

Kinematic Analysis of Biped Robot Forward Jump for Safe Locomotion

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Abstract— In the present work kinematics of a four link biped robot is modeled using body coordinate formulation. The kinematic analysis of the biped robot is done for both stance and flight phases. Apart from joint constraints, constraint equations are introduced on the zero moment point (ZMP) and the centroidal angular momentum of the biped robot during stance phase. While the constraint on ZMP ensures stability during stance phase, constraint on angular momentum makes the biped robot non-holonomic even during the stance phase. During flight phase (during which angular momentum is conserved, and hence the system is non-holonomic by nature), the vertical distance between the foot and the center of mass is planned such that the velocity of the foot reaches zero at the time of landing to ensure a smooth landing without impact. Simulations are done in MATLAB using RK4 method. Simulation results have been presented for both the support and flight phases for various initial configurations of the robot, which resulted in variation of the horizontal distance and vertical height covered by the biped robot.

Keywords—Biped robot; jumping; kinematic analysis; motion planning

I. INTRODUCTION

The primary requirement of any legged locomotion is to avoid obstacles. Jumping is one of the significant motions that humanoid or biped robot needs to perform in working environments with obstacles. There are differences in the motions of jumping, leaping and hopping. *Jump* — jumping from and landing on two feet, *Hop* — jumping from one foot and landing on the same foot, *Leap* — jumping from one foot and landing on the other foot [14]

The major complexities in achieving jumping motion are getting a trajectory for biped jumping considering conservation of angular momentum, the inertial effects of the individual links and also dealing with the high Ground Reaction Force (GRF) or impact at the time of landing. Another issue is obtaining solution to non holonomic constraints which arise in incorporating conservation of angular momentum and Zero Moment Point (ZMP). Active research is going on in recent years to resolve one or the other of the aforementioned complexities. It is still a challenging task for the advanced robots even to achieve a jumping motion, which is at par with the biological systems.

Some of the researchers have focussed on performing vertical jumping with biped [1], [2]. In most of the working environments it is essential for the biped robot to perform forward jumping. Hirano, Sueyoshi and Kawamura (2000), have simulated forward jumping motion of the biped robot in a dynamic simulation environment [3]. Motion planning for biped robot jumping had been done by various researchers. Arikawa and Mita (2002), discussed about a multi- DoF jumping robot with 5 rigid links, for which the required motion planning is achieved by defining various polynomials for the joint angles [4]. Takuro, Ugurlu and Kawamura (2008), performed motion planning for biped robot jumping by formulating the kinematics in joint coordinates. The authors also have tried to get the Center of Gravity (CoG) trajectory by resolved angular momentum [5]. Pavan Kumar, Goswami and Vadakkepat (2008) proposed an intuitive method for gait generation method based on the biped's inverse kinematics. The biped's movement in the sagittal plane is solved in terms of two parameters, the distance between the ankle and the knee joints and the angle that hip makes with the vertical [6]. Wulandari and Komura (2008) have studied the influence of angular momentum in case of forward and backward jumping motion for humanoid robot [7]. Ugurlu and Kawamura (2009) proposed a method for motion planning of 3D-biped robot vertical jump using ZMP and conservation angular momentum equations in spherical coordinates. The method is named as Eulerian ZMP resolution (EZR) method. The authors have not discussed this for forward jumping case [8]. Takaoka, Heerden and Kawamura, (2010) have proposed a method for motion planning biped robot jump based on the Center of Gravity (CoG) trajectory looked from the tip of the toe (i.e the difference between the trajectories defined for the CoG and the foot). The research is aimed at reducing the Ground Reaction Force (GRF) at the time of landing by making the relative velocity between the foot and ground to be zero [9].

Heerden and Kawamura (2012) modeled biped robot leg as an inverted pendulum with compliance, to achieve leaping motion. They proposed a method for automatically finding leap trajectory or motion parameters such as leap velocity and the curvature, for a given leap distance with the help Eulerian ZMP Resolution (EZR) method. They have also used constant force field for trajectory generation

[10]. Goswami and Vadakkepat (2009) have proposed a dynamic model for the landing stability during biped robot jumping when the robot lands with the foot having some orientation [11]. Researchers also have studied jumping of biped robot with artificial muscles [12], [13]. Though the motion planning considering conservation angular momentum and ZMP for biped robot leaping (jumping on leg and landing on the other leg) has been achieved [10], but it has not been studied for forward jumping of biped robot with rigid links. It is essential to study the influence of these parameters to achieve stable forward jumping motion of biped robot.

Though the motion generation with inverse kinematics was done for forward jumping of biped robot, by [5] and [6], but it was done without considering the above mentioned complexities. In the present work motion plan generation for forward jumping of biped robot with 4- rigid links is proposed by kinematic analysis; taking into account the influence of angular momentum, ZMP and landing stability. The forward jumping motion is divided into support phase and flight phase. The present work also studied the effect implementing conservation of angular momentum even when robot is in stance phase. The intention behind this is to keep the angular momentum zero for the entire motion, avoiding unnecessary torso moment. To achieve landing stability the relative velocity between the foot and ground is made zero at the of time landing, similar to [9]. The trajectory is defined for difference in velocities of the center of mass and foot such that velocity of foot reaches zero at the time of landing, thus it aims to gradually decrease the foot velocity from the time when biped robot is in its flight phase.

Inverse kinematic analysis has been performed considering the nonholonomic nature of the ZMP and angular momentum constraint. Simulations are done in MATLAB using RK4 method. Detailed equations of the constraints have been proposed. The simulation results showed a smooth jumping pattern without any kind of undesired motions. The simulations results have been presented for both the support and flight phase. Simulations results are given for various initial configurations of the robot, which resulted in variation of the horizontal and vertical distances travelled by the biped. The simulation results reported a maximum height of 0.97m of the foot and horizontal range of 1.2m.

II. ABBREVIATIONS AND ACRONYMS

- x_0, y_0 - Center of mass coordinates of the foot
- x_1, y_1 - Center of mass coordinates of the lower leg
- x_2, y_2 - Center of mass coordinates of the upper leg
- x_3, y_3 - Center of mass coordinates of the torso
- x_{cm}, y_{cm} - Center of mass coordinates of the biped
- ϕ_0 - Orientation of the foot w.r.to the horizontal axis
- ϕ_1 - Orientation of the lower leg w.r.to the horizontal axis
- ϕ_2 - Orientation of the upper leg w.r.to the horizontal axis
- ϕ_3 - Orientation of the torso w.r.to the horizontal axis
- F_{ext} - Total External force
- m - Sum of the masses of individual links
- t - Time step
- I - Moment of inertia
- T_s - Stance phase time
- R - Ground Reaction force

- g - Acceleration due to gravity
- x_{cm0}, y_{cm0} - Center of mass position coordinates just before the beginning of flight phase

III. KINEMATIC MODELLING

The biped is modelled with four links, numbered from 0 to 3. *Link-0* corresponds to *foot*, *link-1* corresponds to *lower leg*, *link-2* corresponds to *upper leg* and *link-3* corresponds to *torso* as show in Fig. 1. It is assumed that the robot jumps with its legs exactly side by side, the motion details of the second leg are exactly same as the leg model presented here. The kinematics of the body has been modelled using body coordinate formulation, in which the equations of the biped are formed with respect to the centre of mass of the individual links and the angle made by them with respect to the positive x-axis of the inertial frame or origin. The kinematics of the biped has been modelled in two phases, support and flight phases. Support phase is when the foot of the robot is in stationary contact with the ground. The foot loses its contact with ground when the biped gets sufficient upward velocity. Flight phase is defined from the time when foot of biped loses its contact with the ground surface till a sudden impact with ground occurs.

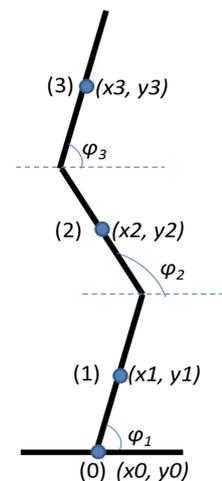


Fig. 1. Model of the Biped robot for jumping

A. Support phase formulation

During the support phase, the foot of the biped is assumed to be fixed to the ground surface. According to the body coordinate formulation since the biped has 4 links, it is defined with 12 parameters. x_0, y_0 and ϕ_0 are the coordinates corresponding to the foot, similarly $x_1, y_1, \phi_1, x_2, y_2, \phi_2, x_3, y_3$ and ϕ_3 are the coordinates corresponding to the lower leg, upper leg and torso respectively as shown in Fig. 1. A total of 12 constraint equations are needed to solve the inverse kinematics of biped robot. As the foot is in stationary contact with the ground surface throughout the support phase without any orientation the corresponding coordinates are equated to zero which are represented as constraint equations from (1) to (3). As the biped has three joints between 4 links, there exist 6 constraint equations corresponding to joints between *link-0*

and *link-1*, *link-1* and *link-2* and *link-2* and *link-3*. Equations from (4) to (9) are corresponding to the joints between links.

The center of mass of biped can be found using the masses and positions of individual links. According to Newton's second Law for a system of constrained particles, the motion of center of mass is governed by the net external force acting on the system. The external forces acting on the biped are the weights of the individual links and the reaction force acting at the foot. 'm' is the sum of the masses of individual links.

$$x_0 = 0 \quad (1)$$

$$y_0 = 0 \quad (2)$$

$$\phi_0 = 0 \quad (3)$$

$$x_1 - (l_1/2)\cos\phi_1 = 0 \quad (4)$$

$$y_1 - (l_1/2)\sin\phi_1 = 0 \quad (5)$$

$$x_2 - (l_2/2)\cos\phi_2 - x_1 - (l_1/2)\cos\phi_1 = 0 \quad (6)$$

$$y_2 - (l_2/2)\sin\phi_2 - y_1 - (l_1/2)\sin\phi_1 = 0 \quad (7)$$

$$x_3 - (l_3/2)\cos\phi_3 - x_2 - (l_2/2)\cos\phi_2 = 0 \quad (8)$$

$$y_3 - (l_3/2)\sin\phi_3 - y_2 - (l_2/2)\sin\phi_2 = 0 \quad (9)$$

$$F_{ext} = m\ddot{y}_{cm} \quad (10)$$

$$-mg + m\ddot{y}_{cm} = R \quad (11)$$

For the biped to move upward the reaction force at the foot need to be positive. The acceleration of the center of mass is assumed as twice the gravity for a fraction of support phase and is adjusted to become equal to gravity such that by the end of the support phase the reaction force at the foot becomes zero. This enables the biped to move under gravity during flight phase. Since biped center of mass in y direction is assumed to maintain certain acceleration, acceleration of the individual links of the biped need to adjust, to synchronize with the center of mass acceleration. This makes the y-center of mass as a mandatory constraint to be followed and is given as (12).

$$\ddot{y}_{cm} = 2g \quad \text{for } t \leq 0.9 T_s$$

$$\ddot{y}_{cm} = \frac{g(t-0.9T_s)}{0.1T_s} + 2g\left(1 - \left(\frac{t-0.9T_s}{0.1T_s}\right)\right)$$

$$\text{for } t > 0.9 T_s$$

$$\sum_{i=0}^3 m_i y_i = m y_{cm} \quad (12)$$

One of the factors to be considered in modeling the kinematics of biped is the angular momentum. It is the property that causes the rotation of the biped, which is not desirable during jumping. Spinning of the base or foot is another problem that arises due to angular momentum. To prevent undesired motions during jumping the angular

momentum needs to be zero. According to Newton's law of conservation, a rotating body will continue to turn about its axis of rotation with constant angular momentum, unless an external couple or eccentric force is exerted upon it. To make the angular momentum zero during flight phase, a constraint on angular momentum conservation is taken from the beginning of the support phase and is continued in the flight phase also. In case of biped angular momentum is the sum of the angular momentums of individual links about the whole body's center of mass and the angular momentums of the links about their center of mass, the constraint equation based on angular momentum obtained by differentiating the angular momentum with respect to time and equating it to zero is given in (13).

$$\sum_{i=0}^3 m_i \{-y_{cm} \ x_{cm}\} \{\ddot{x}_i \ \ddot{y}_i\}' + m x_{cm} g = 0 \quad (13)$$

For stability of the biped during support phase the ZMP should be within the support polygon. For biped resting on a horizontal surface the support polygon is the convex hull of its "footprint" on the base. The stability of the biped during support phase is ensured by taking the ZMP at the center of the foot. By assuming the origin or global coordinate frame to be at the center of the foot, ZMP becomes equal to zero. This is considered as another equation of constraint which is given in (14).

$$\sum_{i=0}^3 l_i \ddot{\phi}_i + \sum_{i=0}^3 m_i \{-y_i \ x_i\} \{\ddot{x}_i \ \ddot{y}_i\}' + m_i x_i g = 0 \quad (14)$$

A total of 12 constraint equations have been formed to solve 12 unknown variables. The constraints formed based on conservation of angular momentum and ZMP are differential equations which are non holonomic i.e., these equations cannot be integrated to get position equations. To solve the system of equations, the equations from 1 to 10 are differentiated twice with respect to time. The modified equations are solved for positions and velocities using Rk-4 method in MATLAB. The center of mass trajectory in support is obtained from coordinates of individual links. The system of equations are solved for the coordinates of individual links and then substituted back into CoM equations. The equations are solved at various initial conditions. The profile of the center of mass obtained for various initial configurations are shown in Fig. 2. The simulation results of the biped for the two initial configurations are shown in Fig. 3. A variation in center of gravity orientation to satisfy the constraints can be observed from figures 3(a) and 3(b). The details of the initial configurations are given below.

Initial configuration1- $\phi_0=0$; $\phi_1=pi/3-0.1$; $\phi_2=pi$; $\phi_3=0$;

Initial configuration 2 - $\phi_0=0$; $\phi_1=pi/3+0.02$; $\phi_2=pi$; $\phi_3=0$;

B. Flight phase formulation

In flight phase, center of mass of the biped is assumed to follow a ballistic trajectory according to the equations of projectile given in (15) and (16). x_{cm0} and y_{cm0} are the coordinates of center of mass at the beginning the flight phase, which are taken as the coordinates of center of mass

at the end of the support phase. Since both x and y center of mass are assumed to follow certain trajectories, the individual links need to adjust according to the center of mass and the constraint equations of center of mass must be followed. The constraint equations of center of mass are given in (17) and (18).

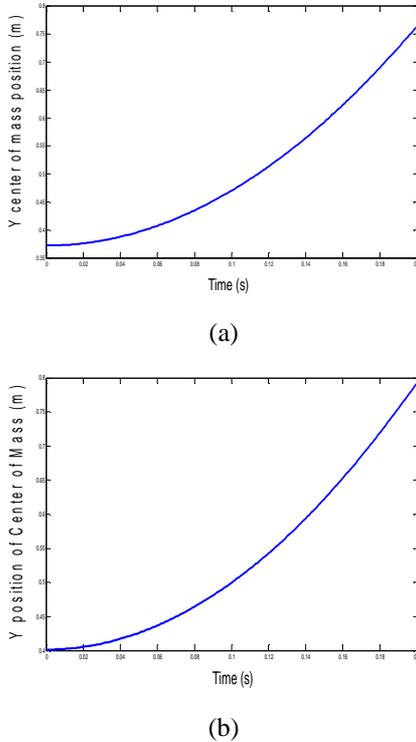


Fig. 2. Center of mass trajectories for (a) initial configurations-1 and (b) initial configurations-2

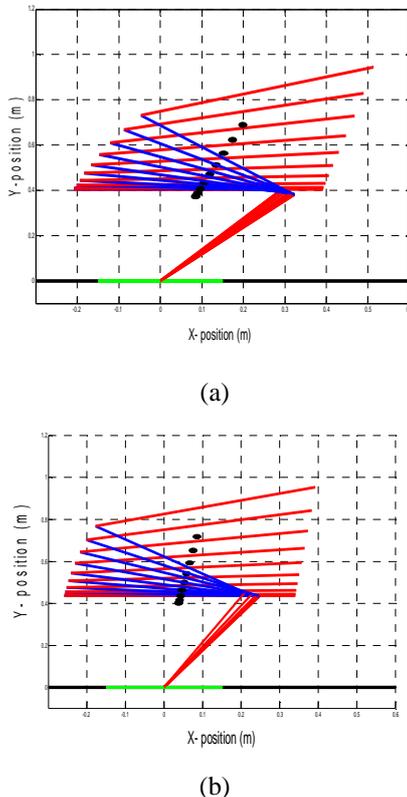


Fig. 3. Simulation results of the biped during support phase for (a) initial configurations-1 and (b) initial configurations-2

$$x_{cm} = x_{cm0} + (\dot{x}_{cm0}t) \quad (15)$$

$$y_{cm} = y_{cm0} + \left(\dot{y}_{cm0} - \frac{1}{2}gt^2 \right) \quad (16)$$

$$\sum_{i=0}^3 m_i x_i = m x_{cm} \quad (17)$$

$$\sum_{i=0}^3 m_i y_i = m y_{cm} \quad (18)$$

In support phase the foot is in stationary contact with the ground, whereas the foot will be above the ground surface in flight phase. The x and y coordinates of the foot are assumed to follow a trajectory, trajectory of the foot is modeled based the difference velocity between the center of mass and the foot coordinates such that foot reaches the ground with zero velocity to reduce impact. And the corresponding trajectory equations to achieve this are given in (19) and (20). The unknown coefficients in the equations are found based on the gap conditions given at various times, during the flight phase. The foot is assumed to be flat throughout the flight phase, and the equation is same as (3). Angular momentum of the biped is assumed as zero throughout the flight phase and the equation is given in (21). The constraint equations specifying joints between links are same in support and flight phases and are given in (4) to (9).

$$\dot{x}_0 = \dot{x}_{cm} - (b_1 + 2b_2t + 3b_3t^2 + 4b_4t^3) \quad (19)$$

$$\dot{y}_0 = \dot{y}_{cm} - (a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4) \quad \dots\dots\dots (20)$$

$$\sum_{i=0}^3 ((-m_i(y_i - y_{cm})\dot{x}_i) + (m_i(x_i - x_{cm})\dot{y}_i) + (J_i\dot{\phi}_i)) = 0 \quad (21)$$

A total of 12 constraint equations, i.e. ., equations 3 to 9, 17 to 21 are formed for flight phase. Since the constraint based on angular momentum is a differential equation in time which are nonholonomic, all the other equations are differentiated with respect to time and are solved using RK-4 method with the initial guess as the configuration of the robot at the end of the support phase. The simulation in flight phase has been continued for the two initial configurations specified in support phase.

IV. RESULTS

It is observed that results were accurate to an order of 10^{-6} . The biped has tucked in its joints during jumping to reduce moment of inertia. The biped is following the required trajectory, any kind of undesired motion is not observed. The biped has landed with good stability due to constraint equations specified on foot. For the initial configuration-1 the biped foot has moved horizontal distance of 1.22m and a maximum vertical height of 0.9m from the ground surface. For the initial configuration-2 it is observed that the horizontal distance travelled by the biped is 0.5m and whereas the maximum vertical distance travelled is almost the same as that obtained for the initial configuration-1. Likewise the initial configurations and the

accelerations during support phase can be changed to get various horizontal and vertical distances to be covered. The simulation results are shown in Figs 4 and 5. It is observed that the torso orientation is almost horizontal at the highest point; this may be avoided by adding arm to the model, which influences the torso movement to a great extent.

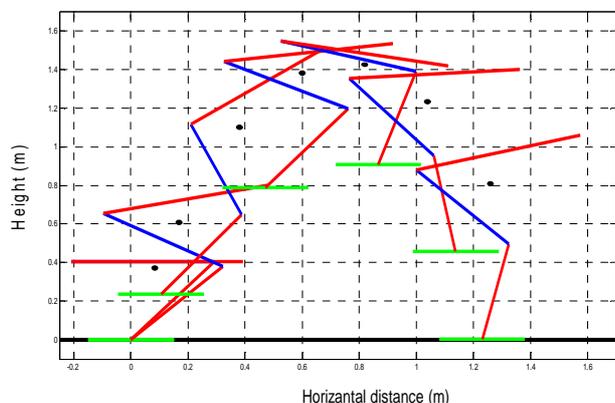


Fig. 4. Simulation result of the biped jumping for the initial configuration

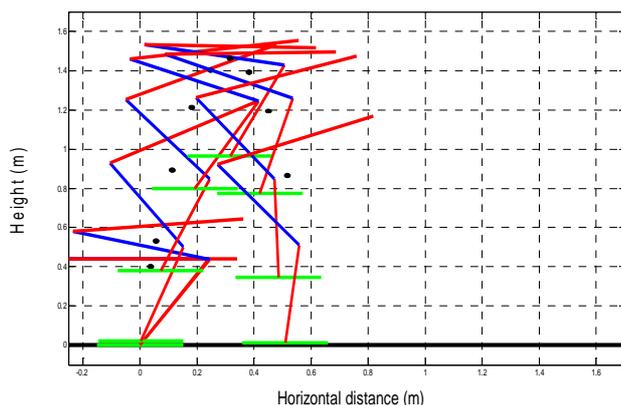


Fig. 5. Simulation result of the biped robot jumping for the initial configuration 2

V. CONCLUSIONS

The biped robot is modeled as a 4-link robot for kinematic analysis using body coordinate formulation. Kinematic analysis has been done for both support and flight phases for various initial configurations of the robot. The constraints for kinematic analysis have been modeled considering ZMP, angular momentum, the acceleration of the biped during support phase and the foot velocity during flight phase. The conservation angular momentum is considered for both the support phase and flight phase. Simulations are performed in MATLAB, and the results obtained for the pure kinematic analysis shows a safe locomotion of the robot during take-off, flight and landing. The simulation results reported a maximum height of 0.97m of the foot and horizontal range of 1.2m. The current work is planned to extend further, to find the initial configuration of the biped and the required acceleration to get desired horizontal and vertical distance to be travelled. This will be useful in generating jumping motion for the

given obstacle size. The inverse dynamic analysis to acquire joint torques is to be performed yet. The simulations need to be improved by including another link representing arm, which will be implemented in future.

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