11}	k_2	c_2
325 Kg	55 Kg	42 kN/ m	180 kN/ m	2000 N s/ m

The On-Off skyhook controller is applied to the quarter car model (*Fig. 2*) when it is subjected to sinusoidal base excitation as $x_0 = X_0 \cos \Omega t$, with amplitude $X_0=0.1$ m and frequency, $\Omega=10$ rad s-1. Numerical simulation has been carried out in MATLAB. Numerical integration is carried out by using appropriate ODE solver. The responses of quarter car models are shown in *Fig. 4*. It is observed that the performance is better when control strategy is applied with a voltage $u=2V$. However, jerks have been observed after control, which is typical of discontinuous control strategy adopted here.

Frequency responses of the same model are shown in *Fig. 5* with the following non-dimensional parameters,

$$x_{1non} = \frac{x_1}{X_0}, \quad x_{rnon} = \frac{x_{rel}}{X_0}, \quad v_{1non} = \frac{\dot{x}_1}{\omega_1 X_0},$$

$$a_{1non} = \frac{\ddot{x}_1}{\omega_1^2 X_0}.$$

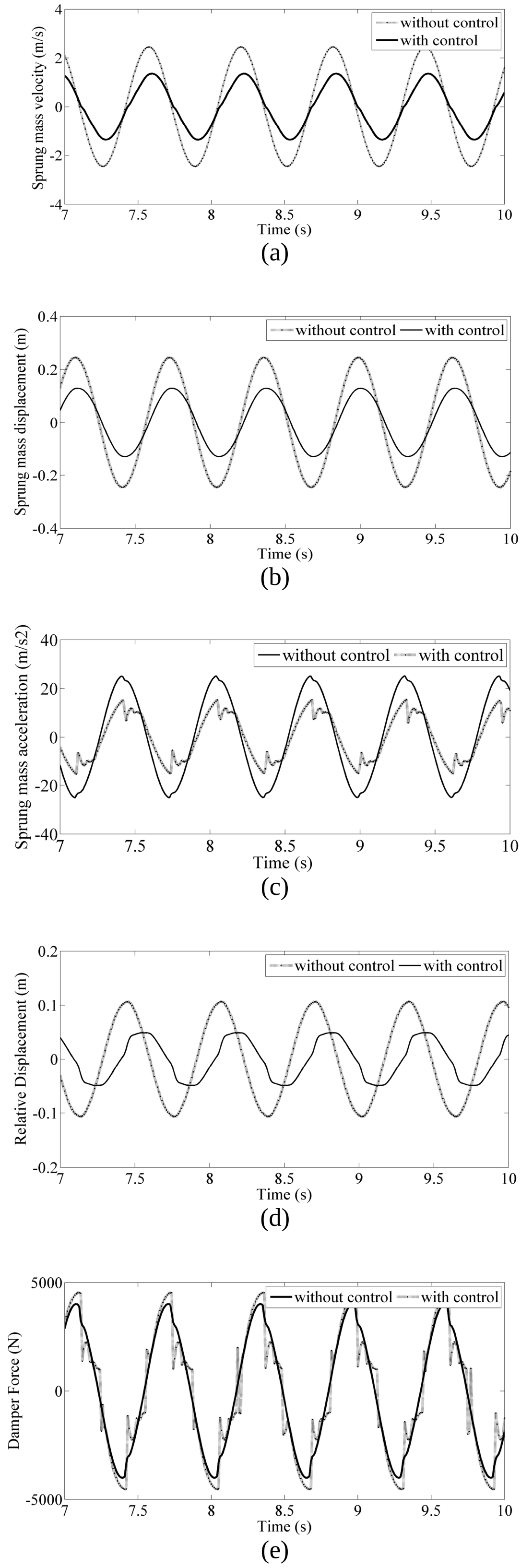
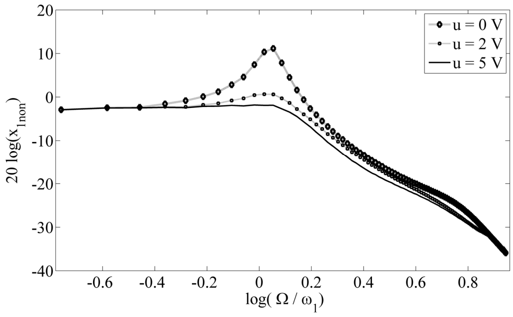


Fig. 4. Time responses of the quarter car system: a) Sprung mass displacement (x_1), b) Sprung mass velocity ($\dot{x}_1$), c) Sprung mass acceleration ($\ddot{x}_1$), d) Relative Displacement ($x_1 - x_2$), e) Damper force.

It is seen from *Fig. 5* that at low frequency excitation, control strategy works better than at higher frequency cases, and also the system performs better with higher values of applied voltage near the resonance zone.



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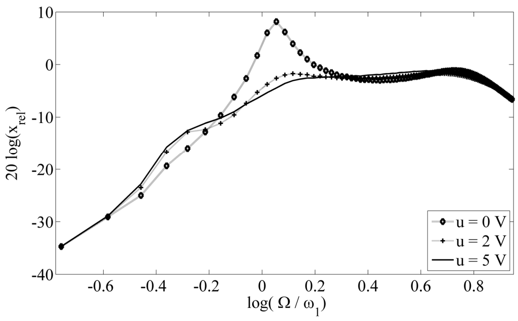

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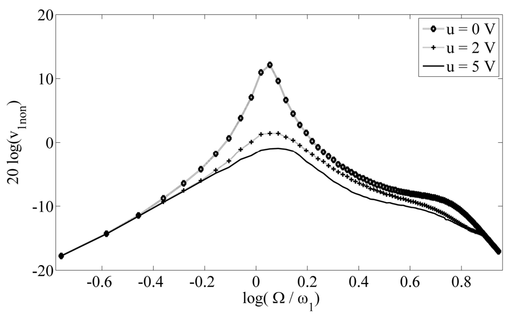

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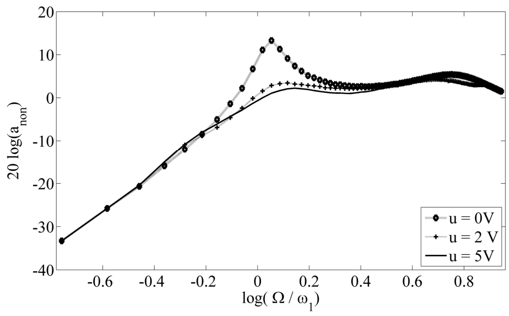
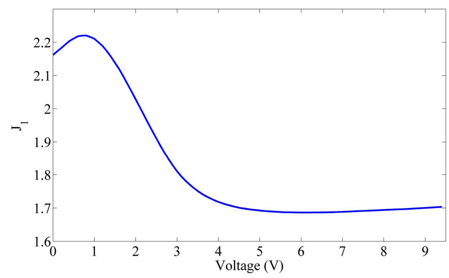


(d)

Fig. 5. Frequency responce of the quarter car model: a) Sprung mass displacement , b) Relative Displacement, c) Sprung mass velocity, d) Sprung mass acceleration.

Fig. 6 shows that the value of J_1 increases initially with the increase in the applied voltage, but at higher voltage attenuation takes place. For J_2, the performance improves as the applied voltage increases, but after a certain value it does not improve much. A different phenomenon is seen in case of J_3. Here, initially the tire force increases and then suddenly it drops and again at higher applied voltage its value increases.

The control logic is then used for random road excitations. A random road profile (*Fig. 7.a*) is generated by the process stated in previous section. The above stated quarter car model with control strategy is tested on this road profile. It is seen from *Fig. 7* that the control strategy works well in this case also. It is found from *Fig. 8* that at low voltage, J_1 and J_2 values, which are associated with acceleration and working space, are low and at higher voltage values, these two remain more or less stable, and the tire force (i.e., J_3) becomes less. It is apparent from *Fig. 8* that the effects of voltage are different for three performance indices. If all the indices are to be minimized then an optimal value of v_0 is to be selected.



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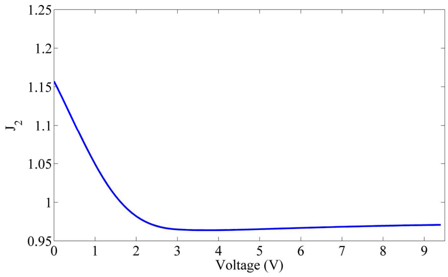

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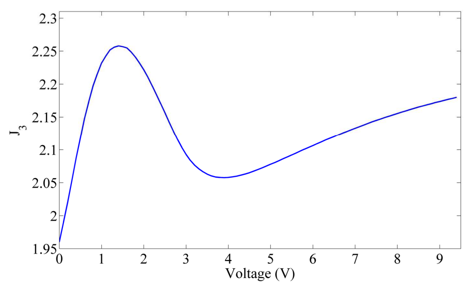

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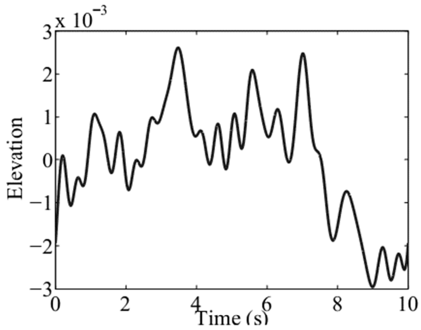
Fig. 6. Dependence of performance indices on applied voltage: a) J_1, b) J_2, c) J_3.



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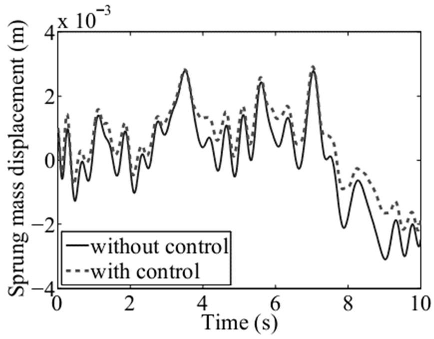

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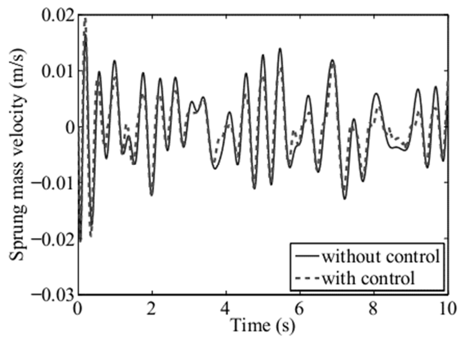

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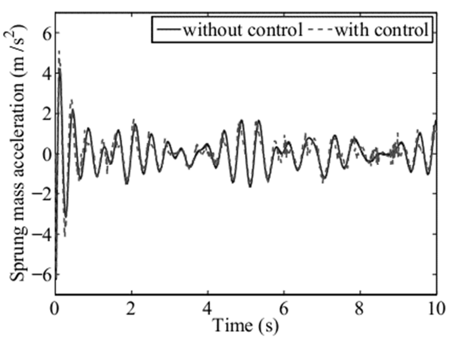
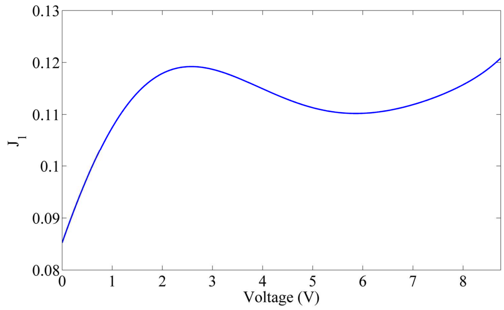


Fig. 7. Time responses of the quarter car system: a) Random road profile, b) Sprung mass displacement (x_1), c) Sprung mass velocity ($\dot{x}_1$), d) Sprung mass acceleration ($\ddot{x}_1$).



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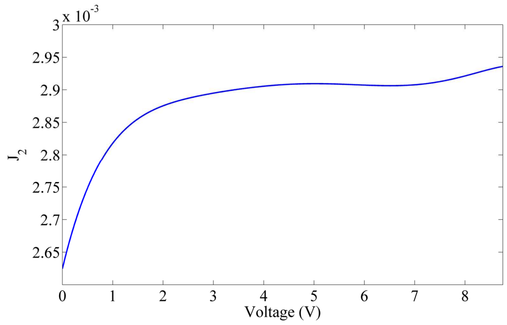

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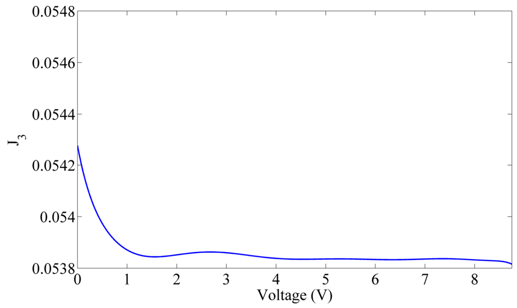

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Fig. 8. Dependence of performance indices on applied voltageunder random excitation: a) J_1, b) J_2, c) J_3.

V. Conclusion

The on-off sky-hook control system is studied for vehicle suspension using two different types of road conditions. The control scheme is found to has been performing well for sinusoidal excitations which are confirmed by estimating the performance indices. In case of sinusoidal excitation, the control strategy works better for low frequency excitations. With the increase of applied voltage the system performs better near the resonance zone. At high frequency excitation, the control strategy does not perform effectively and needs improvement. For random excitation, the chatter phenomenon occurs due to which high values of acceleration are resulted. Thus an anti-chatter strategy is applied to the system. The overall performance index improves under this control strategy. Three separate performance indices are studied in this paper. The effects of voltage on these indices are often antagonistic. An optimal value of voltage is required to minimize all the performance indices at the same time. This optimization problem is currently being studied by the authors.

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