Neural Network Solutions of Forward Kinematics for 3RPS Parallel Manipulator

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***Abstract*— This paper describes how neural networks are much capable to solve the forward kinematics of parallel manipulators. The solution to the forward kinematics involves highly nonlinear equations of motion, which can be solved by numerical methods with great accuracy but the time consuming calculations restrain it in implementing for real time kinematic control. Two types of neural networks namely multilayer perceptron (MLP) and radial basis function (RBF) are considered to solve the forward kinematics of 3RPS (revolute-prismatic-spherical) parallel manipulator. The performance and suitability of both the networks are evaluated for this specific application.**

Keywords*— *workspace volume; MLP; RBF

I. Introduction

Parallel manipulators have become popular in recent years because of their merits like high stiffness, large load carrying capacity and high precision control over the prescribed path of end-effecter. Parallel manipulators are closed loop kinematic structures in which the tool platform is connected to the fixed base by means of several serial legs in parallel. V.E .Gough and S.G.Whitehall [1] has developed a 6 degree of freedom (dof) Universal tire testing machine. The 6 dof manipulator has designed by D.Stewart [2] as an motion simulator. K.H.Hunt [3] has studied the kinematics of distinct types of parallel manipulators. In recent times a fewer dof mechanisms has attracted the attention of researchers because of reduction in manufacturing cost and easiness in control. The 3dof translational DELTA robot has been investigated by R.Clavel Delta [4], the 3dof CapaMan manipulator by X.J.Liu et al [5, 6]. The kinematics of 3-RPS parallel manipulator with three identical legs have been studied by K.Lee and D.K.Shah [7], L.W.Tsai [8] . S.A.Joshi and L.W.Tsai [9], C.H.Liu et al [10] has studied the singular configurations of 3RPS manipulator through inversion of jacobian matrix. The inverse kinematics for parallel manipulators is usually simple, in which joint displacements are computed for the given end-effecter position, but the solution for direct kinematics is complicated, in which a set of nonlinear equations is to solved, however it is difficult to find a unique solutions. Many researchers have solved the direct kinematics problem using the special methods such as numerical approaches; algebraic elimination and continuation methods. J.Gallardo et al [11] have carried out the forward position analysis of parallel manipulators with identical limb type revolute-prismatic-spherical (R-P-S) leg by applying the Sylvester dialytic elimination method. C.R.Rad et al [12] addressed the forward kinematics of a 3-dof medical robot with R-P-S joints through Newton-Kantorovich (N-K) method. Y.Li and Q.Xu [13] has performed forward kinematics analysis for a 3PRS spatial parallel mechanism using Newton iterative algorithm. T.Y.Lee and J.K.Shim [14] have solved FKP (forward kinematics problem) of a 6 dof Stewart manipulator through algebraic elimination method, M.Raghavan [15] by continuation method. Generally different methods (numerical approaches, algebraic methods) will give different solutions for a specific direct kinematic problem, but these methods take much time for computation so not applied for a real time control of manipulators.

Some researchers have put their efforts towards FKP solving of parallel robots using neural networks. H.Sadjadian et al [16] has applied the NN approach for solving the forward kinematics of a 3-dof redundant manipulator, M.Dehghani et al [17] for hexa parallel robot. The main objective of the paper is to find out the direct kinematic solution of a parallel manipulator with the help of neural network method. The work has been under taken to find the best ANN configuration for the problem. In this paper two types of NN namely multilayer perceptron (MLP) and (RBF) are used to solve FKP of 3RPS parallel manipulator, the performance of both the networks are compared and simulation of networks have been performed.

The organization of the paper is followed as section 2 describes the geometry of the manipulator and the kinematic analysis of mechanism, section 3 explains the workspace analysis. A brief introduction to NN approach is given in section4; NN solution for FKP in section5 and the performance of the networks are compared in section6. Section 7 describes the simulated results of networks. Finally, the paper is concluded in Section 8.

II. KINEMATIC ANALYSIS

A. *Geometric description and position analysis of manipulator*

The geometry of the 3-RPS parallel manipulator is shown in Fig.1, in which the tool platform $\{B\}$ is connected to the fixed base $\{A\}$ by three identical serial chains having R-P- S joints. The revolute joints are located at the corners (A_i) of a fixed base which is an equilateral triangle; the spherical joints are located at the corners (B_i) of the tool platform.

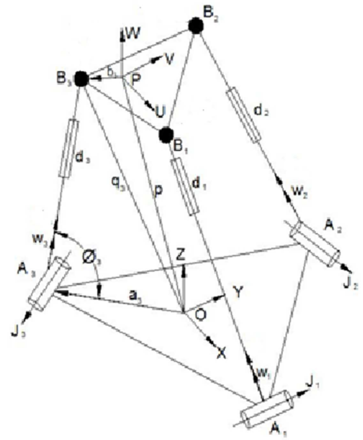

Fig.1 3RPS spatial manipulator

The axes of revolute joints are J_i $(i = 1,2,3)$, $a_i = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}^T$ is the position vector of corner points A_i, and $b_i = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T$ is the position vector of the spherical joint B_i with respect to the moving frame uvw which is located at p the centre point of moving platform. The magnitude of the position vector a_i is represented by g and the magnitude of the position vector b_i is represented by h. The i_{th} leg length and unit vector are represented by d_i and w_i respectively. The rotational transformation matrix ${}^{O}R_P$ from the moving frame to the fixed frame can be expressed by

$$ {}^{O}R_{P} = R_y(\beta)R_x(\alpha)R_z(\gamma) \qquad (1)$$

The three Euler angles γ, α, β rotating about the Z, X and Y-axes of the fixed reference frame in sequence that is (Y-X-Z) Euler angle system is considered here. The rotation matrix can also be expressed in terms of the direction cosines of u , v and w as follows

$$ {}^{O}R_{P} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \qquad (2)$$

Where u , v and w be three unit vectors along u, v and w axes of the moving frame $\{B\}$.

The position vector of point B_i can be expressed as

$$ q_i = p + b_i \qquad (3)$$

Where $b_i = {}^{O}R_p\, {}^{P}b_i$, $q_i = \begin{bmatrix} q_{ix} & q_{iy} & q_{iz} \end{bmatrix}^T$ considering the mechanical constraints imposed by a revolute joint R_i the motion of the spherical joint S located at B_i is constrained to move in one of the following three planes:

$$ q_{1y} = 0 \qquad (4a)$$

$$ q_{2y} = -\sqrt{3}q_{2x} \qquad (4b)$$

$$ q_{3y} = +\sqrt{3}q_{3x} \qquad (4c)$$

On substitution of Eq. (3) into Eq. (4) ,we can get

$$ p_y + hu_y = 0 \qquad (5a)$$

$$ v_x = u_y \qquad (5b)$$

$$ p_x = \frac{h}{2}(u_x - v_y) \qquad (5c)$$

B. *Inverse kinematic analysis*

In an inverse kinematic analysis one can get the actuated joint variables for a given configuration of end-effecter. The vector loop closure equation for i_{th} limb is given as

$$ d_i = p + {}^{O}R_P\, {}^{P}b_i - a_i \qquad (6)$$

Operate the dot product to the above equation yields

$$ d_i^2 = [q_i - a_i]^T [q_i - a_i] \quad \text{for } i = 1,2,3 \qquad (7)$$

Taking square root of the above equation can find the actuator displacements; in which there are a total of eight possible solutions for a given end-effecter pose.

C. *Forward kinematics problem*

In direct kinematics problem can find the position and orientation of the end-effecter for the actuator displacements d_1,d_2,d_3.The position and orientation of the mechanism can be obtained by taking the distance between any two consecutive spherical joints B_i and B_{i+1} is equals to a constant $\sqrt{3}h$, which can be expressed mathematically as

$$ \left\| B_i B_{i+1} \right\| = \sqrt{3}h \qquad (8)$$

$$ [q_i - q_{i+1}]^T [q_i - q_{i+1}] - 3h^2 = 0 \quad i = 1,2,3 \qquad (9)$$

The above Eq. (9) gives a set of three nonlinear equations which can be solved either by Sylvester dialytic elimination method or by any numerical methods but which requires a lot of computational efforts.

III. WORKSPACE ANALYSIS

For effective utilization of parallel manipulators, it is necessary to determine the size and shape of workspace of the manipulators. The dexterous workspace of manipulator signifies its working potential. While designing a practical manipulator the physical constraints interns of the range of parasitic motions of the end-effecter and limits of the prismatic actuators, interference of legs and limitations on the passive joints are to be considered. C.Gosselin [18], J.P.Merlet [19] presented an algorithm enabling to compute the possible rotation of the end-effecter around a fixed point.

A. Workspace simulation

Since symmetric architectures are commonly considered in literature, 3RPS manipulator with both $\Delta A_1 A_2 A_3$ and $\Delta B_1 B_2 B_3$ as equilateral triangles with $|OA_1| = |OA_2| = |OA_3| = g = 2m, |PB_1| = |PB_2| = |PB_3| = h$=1m) and has been taken up as an example. The possible maximal leg lengths range, for each leg $i = 1,2,3$ considered here is [1 3] m and the two rotational Euler angles α, β of the moving platform are bounded in the range [-60^0 60^0]. The workspace volume of 3RPS manipulator is shown in Fig 2, using the inverse kinematic solutions instead of using the direct kinematics in which a multiple solutions are to be obtained for the given input joint positions. The workspace volume is continuous and there are no vertical gaps along the z-coordinate. The bottom, middle portion of the reachable workspace volume is cylindrical in shape and the top portion is in the shape of a triangular pyramid. Larger workspace size at the middle sections and smaller workspace size at top sections is observed.

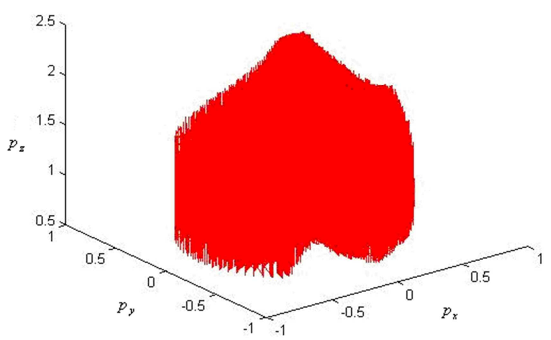

Fig.2 Workspace volme of 3RPS parallel manipulator

B. Neural networks

A neural network resembles the functioning of a human brain. The key elements in NN are neurons which are the information processing units interconnected with each other, the connections between elements determine the network function. In NN knowledge is represented as numeric weights; these are iteratively adjusted to minimize the sum of the squared approximation errors using the specified training algorithm. NN can be used to input-output mapping, generalization, parallel and high speed information processing.

C. Multi layer perceptron neural networks

A.Jain et al [20**]** extensively studied MLP network s**tructure**s**;** MLP is the most common type of feed-forward network which consists of an input layer, output layer and some hidden layers as shown in Fig.3. A vector of predictor variable values $(x_1 \ldots.. x_n)$ is supplied to the input layer. The input layer only transmits these values to each neuron in the hidden layer. The neurons in hidden layer adjust the weights until the desired error between the output vector and desired vector is achieved.

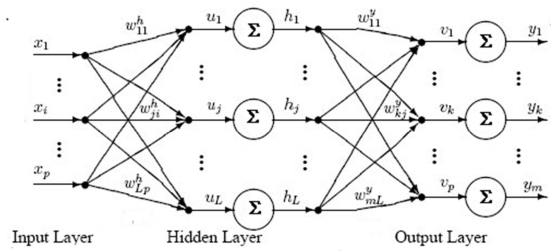

Fig.3 MLP Network structure

D. Radial basis neural networks

Radial basis function (RBF) networks consist of two layers, in which one is hidden layer and other is the output layer [21], [22]. The input layer has neurons with a linear function that simply feed the input signals to the hidden layer. Moreover, the connections between the input and hidden layer are not weighted. The hidden layer uses neurons with RBF activation that perform the radial basis function, which is expressed mathematically as

$$\varphi_i(x) = \varphi \| x - x_i \| \quad i = 1,2,3 \qquad (10)$$

i_{th} input data point x_i denotes the centre of radial basis function and x is the vector pattern applied to the input layer.

IV. NN SOLUTION FOR FKP

In order to model NN structure for solving FKP of 3RPS parallel manipulator, have chosen 4000 data points randomly while it covers the entire reachable workspace of the manipulator which is shown in Fig.2.Then inverse kinematics problem (IKP) is solved for these chosen workspace points , the MLP and RBF NN are trained for these IKP solutions.

A. *Multi layer perception network*

A Multi-layer feed forward network with back propagation (BP) learning and Levenberg -Marquardt training algorithm is considered to solve the FKP of manipulator. The input layer has three nodes which represent the three input actuator displacements (d_1, d_2, d_3), similarly the output layer has three nodes which represents three position variables (p_x, py, pz) are the outputs of the mechanism. The desired behavior can be generalized by a set of inputs and output pairs. Different MLP structures were tested by varying the design parameters such as number of hidden layers, number of neurons in each layer for finding the efficient network configuration. The MLP network with larger number of hidden layers does not perform well, so two MLP structures one having one hidden layer and other with two hidden layers are considered for further testing. The performance of two NN structures one with one hidden layer and other with two hidden layers for various configurations are given in Table.1and Table.2 respectively. Three performance indices mean square errors (MSE), sum squared error (SSE) and auto correlation coefficient (R) are used to evaluate the NN. The MLP with two hidden layers having neurons 15, 20 in first and second hidden layers respectively is observed as the best MLP NN structure with performance measures MSE=0.000537, SSE=7.42 and auto correlation coefficient R=0.99844.The performance plot of this MLP is shown in Fig.4, it is observed that the training, testing and validation curves are followed the consistently decreasing trend. The regression plot for training, validation and testing is shown in Fig.5. The MLP structure with one hidden layer having 100 neurons results a considerable performance of MSE=0.00311, SSE=43 and R=0.99559 among all the neuron configurations. It is observed that networks with less number of nodes are the good choice because of the number of weights and also training time is reduced considerably.

Table.1 Performance of MLP with one hidden layer

Network Structure	Multilayer Feed Forward One hidden layer				
Network Performance	Hidden Layer neuron	Training Time (sec)	MSE	SSE	R
	S=10	19	0.0076	106	0.9900
	S=30	47	0.0054	75	0.9935
	S=50	42	0.0042	59	0.9955
	S=100	138	0.0031	43	0.9955
	S=300	120	0.0032	44	0.9954

Table.2 Performance of MLP with two hidden layers

Network structure	Multilayer feed forward Two hidden layer				
Network Performance	No. Hidden Layer neurons	Training Time (sec)	MSE	SSE	R
	S1=5,S2=10	21	0.0041	57	0.9952
	S1=10,S2=15	38	0.0032	44	0.9959
	S1=15,S2=20	139	0.0005	07	0.9984
	S1=20,S2=25	43	0.0031	43	0.9954
	S1=25,S2=30	80	0.0036	51	0.9953
	S1=30,S2=35	94	0.0028	39	0.9957
	S1=40,S2=45	435	0.0029	40	0.995

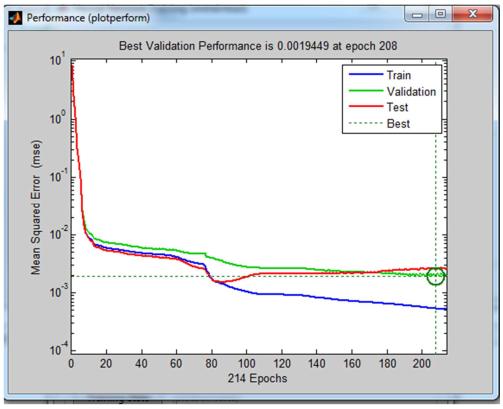

Fig.4 Performance plot for MLP Network

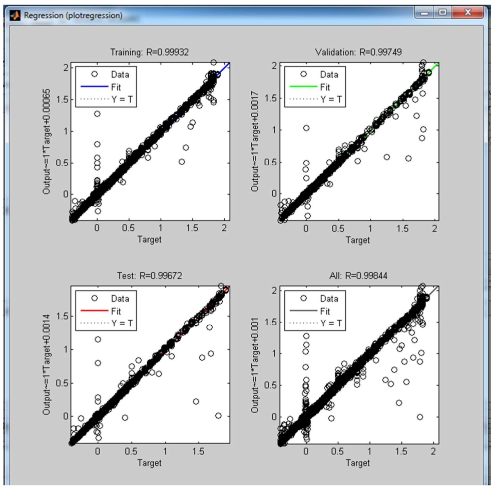

Fig.5 Regression plot for MLP Network

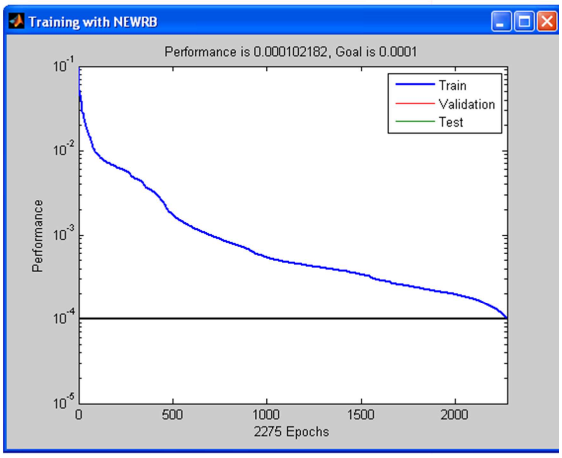

Fig.6 Training performance for RBF Network

B. *RBF networks*

Radial basis function is chosen as the second alternative NN structure to solve the FKP of a 3RPS parallel manipulator. The input and output patterns were generated in same manner as in the MLP. In RBF the neurons are added continuously to the network until the desired accuracy will be reached. The network is trained for different configurations with different spread parameters varied between 0.04 and 4, among all the configurations only a two networks with best performance are selected, results are given in Table.3. The performance of the RBF1 for the spread parameter of 0.15, 2275 neurons in hidden layer is shown in Fig.6.The network is trained over 2275 epochs using 2275 neurons for attaining the desired MSE of 0.00010, the linear regression plot between targets and the outputs for RBF1 with a regression coefficient of R=0.99943 is shown in Fig.7.The desired MSE of 0.00001 with a regression coefficient of R=0.99996 is achieved for the RBF2 in a 14400 sec long time.

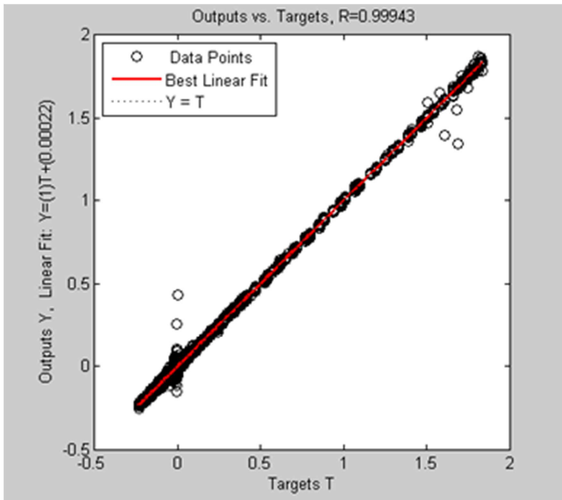

Fig.7 Regression plot for RBF Network

Table.3 Performance of RBF Networks

Network performance	Training Time (sec)	MSE	R
RBF1	2220	0.00010	0.9994
RBF2	14400	0.00001	0.9999

V COMPARISION OF MLP AND RBF

Even though both NN approaches yields good results for training and testing of given set of data points, some comparison between the MLP and RBF is necessary to discuss briefly. A two hidden layer MLP with neurons 15, 20 yields a lowest MSE of 0.000537, so the average error of FKP solution in the typical workspace is less than 0.5mm. In RBF Network the lowest MSE of 0.00001 is attained, so the average error of FKP solution is less than 0.01mm. The training time for MLP is 139 seconds where as in RBF the training time is 14400 sec. It is apparent from the results discussed above that the errors occurred in RBF is less when compared with MLP, but the training time for RBF is much more than the MLP.

VI SIMULATION OF NN FOR FKP SOLUTION

The best NN configurations for both MLP and RBF were tested for 100 chosen workspace points. The simulated network outputs using MLP for the chosen workspace points (targets) are shown in Fig.8-10 for each position variable (p_x,p_y,p_z) separately. The approximate errors for p_x , p_y , p_z coordinates are found to be 3mm,1mm and 2mm respectively using the MLP structure. Similarly the desired positions and estimated outputs for p_x , p_y , p_z are shown in Fig.11-13 using the RBF networks. It is observed that the approximation errors in x, y, z coordinates are 0.4mm, 0.3mm and 0.5mm respectively. It can be observed that the performance of RBF is better than the MLP structure.

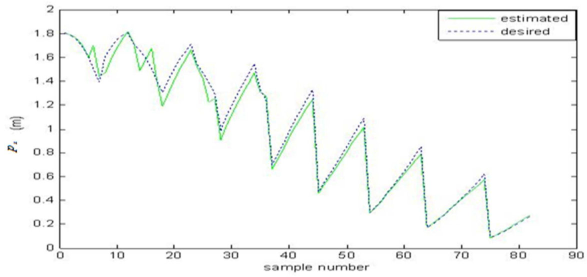

Fig.8 Tracking Performance of p_x for MLP

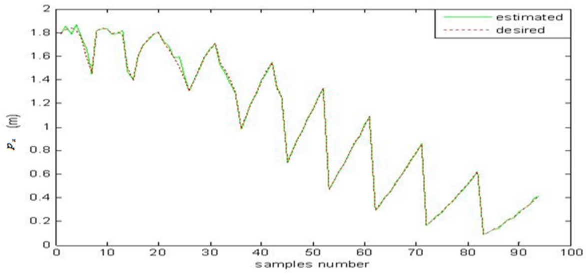

Fig.9 Tracking Performance of p_y for MLP

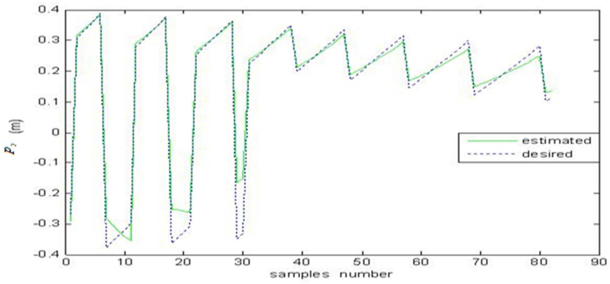

Fig.10 Tracking Performance of p_z for MLP

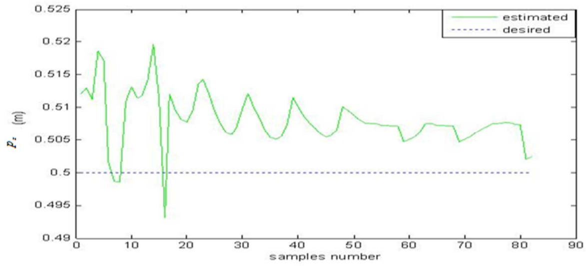

Fig.11 Tracking Performance of p_x for RBF

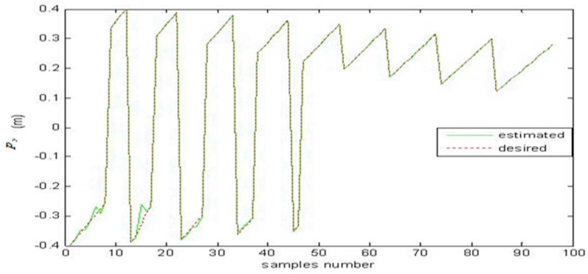

Fig.12 Tracking Performance of p_y for RBF

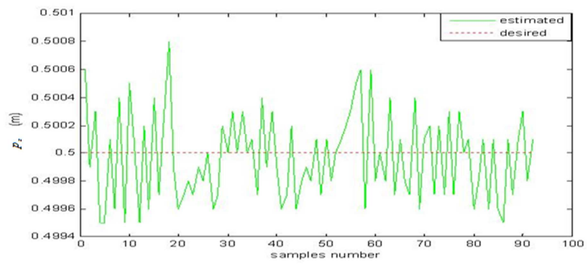

Fig.13 Tracking Performance of p_z for RBF

CONCLUSIONS

In this work proposed the application of neural networks for solving forward kinematics of a 3RPS parallel manipulator. The accuracy and tracking performance of both the MLP and RBF structures are compared; simulation results revealed that RBF has better performance than MLP. The NN method can also be applied for other parallel manipulators which have no unique solutions.

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