A New Method for Detection of Structural Properties of Planar Kinematic Chain

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***Abstract*— Isomorphism had been keen area of research since long ago. Many methods are available to the kinematician to detect isomorphism among planar kinematic chains, but each has its own shortcomings. In this paper, an attempt is made to develop a technique to fulfill this need. A method is proposed for detection of isomorphism of the planar kinematic chains based on structural properties. In this approach, kinematic chains are represented by matrices, and the elements of matrices are used for detection of isomorphism. The kinematic chains of 1- D.O.F., 6- Links and 8-Links has been tested and the results are in complete agreement with the available literature. This method is simple, reliable and at the same time, it is capable to detect unique links of kinematic chains for identifying the different kinematic inversions.**

Keywords— Isomorphism, Inversions, Distance Matrix.

I. Introduction

One of the major problems in generating distinct planar kinematic chains is the detection of isomorphism. A lot of time and effort has been devoted to develop a reliable and computationally efficient technique. Undetected isomorphism results in duplicate solutions and unnecessary effort, and falsely identified isomorphism eliminates possible condition for new mechanisms. Researchers in the mechanisms community follow different approaches to specify the non-isomorphic kinematic chains. Furthermore, there are several discrepancies in the results obtained by researchers in the mechanisms community. However, most of these codes or indices are either computationally inefficient or unreliable. Much research has also been devoted in graph theory to the more general graph isomorphism problem. This problem was so popular that in 1977 it was named "The Graph Isomorphism Disease". In the course of development of kinematic chains and mechanisms, duplication may be possible. For this reason, many methods have been proposed by many researchers to check for duplication or, in other words, to detect the isomorphism among the kinematic chains. The papers given in Reference [1] and [2] Crossley and J.J. Uicker made a comparative study of various methods available until then for their strengths and weaknesses. The methods based on characteristic polynomials have the disadvantage of dealing with large numerical and graph theorists noted the existence of counter examples while applying these methods. In code approaches [3, 4] Mruthyunjaya and Yan & Hall, the Max–Min code of a kinematic chain is unique and decodable. However, this method requires highly sophisticated algorithms when applied to large kinematic chains. The degree code, [5] Ambekar and Agrawal, based test overcomes the drawback of Max–Min code method but at the cost of computational efficiency. In characteristic polynomial approach methods, considerable amount of additional computation has to be undertaken to get this information. Most of the methods discussed above are generally based on link adjacencies and the connectivity of the links. In this paper, a very new approach has been adopted for detection of structural properties of planar kinematic chains. Mruthyunjaya and Balasubramanian [7] proposed a vertex-vertex degree matrix whose ijth entry is the sum of degrees of links i and j if i and j are adjacent and is equal to 1 otherwise. The characteristic polynomial of this matrix successfully identified all the 10-link kinematic chains with up to 3 degrees of freedom. According to Ambekar and Agrawal [9] two canonical forms of any adjacency are possible – one yielding a maximum code – Max Code and the other yield a minimum code – Min Code. The Min Code is used as a canonical number to identify the kinematic chains. The method is used in Watt's and Stephenson's chain. Wen-Miin Hwang et al [11] proposed a straight forward approach for the computer-aided structural combination of planar kinematic chains with simple joints, which consists of systematic generation of possible contracted link adjacency matrices, detection of degenerate chains and identification of isomorphic chains. Based on the proposed algorithm, a computer program is developed such that the catalogues of planar kinematic chains with the given number of links and degrees of freedom can be synthesized. .Huafeng Ding et al [12] proposed some new concepts, such as the perimeter loop, the maximum perimeter degree sequence, and the perimeter topological graph, and the method for obtaining the perimeter loop is also involved. Then, based on the perimeter topological graph and some rules for relabeling its vertices canonically, a one-to-one descriptive method, the canonical adjacency matrix set of kinematic chains, is proposed. Another characteristic of the method is that in the canonical adjacency matrix set the element number is reduced, usually to only one. After that, an effective method to identify isomorphism of kinematic chains is given. Rao et al [21] introduced the concept of Hamming distance to the structural studies of kinematic chains. The row of the adjacency matrix corresponding to a link is called the Hamming code of that link. The Hamming distance between two links is defined to be number of places where the Hamming codes for the two links differ. The Hamming matrix is the matrix of the same size as the adjacency matrix when the i j^{th} entry corresponds to the Hamming distance between the links i and j. The link Hamming number is the sum of the corresponding row of the Hamming matrix. The chain Hamming number is the sum of the entire link Hamming numbers. The link Hamming string is defined as the concatenation of the link Hamming number with the frequency of the rate of all the integers from n down to zero. The chain Hamming string is defined as the concatenation of the chain Hamming value and the entire link Hamming strings arranged in decreasing order. A.C. Rao [27] suggested utility of fuzzy logic to investigate isomorphism among kinematic chains and inversions. Chang et al [29] anticipated a new method based on Eigen vectors and Eigen values to identify Isomorphism among kinematic chains. Kinematic chains are firstly represented by Adjacent Matrices. By comparing the Eigen values and corresponding Eigen vectors of Adjacent Matrices, the Isomorphism of mechanism kinematics chain can be identified. Srinath and Rao [30] presented a method based on correlation concept for detection of isomorphism among kinematic chains and their inversions. Correlation between two links shows the number of links usually connected to them. The method uses the adjacency matrix to obtain the correlation matrix

II. TERMINOLOGY

The following definitions are to be understood clearly before applying the method. Various definitions with their abbreviations are given below.

1. Degree of link: It is a numerical value for the link, based on its connectivity to other links therefore quaternary link has degree equal to four and ternary link has degree equal to three.

2. Node value (NV): It is defined as inverse of degree of a link.

3. Joint Value (JV): It is the sum of all node values of the links connected at a joint.

4. Value of Link (VL): For a link, it defined as the sum of all elements of each row or column of matrix.

5. Value of the Chain (VC): It is defined as the sum of all link values.

6. Path Value (PV): It is the least sum of all joint values between two links under consideration.

III. DISTANCE MATRIX

Distance Matrix, for an n-link kinematic chain it is defined as an n X n square matrix,

$$\mathbf{DM} = \{\mathbf{D_{ij}}\}_{\mathbf{n\times n}}$$

The elements of matrix are defined by D_{ij} which is the least sum of all joint values between two links i and j, and is equal to zero if i is equal to j, of course, all the diagonal elements $D_{ii} = 0$.

$$DM = \begin{bmatrix} 0 & D_{12} & D_{13} & \ldots.. & D_{1n} \\ D_{21} & 0 & D_{23} & \ldots.. & D_{2n} \\ \ldots.. & \ldots.. & \ldots.. & \ldots.. & \ldots.. \\ \ldots.. & \ldots.. & \ldots.. & \ldots.. & D_{12} \\ D_{n1} & D_{n2} & D_{n3} & \ldots.. & 0 \end{bmatrix}$$

IV. METHODOLOGY

In this approach, it is proposed to write the elements in the matrix by the path value. It is a square and symmetric matrix. The sum of each row elements of the matrix is the value of the link, while the sum of all the elements in the matrix is the value of the chain. The numerical scheme for the chain is developed with the help of values of the links. Similarly, the numerical schemes for the links are also developed with the help of each row elements of the matrix. For example, consider a kinematic chain with, six bar, seven joints, and single degree-of-freedom Figure. 1; the links are labelled as A, B, C, D, E and F. Links A, B, D and E are the binary links so that, their node values are 1/2. Similarly, the links C and F are the ternary links, so that their node values are 1/3. Now the value of the joint, which is generated by link A, and link B is 1/2 + 1/2 = 1. Similarly the value of the joint, which is generated by link A, and link F is 1/2 + 1/3 = 5/6. All the joint values are shown in fig 1

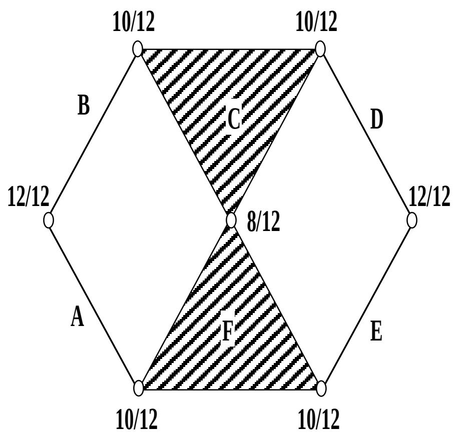

Figure 1 (Watt Chain)

Path Value (PV) is the least sum of all joint values between two links under consideration. For example, considering Link A and B, all possible paths between the links are

Path No.	Pathway	Path Value
1	**A**-**B**	**12/12**
2	**A**-F-C-**B**	1/12 (10 + 8 + 10) = **28/12**
3	**A**-F-E-D-C-**B**	1/12(10+10+12+10+10) = **52/12**

The least Path Value between link A and B is 12/12, this value is represented by element D_{12} in the matrix, similarly the Path Value between A, C is 18/12, and the value of element D_{13} is represented by 18/12 in the matrix and so on. It is the square and symmetric matrix because value of the elements $D_{12} = D_{21}$, $D_{13} = D_{31}$ and so on. By the similar procedure the complete matrix for the watt chain (Figure 1) given by DM_1

$$DM_1 = 1/12 \begin{bmatrix} 0 & 12 & 18 & 28 & 20 & 10 \\ 12 & 0 & 10 & 20 & 28 & 18 \\ 18 & 10 & 0 & 10 & 18 & 8 \\ 28 & 20 & 10 & 0 & 12 & 18 \\ 20 & 28 & 18 & 12 & 0 & 10 \\ 10 & 18 & 8 & 18 & 10 & 0 \end{bmatrix}$$

For the Six - bar chain Figure 1, the values of the link A are given by $D_{11} + D_{12} + \ldots\ldots D_{16} = 88/12$. Similarly, the values of the links B, C, D, E and F, are 88/12, 64/12, 88/12, 88/12 and 64/12 respectively, and the value of the chain is given by summation of all value of link and it is 480/12. Now considering the values of the links from the matrix, the numerical scheme for this chain can be written as follows,

[88/12, 88/12, 64/12, 88/12, 88/12, 64/12]

Or 1/12[2(64), 4(88)]

The number, for example 2, before the bracket indicates, two links with identical values. Reduction in length of the numerical scheme will be more evident in case of chains with more number of links.

For six bars, chain Fig 1
Value of the chain = 480/12
Numerical scheme for chain = 1/12[2(64), 4(88)]

By the similar procedure the complete matrix for the Stephenson chain (Figure 2) given by DM_2

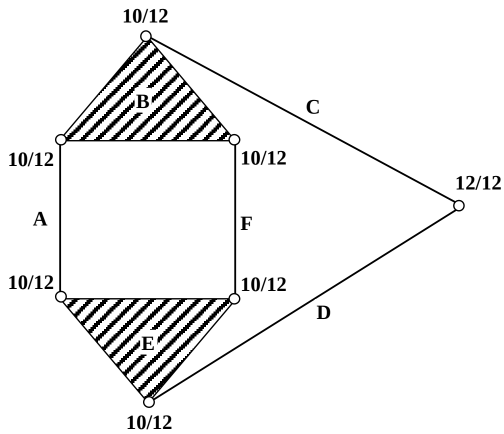

Figure 2 (Stephenson Chain)

$$DM_2 = 1/12 \begin{bmatrix} 0 & 10 & 20 & 20 & 10 & 20 \\ 10 & 0 & 10 & 22 & 20 & 10 \\ 20 & 10 & 0 & 12 & 22 & 20 \\ 20 & 22 & 12 & 0 & 10 & 20 \\ 10 & 20 & 22 & 10 & 0 & 10 \\ 20 & 10 & 20 & 20 & 10 & 0 \end{bmatrix}$$

Value of the chain = 472/12
Numerical scheme for chain = 1/12[2(72), 2(80), 2(84)]

V. ISOMORPHISM

The values of the chains are compared for detection of isomorphism, it is the first stage of comparison, and the second stage is to compare one-to-one similarity among their numerical schemes. When both the values and numerical schemes for two kinematic chains are identical, the test is positive or the chains are isomorphic, otherwise not. This method reports that kinematic chain, Figure 1 and 2, are non-isomorphic as the values of the chain and numerical schemes, are different for both the kinematic chain.

The **counter examples** appeared in reference [23] the chains shown in Figs. 3 and 4 were predicted as isomorphic. When these chains are checked by proposed method for detection of isomorphism, the results obtained are in agreement with the results reported in reference [23]

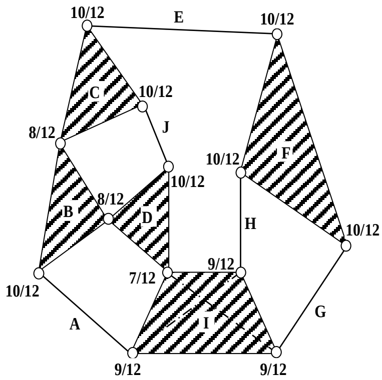

Figure 3

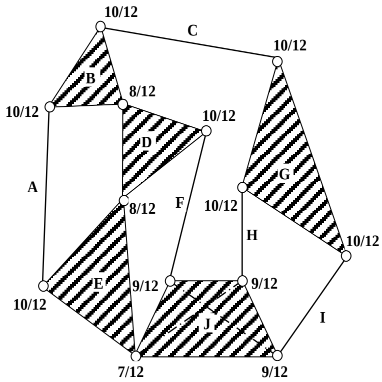

Figure 4

For kinematic chain (Figure 3)

Value of the chain = 1654/12

Numerical scheme for chain = 1/12[(137), (141), (153), (165), 3(171), 2(181), (183)]

For kinematic chain (Figure 4)

Value of the chain = 1654/12

Numerical scheme for chain = 1/12[(137), (141), (153), (165), 3(171), 2(181), (183)]

Proposed method reports that kinematic chain 3 and 4 are isomorphic as the values as well as their numerical schemes for both the kinematic chain are same.

The **counter examples** appeared in Reference [7] the chains shown in Figures 5 and 6 were wrongly predicted as isomorphic but these two chains have different value and numerical scheme and hence they are non-isomorphic

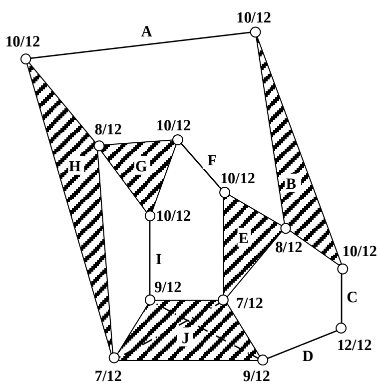

Figure 5

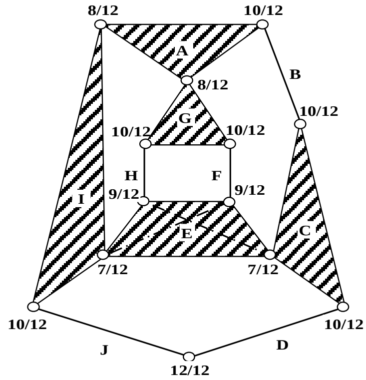

Figure 6

For kinematic chain (Figure 5)

Value of the chain = 1574/12

Numerical scheme for chain = 1/12[(117), (127), (137), (145), (155), (169), (171), (173), (175), (205)]

For kinematic chain (Figure 6)

Value of the chain = 1662/12

Numerical scheme for chain= 1/12[(129), (137), (141), (157), (165), (167), (179), (189), 2(199)]

Proposed method reports that kinematic chain 5 and 6 are non-isomorphic because the values as well as their numerical schemes for both the kinematic chains are different.

The **counter examples** appeared in Reference [11] the chains shown in Figs. 7, 8, 9 and 10, were predicted as non-isomorphic

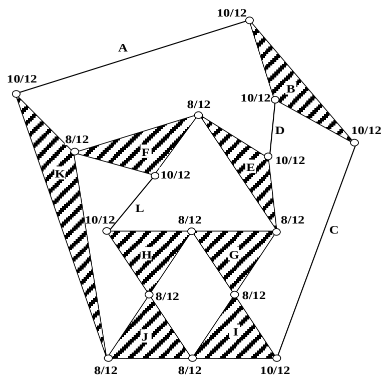

Figure 7

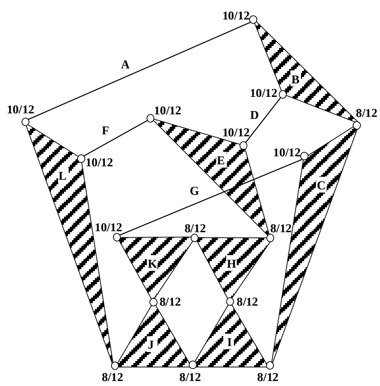

Figure 8

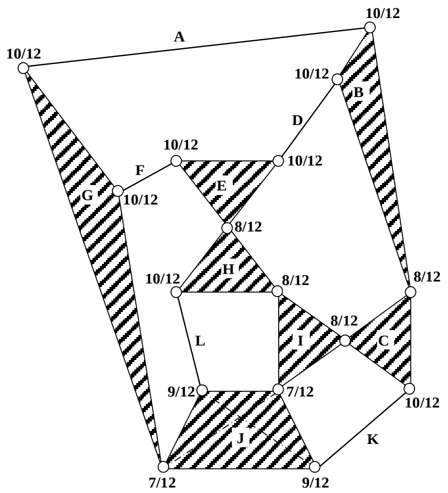

Figure 9

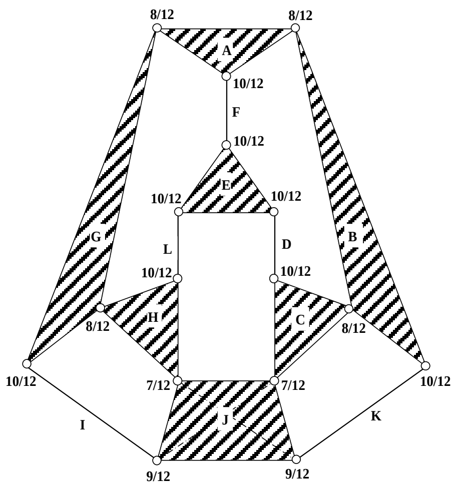

Figure 10

For kinematic chain (Figure 7)
Value of the chain = 2576/12
Numerical scheme for chain = 1/12[(188), 2(192), 3(196), (208), 2(236), (240), 2(248)]

For kinematic chain (Figure 8)
Value of the chain = 2544/12
Numerical scheme for chain = 1/12[(184), 2(192), (196), (200), 2(204), (208), (236), 2(240), (248)]

For kinematic chain (Figure 9)
Value of the chain = 2523/12
Numerical scheme for chain = 1/12[2(175), (191), (197), (202), 2(211), 2(223), (235), (239), (241)]

For kinematic chain (Figure 10)
Value of the chain = 2508/12
Numerical scheme for chain = 1/12[3(179), (189), 2(199), 2(227), 2(229),(237),(245)]

Proposed method reports that kinematic chain 7, 8, 9 and 10 are non-isomorphic as the values and their numerical schemes are different for all kinematic chains.

Fig 11 shows two graphs of 28 links kinematic chain in Reference [38]. By using proposed method on these graphs, they found isomorphic.

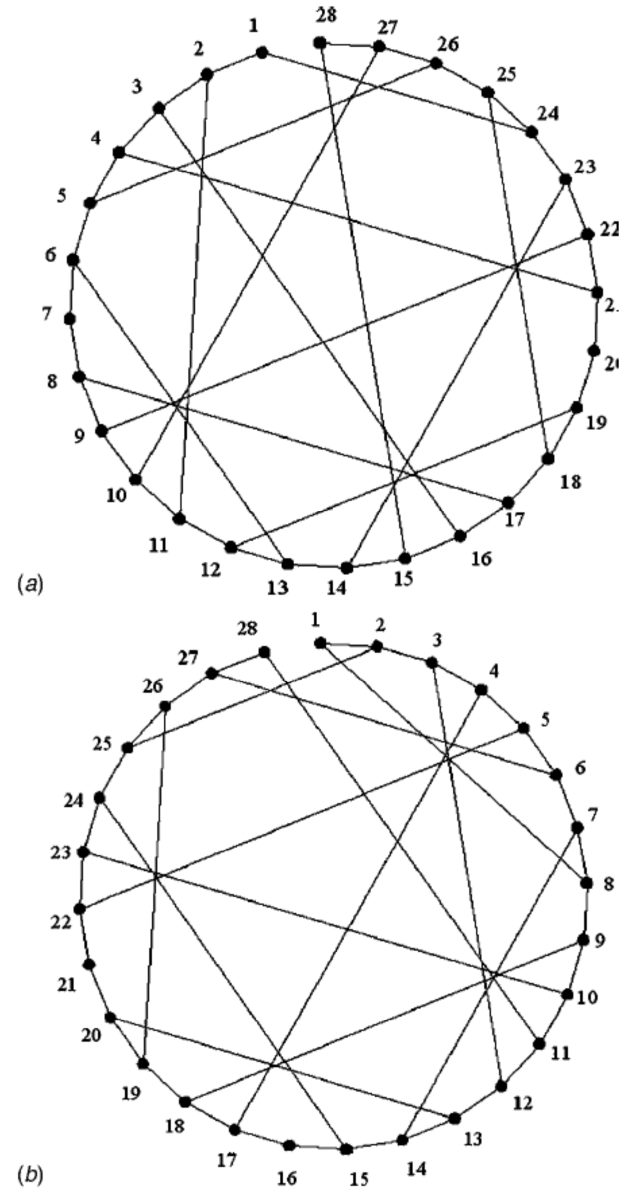

Figure 11

For kinematic chain (Figure 11 a)
Value of the chain = 18384/12
Numerical scheme for chain = 1/12[4(796), 8(660), 16(620)]

For kinematic chain (Figure 11 b)
Value of the chain = 18384/12
Numerical scheme for chain = 1/12[4(796), 8(660), 16(620)]

VI. INVERSIONS

The distinct inversions of a kinematic chain depend on the detection of distinct links. Once the distinct links are, identified then distinct inversions can established. As per the finding of earlier works there exist 5, 71, 1834 distinct mechanisms for six, eight and ten link of single degree of freedom kinematic chains respectively. To find inversions of a kinematic chain, the first stage is the comparison of values of the links, and the second stage is to compare one-to-one similarity among their numerical schemes. When both, the values and their numerical schemes are identical, the test is positive or they will give same kinematic inversions. For six bar chains, Figure 1 and Figure 2 the values of each link with their numerical schemes are tabulated in table 1 and 2 respectively with the help of matrices.

Links	Value of Link	Numerical Scheme
A	88/12	1/12[(10),(12),(18),(20),(28)]
B	88/12	1/12[(10),(12),(18),(20),(28)]
C	64/12	1/12[(8),2(10),2(18)]
D	88/12	1/12[(10),(12),(18),(20),(28)]
E	88/12	1/12[(10),(12),(18),(20),(28)]
F	64/12	1/12[(8),2(10),2(18)]

Distinct links	Value of Link	Numerical Scheme For Distinct Link
(A,B,D,E)	88/12	1/12[(10),(12),(18),(20),(28)]
(C,F)	64/12	1/12[(8),2(10),2(18)]
Total Distinct Inversion - 02		

Table 1

Distinct inversions of Six Bar Watt chain.

Links	Value of Link	Numerical Scheme
A	80/12	1/12[2(10),3(20)]
B	72/12	1/12[3(10),(20),(22)]
C	84/12	1/12[(10),(12),2(20),(22)]
D	84/12	1/12[(10),(12),2(20),(22)]
E	72/12	1/12[3(10),(20),(22)]
F	80/12	1/12[2(10),3(20)]

Distinct links	Value of Link	Numerical Scheme For Distinct Link
(A,F)	80/12	1/12[2(10),3(20)]
(B,E)	72/12	1/12[3(10),(20),(22)]
(C,D)	84/12	1/12[(10),(12),2(20),(22)]
Total Distinct Inversion - 03		

Table 2

Distinct inversions of Six Bar Stephenson chain

The value of link and numerical schemes for each distinct links of all 16 eight-link chains are tabulated in table 3. Total 71 distinct inversions are found from all eight links, single degree of freedom kinematic chains that comply with earlier findings.

RESULTS: The proposed method is very simple and able to detect isomorphism among the Kinematic chains and number of inversions derived from a kinematic chain. All the simple jointed, single degree of freedom; 6-links 2 kinematic chains, 8-links 16 kinematic chains, 10-link 4 kinematic chains and 12-link 4 kinematic chains have been tested successfully for their non-isomorphism.

CONCLUSION: In the present paper, a simple, efficient, and reliable method is proposed to identify isomorphism. The results obtained are compared with the earlier findings and are in agreement. The proposed method of detection of isomorphism is based on link connectivity, and successfully identifies the different planar kinematic chains with single degree of freedom. The method is simple and involves less mathematical calculations. Further, it is directly applicable to kinematic chains and does not require converting the kinematic chain to their graphs.

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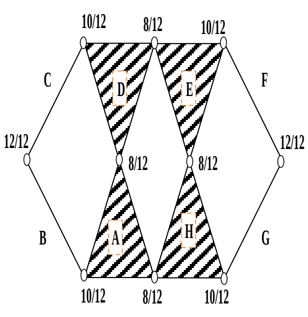
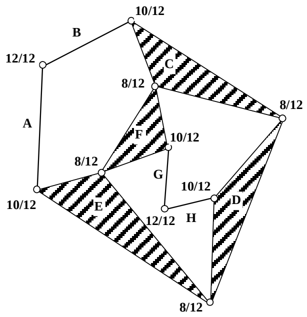
Chain No.	Distinct Link	Value of Link	Numerical String	Distinct Inversion
1	(A,D,E,H)	104/12	1/12[2(8), (10), (16), 2(18), (26)]	2
	(B,C,F,G)	148/12	1/12[(10), (12), 2(18), (26), (28), (36)]	
2	(A,B,G,H)	136/12	1/12[(10), (12), 2(18), (22), 2(28)]	2
	(C,D,E,F)	100/12	1/12[2(8), (10), (16), 2(18), (22)]	
3	(A)	108/12	1/12[(2(8), (10), (16), (18), (22), (26)]	8
	(B)	140/12	1/12[(10), (12), 2(18), (22), (28), (32)]	
	(C)	140/12	1/12[(10), (12), (18), (20), (22), (28), (30)]	
	(D)	104/12	1/12[(8), 2(10), (16), (18), (20), (22)]	
	(E)	136/12	1/12[2(10), (18), 2(20), (26), (32)]	
	(F)	116/12	1/12[(8), 2(10), (18), 2(20), (30)]	
	(G)	100/12	1/12[2(8), (10), 3(18), (20)]	
	(H)	132/12	1/12[2(10), 2(18), (20), 2(28)]	
4	(A,H)	132/12	1/12[2(10), 2(18), (20), 2(28)]	4
	(B,E)	108/12	1/12[(8), 2(10), (16), (18), (22), (24)]	
	(C,D)	136/12	1/12[(10), (12), (18), (20), (22), (26), (28)]	
	(F,G)	104/12	1/12[2(8), (10), (16), 2(18), (26)]	
5	(A,H)	132/12	1/12[2(10), 2(18), (20), 2(28)]	4
	(B,G)	108/12	1/12[(8), 2(10), (16), (18), (20), (26)]	
	(C,F)	96/12	1/12[2(8), (10), (16), 3(18)]	
	(D,E)	140/12	1/12[(10), (12), 2(18), (26), 2(28)]	
6	(A,F)	136/12	1/12[2(10), (18), 2(20), (26), (32)]	5
	(B,E)	108/12	1/12[(8), 2(10), 2(16), (22), (26)]	
	(C,D)	140/12	1/12[(10), (12), (18), (20), (22), (26), (32)]	
	(G)	112/12	1/12[(8), 2(10), 2(16), 2(26)]	
	(H)	96/12	1/12[3(8), 4(18)]	
7	(A)	136/12	1/12[2(10), (18), 2(20), (28), (30)]	6
	(B,F)	108/12	1/12[(8), 2(10), (16), (18), (20), (26)]	
	(C,E)	124/12	1/12[2(10), (18), 3(20), (26)]	
	(D)	120/12	1/12[3(10), 3(20), (30)]	
	(G)	100/12	1/12[2(8), (10), 3(18), (20)]	
	(H)	124/12	1/12[2(10), 2(18), 2(20), (28)]	
8	(A,D,G,H)	132/12	1/12[2(10), 2(18), (20), 2(28)]	2
	(B,C,E,F)	112/12	1/12[(8), 2(10), 2(18), (20), (28)]	
9	(A,B,D,G)	104/12	1/12[(8), 2(10), 2(18), 2(20)]	2
	(C,E,F,H)	124/12	1/12[2(10), 2(18), 2(20), (28)]	
10	(A)	103/12	1/12[4(9), 2(19), (29)]	4
	(B,F,G,H)	121/12	1/12[(9), (10), 3(18), (20), (28)]	
	(C,E)	125/12	1/12[3(10), (19), (20), 2(28)]	
	(D)	129/12	1/12[2(10), 4(20), (29)]	
11	(A)	103/12	1/12[(8), 2(10), 2(18), (19), (20)]	7
	(B)	133/12	1/12[2(10), (18), (19), (20), 2(28)]	
	(C)	121/12	1/12[(9), (10), 3(18), (20), (28)]	
	(D)	93/12	1/12[4(9), 3(19)]	
	(E,H)	119/12	1/12[(9), (10), 4(18), (28)]	
	(F)	111/12	1/12[(8), 2(10), 2(18), (19), (28)]	
	(G)	111/12	1/12[(9), (10), 4(18), (20)]	
12	(A)	127/12	1/12[3(10), (19), (20), (28), (30)]	7
	(B,H)	117/12	1/12[(9), (10), (16), 2(18), (20), (26)]	
	(C)	87/12	1/12[(7), 3(9), 2(17), (19)]	
	(D)	127/12	1/12[(9), (12), (16), 2(18), (26), (28)]	
	(E)	141/12	1/12[(10), (12), (17), (20), 2(26), (30)]	
	(F)	95/12	1/12[(7), 2(10), 3(16), (20)]	
	(G)	123/12	1/12[2(10), (17), 3(20), (26)]	
13	(A)	91/12	1/12[(7), (8), (10), 3(16), (18)]	8
	(B)	137/12	1/12[(10), (12), (17), (18), 2(26), (28)]	
	(C)	127/12	1/12[(9), (12), (16), 2(18), (24), (30)]	
	(D)	87/12	1/12[(7), 3(9), (15), (17), (21)]	
	(E)	121/12	1/12[(9), (12), (16), 2(18), (22), (26)]	
	(F)	139/12	1/12[(10), (12), (18), (20), (21), (28), (30)]	
	(G)	107/12	1/12[(8), 2(10), (15), (18), (22), (24)]	
	(H)	117/12	1/12[(9), (10), (16), 2(18), (20), (26)]	
14	(A,E)	143/12	1/12[(10), (12), (17), (20), (24), (26), (34)]	5
	(B,D)	123/12	1/12[(9), (12), 2(16), (18), 2(26)]	
	(C)	83/12	1/12[2(7), 2(9), 3(17)]	
	(F,H)	97/12	1/12[(7), 2(10), (14), 2(16), (24)]	
	(G)	129/12	1/12[2(10), (17), 2(20), 2(26)]	
15	(A,D)	78/12	1/12[(6), 3(9), 3(15)]	2
	(B,C,E, F,G,H)	120/12	1/12[(9), (12), (15), 2(18), 2(24)]	
16	(A,B,D,E)	126/12	1/12[(9), (12), 3(18), (21), (30)]	3
	(C,F)	96/12	1/12[4(9), (18), 2(21)]	
	(G,H)	108/12	1/12[2(9), 5(18)]	
			Total Inversion	71

Appendix 1

Eight Link, Single Degree of Freedom Kinematic Chain



(1)



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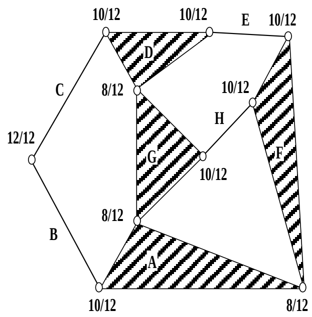

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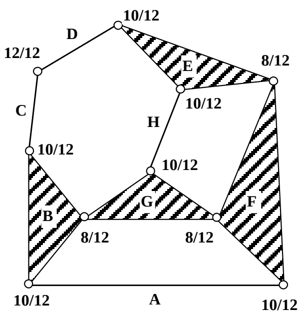

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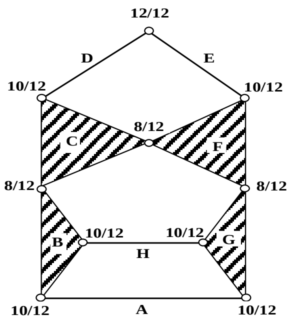

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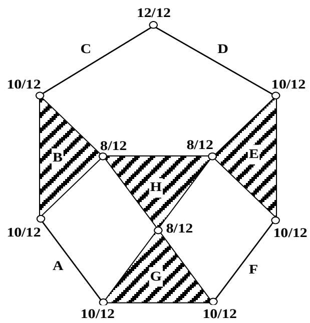
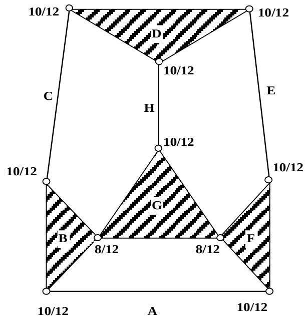
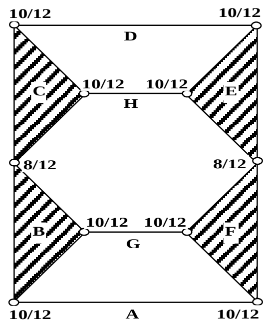


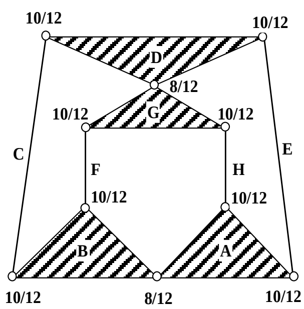
(6)



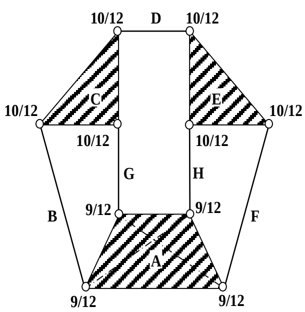
(7)



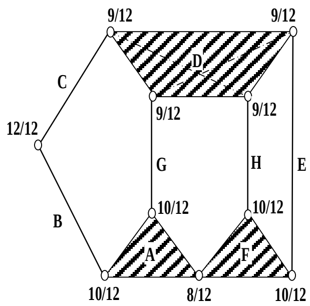
(8)



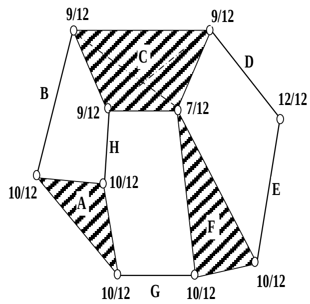
(9)



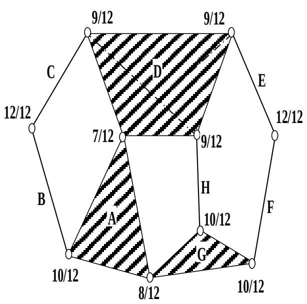
(10)



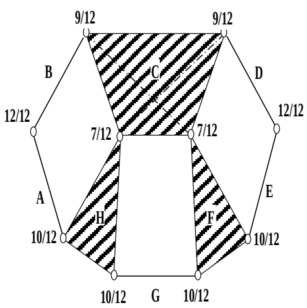
(11)



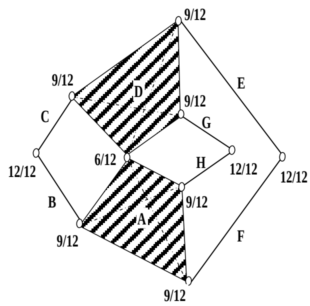
(12)



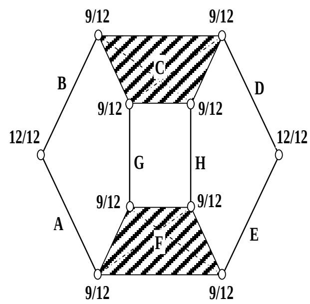
(13)



(14)



(15)



(16)